Data, Logic, and Computing

ECS 17 (Winter 2024)

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Homework 8 - For 3/06/2024

Exercise 1: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Exercise 2: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.

Exercise 3: 10 points

Show that $\forall n \in \mathbb{N}, n > 1, \sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$.

Exercise 4: 10 points

Use a proof by induction to show that $\forall n \in \mathbb{N}, n > 3, n^2 - 7n + 12 \geq 0$.

Exercise 5: 10 points

A sequence $a_0, a_1, \ldots, a_n$ of natural numbers is defined by $a_0 = 2$ and $a_{n+1} = (a_n)^2, \quad \forall n \in \mathbb{N}$. Find a closed form formula for the term a_n and prove that your formula is correct.

Exercise 6: 10 points

Show that $\forall n \in \mathbb{N} f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ where f_n are the Fibonacci numbers.

Exercise 7: 10 points

Show that $\forall n \in \mathbb{N} f_0 - f_1 + f_2 - \ldots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ where f_n are the Fibonacci numbers.

Exercise 8: 10 points

Use the method of proof by induction to show that any amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.