Data, Logic, and Computing

ECS 17 (Winter 2022)

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Midterm 1: solutions

Part 1 *(6 questions, each 5 points; total 30 points)*

1) *How much space would you need to store a 5 min song that has been sampled at 44.1 kHz, with each data point stored on 24 bits, in mono (i.e. with a single microphone)? Assume no compression.*

   a) *About 40 Gbytes*
   b) *About 40 Mbytes*
   c) *About 53 Gbytes*
   d) *About 400 Kbytes*

$$S = 5[mins] \times 60\frac{[s]}{[mins]} \times 44100\frac{1}{[s]} \times 24[bits] \quad (1)$$

$$S = 317,520,000[bits] \quad (2)$$

$$= 39,690,000[bytes] \quad (3)$$

$$\approx 40[Mbytes] \quad (4)$$

2) *Let X be the number with the hexadecimal representation AA and Y the number whose hexadecimal representation is 9D; which of these numbers T (in hexadecimal form) satisfies $X - T = Y$?*

   a) *A*
   b) *B*
   c) *C*
   d) *D*

Note that:

a) $X = \#AA = 10 * 16 + 10 = 170$ (decimal)

b) $Y = \#9D = 9 * 16 + 13 = 157$ (decimal)

Therefore $T = X - Y = 170 - 157 = 13$ (decimal) $= \#D$.

3) *Which of these bytes represents the letter P (uppercase) based on the ASCII code?*

a) *01010000*

b) *10100000*

c) *01010010*

d) *10100010*

Note that:

a) $01010000 = 0101 \quad 0000 = \#50$

a) $10100000 = 1010 \quad 0000 = \#A0$

a) $01010010 = 0101 \quad 0010 = \#52$

a) $10100010 = 0101 \quad 0000 = \#A2$

and the letter P is represented by the hexadecimal #50. Therefore the correct answer is a).

4) *The heart rate of a young athlete can go as high as 180 beats per minute. What is the most appropriate sampling rate to use if you want to monitor heart rate during exercise?*

a) *1 Hz*

b) *8 Hz*

c) *5 Hz*

d) *3 Hz*

180 beats per minute = 3 beats per second = 3 Hz. Therefore we need a sampling rate greater than 2×3 Hz; the correct answer in b).

5) *Multiplying two numbers on a computer requires 20 cycles of computing time. How long would it take to perform a calculation that involves 5 million multiplications on a 2 GHz processor?*

a) *0.5 s*

b) *0.05 s*

c) *0.005 s*

d) *0.0005 s*

5 million multiplications require 100 million cycles. In one second, the computer can

6) *Which binary number comes right after the binary number 101111?*

a) *101112*

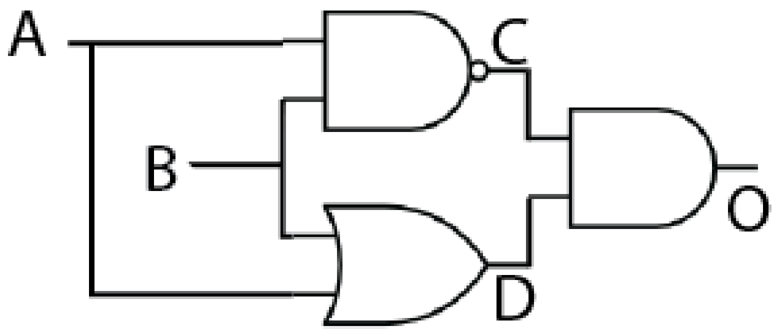
b) *111111*

c) *101110*

d) *110000*

Part II *2 problems, each 10 points; total 20 points)*

1) *Complete the logic table corresponding to the logic gate shown below. Convert it into a Boolean expression* (10 points)



A	B	$C = \overline{AB}$	$D = A + B$	$0 = CD = \overline{AB}(A + B)$
1	1	0	1	0
1	0	1	1	1
0	1	1	1	1
0	0	1	0	0

The corresponding Boolean expression is $\overline{AB}(A + B)$. Note however that the output of this logic gate is exactly the output of the XOR logic gate.

2) *You encounter a problem on an exam with only answer choices:*

   a) *Option 1*
   b) *Option 1 or Option 2*
   c) *Option 2 or Option 3*

   You do not know what those options are, as the question has been omitted, but you know that only one answer (a, b, or c) is possible. Can you find that answer? Explain your reasoning.

   i) If option 1 was correct, answers a) and b) would be valid: No, as only one answer is possible. Therefore option 1 is wrong.
   ii) If option 2 was correct, answers b) and c) would be valid: No, as only one answer is possible. Therefore option 2 is wrong.
   iii) Therefore, only option 3 is correct, and the answer is c).

Part III *2 problems, each 10 points; total 20 points)*

a) *Let p and q be two propositions. The proposition p $NAND$ q is false when both p and q are true, and it is true otherwise. It is denoted $p \uparrow q$. Show that $p \uparrow q \Leftrightarrow \neg(p \wedge q)$*

p	q	$p \uparrow q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

Therefore $p \uparrow q$ is logically equivalent to $\neg(p \wedge q)$

2) *Find a compound proposition logically equivalent to $p \vee q$ using only the logical operator $\uparrow$.*

Note first that since $p \uparrow q \Leftrightarrow \neg(p \wedge q)$, we get $p \uparrow p \Leftrightarrow \neg p$ and therefore we know how to express the negation operator using only the NAND operator.

Now, notice that:

$$\begin{aligned} p \uparrow q &\Leftrightarrow \neg(p \wedge q) \\ &\Leftrightarrow \neg p \vee \neg q \end{aligned}$$

where the second logical equivalence comes from de Morgan's law. Let $A \Leftrightarrow \neg p$, then $\neg A \Leftrightarrow p$. Similarly, Let $B \Leftrightarrow \neg q$, then $\neg B \Leftrightarrow q$.

We get:

$$\neg A \uparrow \neg B \quad \Leftrightarrow \quad A \vee B$$

We have seen that $\neg A \Leftrightarrow A \uparrow A$, and $\neg B \Leftrightarrow B \uparrow B$. We conclude that:

$$A \vee B \Leftrightarrow (A \uparrow A) \uparrow (B \uparrow B)$$

We verify using a table of truth:

A	B	$A \vee B$	$A \uparrow A$	$B \uparrow B$	$(A \uparrow A) \uparrow (B \uparrow B$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	T	F