Data, Logic, and Computing

ECS 17 (Winter 2024)

Patrice Koehl
koehl@cs.ucdavis.edu

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Homework 4 - For 2/7/2024

Exercise 1 *(4 problems, each 5 pts; total 20 points)*

Build the truth tables for the Boolean expressions:

a) $\bar{A}B$

A	B	$\bar{A}$	$\bar{A}B$
1	1	0	0
1	0	0	0
0	1	1	1
0	0	1	0

b) $\overline{A\bar{B}}$

A	B	$\bar{B}$	$A\bar{B}$	$\overline{A\bar{B}}$
1	1	0	0	1
1	0	1	1	0
0	1	0	0	1
0	0	1	0	1

c) $A + \bar{B}$

A	B	$\bar{B}$	$A + \bar{B}$
1	1	0	1
1	0	1	1
0	1	0	0
0	0	1	1

d) $\overline{A + \bar{B}}$

A	B	$\bar{B}$	$A+\bar{B}$	$\overline{A+\bar{B}}$
1	1	0	1	0
1	0	1	1	0
0	1	0	0	1
0	0	1	1	0

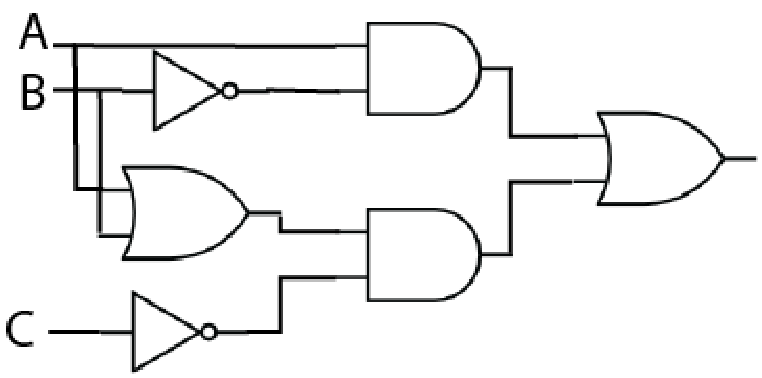
Exercise 2 *(10 points)*

An engineer hands you a piece of paper with the following Boolean expression on it, and tells you to build a gate circuit to perform that function:

$$A\bar{B} + \bar{C}(A+B)$$

Draw a logic gate circuit for this function. Build its truth table

One solution is:



We build its truth table:

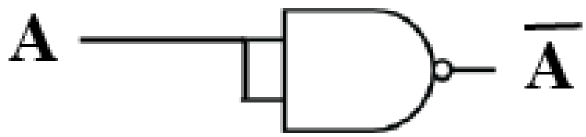
A	B	C	$\bar{B}$	$A\cdot\bar{B}$	$\bar{C}$	$A+B$	$\bar{C}(A+B)$	$A\bar{B}+\bar{C}(A+B)$
1	1	1	0	0	0	1	0	0
1	1	0	0	0	1	1	1	1
1	0	1	1	1	0	1	0	1
1	0	0	1	1	1	1	1	1
0	1	1	0	0	0	1	0	0
0	1	0	0	0	1	1	1	1
0	0	1	1	0	0	0	0	0
0	0	0	1	0	1	0	0	0

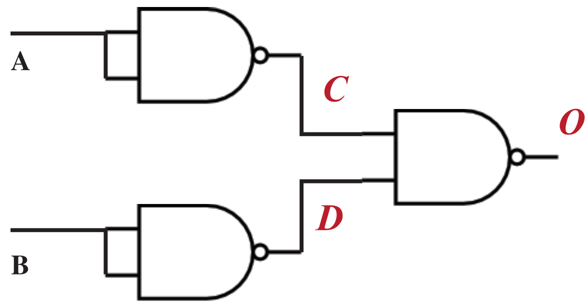
Exercise 3 *(10 points)*

Suppose we wished to have an OR gate for some logic purpose, but did not have any OR gates on hand. Instead, we only had NAND gates in our parts collection. Draw a diagram whereby multiple NAND gates are connected together to form an OR gate.

