

January 8th
Good practices

1) Problem 1:

Solve $x^2 = 2$

$$x = +\sqrt{2} \text{ or } x = -\sqrt{2}$$

2) Problem 2:

Let x be a real number.

Find such x that satisfies

$$\sqrt{1-x} = x+1$$

Square both sides

$$(\sqrt{1-x})^2 = (x+1)^2$$

$$1-x = x^2 + 2x + 1$$

$$0 = x^2 + 2x + \cancel{1} - 1 + x$$

$$0 = x^2 + 3x$$

$$0 = x(x+3) \quad S = \{0, -3\}$$

Problem 1

Let x be a real number.

Show that

$$(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2$$

Proof:

~~$$(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2$$

$$x^2 + 2x - x - 2 - x^2 + 2 = 2x + 2 - x - 2$$

$$x = x$$~~

True

Let us define:

$$\text{LHS}(x) = (x-1)(x+2) - x^2 + 2$$

$$\text{RHS}(x) = 2(x+1) - x - 2$$

Simplification:

$$\text{LHS}(x) = x^2 + 2x - x - 2 - x^2 + 2$$

$$\text{RHS}(x) = 2(x+1) - x - 2 = 2x + 2 - x - 2 = x$$

Therefore, $\text{LHS}(x) = \text{RHS}(x)$.

(2)

Problem 2

Let n be a natural number.

Show that $n(n+1) > n^2$

Proof:

Let us define

$$\text{LHS}(n) = n(n+1)$$

$$\text{RHS}(n) = n^2$$

Let us compute

$$\begin{aligned} \text{LHS}(n) - \text{RHS}(n) &= n(n+1) - n^2 \\ &= n^2 + n - n^2 \\ &= n \end{aligned}$$

n is a natural number, therefore $n > 0$

$$\text{LHS}(n) - \text{RHS}(n) > 0$$

$$\text{LHS}(n) > \text{RHS}(n)$$

Problem 3

(3)

A bottle of wine costs \$11.00

The wine itself costs \$10.00 more than the bottle.

How much costs the wine alone?

Let W be the cost of wine.

B be the cost of the bottle.

I know:

$$W + B = 11$$

$$W = B + 10$$

$$B + 10 + B = 11$$

$$2B = 1$$

$B = 0.5$
$W = 10.5$

Problem 4

(4)

We are on the island of knights and knaves, where knights always tell the truth and knaves always lie.

We meet two people, Bill and Sophie. Bill tells you that: "We are both of the same type".

Sophie tells you that: "We are different".

Bill	Sophie	Bill says	Sophie says
Knight	Knight	True	False
Knight	Knave	False	True
Knave	Knight	False	True
Knave	Knave	True	False

There is enough information to solve the problem: Bill is a knave and Sophie is a knight.

Problem 5

(5)

We are still on the island of knights and knaves. We meet again two people; Bill and Sophie.

Bill: "We are both knaves"

Sophie: "I like chocolate"

Bill	Sophie	Bill
Knight	Knight	False
Knight	Knave	False
Knave	Knight	False
Knave	Knave	True

Sophie is a knight. She is telling the truth, therefore she likes chocolate.