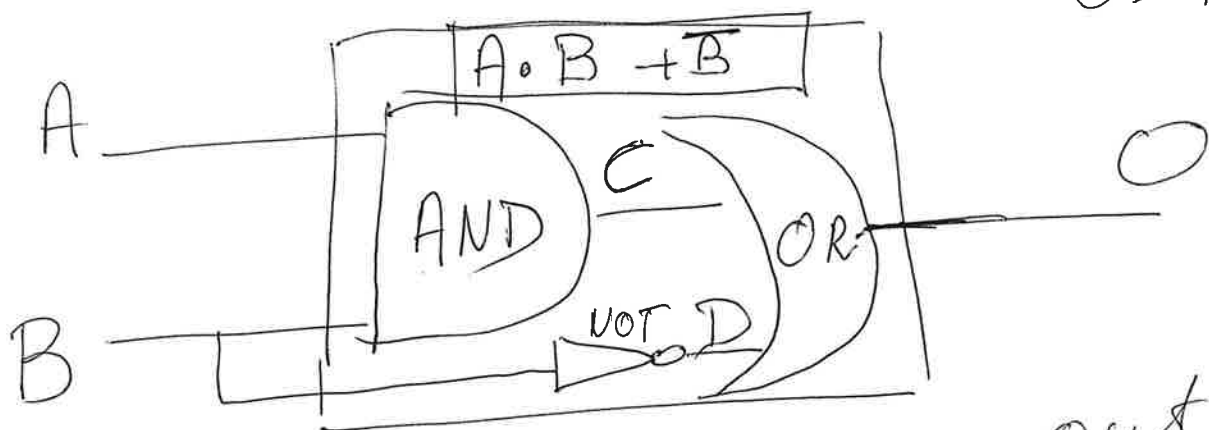


Exercise 1

"Machine"