(*Hint: the NOT gate can be formed using:)*

We build the truth table for this gate:





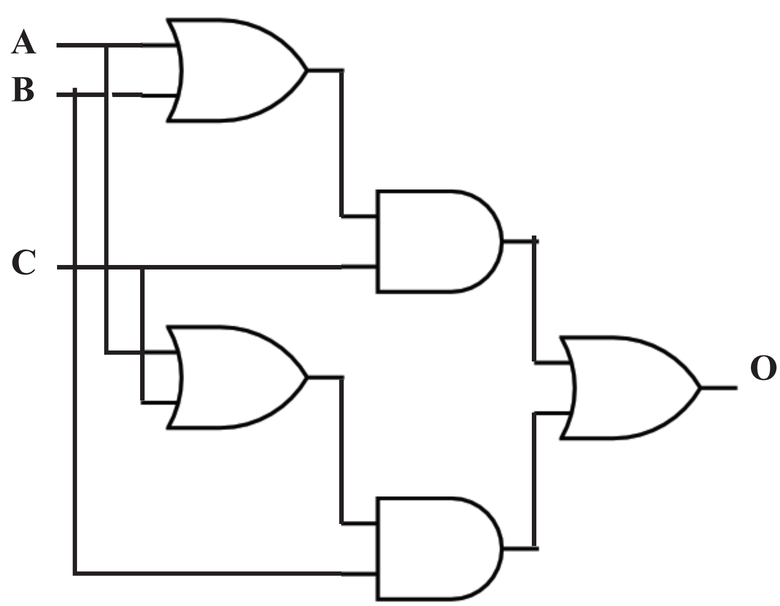
A	B	C	D	O	$A+B$
1	1	0	0	1	1
1	0	0	1	1	1
0	1	1	0	1	1
0	0	1	1	0	0

The output of this gate is fully equivalent to the OR gate.

Exercise 4 *(10 points)*

*Design a circuit that implements majority voting for three individuals (i.e. the ouput of the circuit is 1 if two at least of the inputs are 1, and 0 otherwise). Build its truth table. (*Hint: consider the Boolean expression $(A+B) \cdot C + (A+C) \cdot B$*).*

One solution is:



To check that this is what we need, we build its truth table:

A	B	C	Expected output	$A+B$	$(A+B)\cdot C$	$A+C$	$(A+C)\cdot B$	$(A+B)\cdot C+(A+C)\cdot B$
1	1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1	1
1	0	1	1	1	1	1	0	1
1	0	0	0	1	0	1	0	0
0	1	1	1	1	1	1	1	1
0	1	0	0	1	0	0	0	0
0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0

$0 = (A+B)\cdot C + (A+C)\cdot B$ is the expected output.

Exercise 5 *(10 points)*

On the fabled island of Knights and Knaves, we meet three people, John, Kari, and Tania, one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth.

John says: "Tania is a knave"
Kari says: "John is the knight"
Tania says: "I am the spy"

What are John, Kari, and Tania?

We check all possible "values" for John, Kari, and Tania, as well as the veracity of their statements:

Line number	John	Kari	Tania	John says	Kari says	Tania says
1	Knight	Knave	Spy	F	T	T
2	Knight	Spy	Knave	T	T	F
3	Knave	Knight	Spy	F	F	T
4	Knave	Spy	Knight	F	F	F
5	Spy	Knight	Knave	T	F	F
6	Spy	Knave	Knight	F	F	F

We can eliminate:

- Line 1, as John would be a knight but he lies
- Line 3, as Kari would be a knight but she lies
- Line 4, as Tania would be a knight but she lies
- Line 5, as Kari would be a knight but she lies
- Line 6, as Tania would be a knight but she lies

Line 2 is valid, and it is the only one. Therefore, John is the knight, Kari is the spy, and Tania is the knave.