*Name:*___________________________________
*ID:*______ ____________________________

ECS 17: Data, Logic, and Computing
Midterm
February 23, 2022

Notes:

1) Midterm is open book, open notes…
2) You have 50 minutes, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Proofs

Exercise 1 (1 question, 10 points)
Let a and b be two real numbers, with $a \neq 0$ and $b \neq 0$. Use a **proof by contradiction** to show that if $ab > 0$, then $\frac{a}{b} + \frac{b}{a} \geq 2$.

*Name:*___________________________________
*ID:*______ ____________________________

Exercise 2 (2 questions, each 10 points; total 20 points)

1) Let a and b be two integers. Show that if $\mathrm{a} + \mathrm{b}\sqrt{2} = 0$, then a =0 and b=0.

2) Let m, n, p, q be four integers. Show that $m + n\sqrt{2} = p + q\sqrt{2}$ if and only if m=p and n=q.

Name:____________________________________
ID:______   ______________________________

Exercise 3 (1 question, 10 points)

Let x be a real number. Show that $|x - 1| \leq x^2 - x + 1$,

where $|\quad|$ stands for the absolute value, defined as

$$|y| = \begin{cases} y & \text{if } y \geq 0 \\ -y & \text{if } y < 0 \end{cases}$$

for y real number.

*Name:*___________________________________
*ID:*______ ____________________________

Exercise 4 (1 question, 10 points)
Let *a* and *b* be two integers. Show that if $(a^2 + b^2)^2$ is even, then *a*+*b* is even.

Exercise 5 (1 question, 10 points)
Let *a* and *b* be two real numbers. Show that if $a \neq -1$ AND $b \neq -1$ then $a + b + ab \neq -1$
(*Hint:* $a + b + ab = (a + 1)(b + 1) - 1$).

*Name:*___________________________________
*ID:*______ ____________________________

Exercise 6 (1 question, 10 points)
Let a and b be two integers. Show that $a^2 - 4b \neq 2$

*Name:*____________________________________
*ID:*______ ____________________________

Appendix

Theorems that you are allowed to use:

a) Let n be an integer. n^2 is even if and only if n is even

b) Let a and b be two integers. If $a^2 + b^2$ is even, then $a + b$ is even

c) $\forall n \in \mathbb{N}$, $n(n+1)$ is even

d) $\sqrt{2}$ is irrational

e) Let n be an integer. n is even if and only if there exists an integer k such that $n = 2k$.

f) Let n be an integer. n is odd if and only if there exists an integer k such that $n = 2k+1$.

g) Let n be a positive integer. n is a perfect square if and only if there exists an integer k such that $n = k^2$.