$$O = C + D$$

$$O = A \cdot B + \overline{B}$$



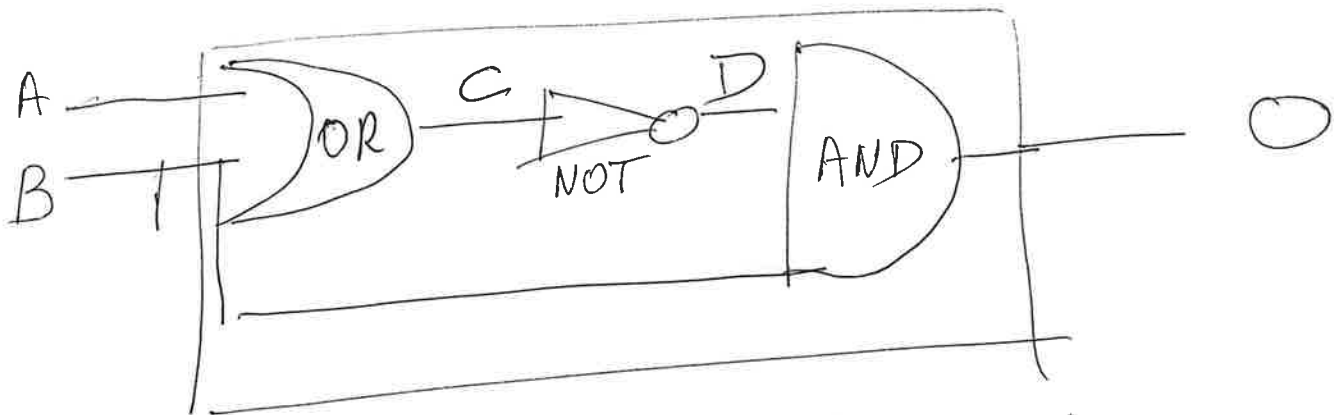
input

output

To understand this "machine", I build a logic table for its functions

A	B	C	D	O
1	1	1	0	1
1	0	0	1	1
0	1	0	0	0
0	0	0	1	1

# Exercise 2



A	B	C	D	O
1	1	1	0	0
1	0	1	0	0
0	1	1	0	0
0	0	0	1	0

$$O = D \cdot B$$

$$O = \overline{C} \cdot B$$

$$O = \overline{(A+B)} \cdot B$$

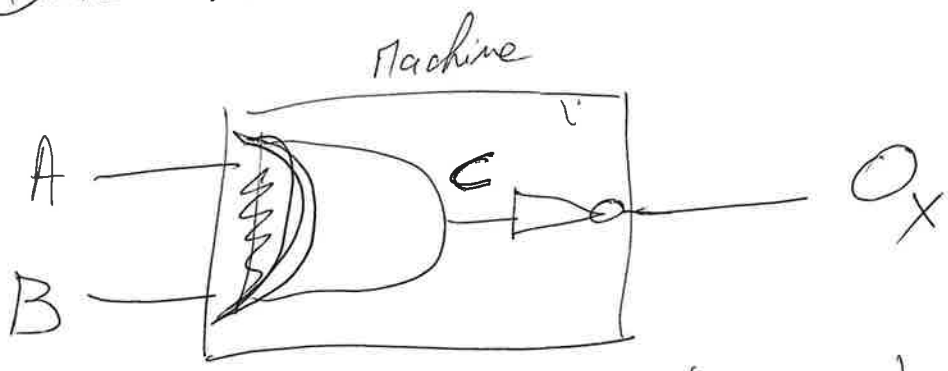
# Exercise 3

(3)

Is it true that  $\underbrace{\overline{A+B}}_X = \underbrace{\overline{A} + \overline{B}}_Y$ ?

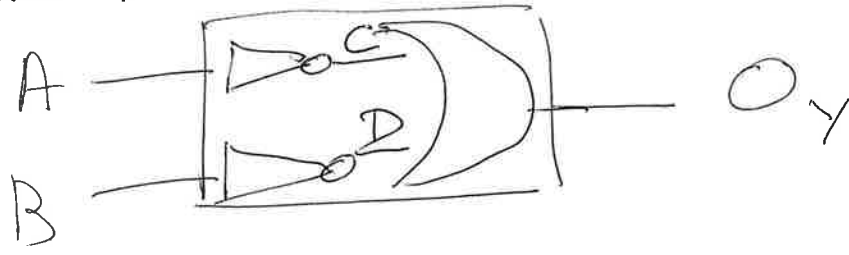
Do the gates X and Y have the same logic values?

Gate X:



A	B	C	$O_X$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Gate Y:



A	B	C	D	$O_Y$
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

A	B	$\overline{A+B}$
1	1	0
1	0	0
0	0	0
0	1	1

(4)

N Gate:

A	B	$\overline{A}$	$\overline{B}$	$O_y = \overline{\overline{A+B}}$	$\overline{A} \cdot \overline{B}$
1	1	0	0	0	0
1	0	0	1	1	0
0	0	1	1	1	0
0	1	1	0	1	1

We just showed that  $\overline{\overline{A+B}} = \overline{A} \cdot \overline{B}$

You can show that  $\overline{A \cdot B} = \overline{A} + \overline{B}$

These properties are referred to as De Morgan's laws of boolean algebra.

On the island of knights and knaves, 5  
 you meet John, Kai and Tania.

John says: "Tania is not a knave"

Kai says: "John and Tania are both knights"

Tania says: "John is a knight, or Kai is a knave, or both"

Let us define: K: Knights, k: knaves

	John	Kai	Tania	John Says	Kai Says	Tania Says
①	K	K	K	T	T	T
②	<del>K</del>	K	k	<del>F</del>	F	T
③	K	<del>k</del>	K	T	<del>T</del>	T
④	<del>K</del>	k	k	<del>F</del>	F	T
⑤	<del>k</del>	K	K	<del>T</del>	F	F
⑥	k	<del>K</del>	k	F	<del>F</del>	F
⑦	<del>k</del>	k	K	<del>T</del>	F	T
⑧	k	k	<del>k</del>	F	F	<del>T</del>

- ① possible.
- ②, ④ John K that is lying
- ③ Kai k telling the truth.
- 5, 7 John k telling the truth.
- ⑥ Kai knight, lying.
- ⑧ Tania is a knave lying.

