Bad chatGPT

Let a and b be 2 integers.

Show that if ab is odd, then a is odd and b is odd.

p: ab is odd $\neg p$: ab is even

q: a is odd and b is odd $\neg q$: a is even OR b is even.

Indirect proof: we show $\neg q \to \neg p$,

i.e

if a is even or b is even then ab is even.

a is even: there exists an integer m such that $a = 2m$

b is even there exists an integer n such that $b = 2n$

Look at 2 cases:

Case 1: a is even;

$ab = 2\underbrace{mb}_{\text{integer}}$ ab is even.

Case 2: b is even

$$ab = a\,2n = 2\underbrace{(an)}_{\text{integer}}$$

ab is even

In both cases, ab is even, therefore $\neg p$ is true.

Exercise 2:

Let n be an integer. Show that if $2n^2 + n + 9$ is odd, then n is even.

p: $2n^2+n+9$ is odd $\qquad \neg p$: $2n^2+n+9$ is even

$\downarrow ? \qquad\qquad \uparrow ?$

q: n is even $\qquad \neg q$: n is odd

③

We use an indirect proof.

We assume $\neg q$ is true: n is odd.

There exists an integer k such that $n = 2k+1$

$$\begin{aligned}
2n^2 + n + 9 &= 2(2k+1)^2 + 2k+1 + 9 \\
&= 2(4k^2+4k+1) + 2k + 10 \\
&= 2[\underbrace{4k^2+4k+1 + k + 5}_{\text{integer}}]
\end{aligned}$$

$2n^2 + n + 9$ is even: $\neg p$ is true

I showed $\neg q \to \neg p$ is true, therefore $p \to q$ is true.

Direct proof:

I assume p is true,

$2n^2 + n + 9$ ~~is~~ odd.

There exists an integer k such that

$$2n^2 + n + 9 = 2k+1$$

$$\begin{aligned} n &= -2n^2 - 9 + 2k + 1 \\ &= -2n^2 + 2k - 8 \\ &= 2(\underbrace{-n^2 + k - 4}_{\text{integer}}) \end{aligned}$$

Therefore n is an even number.

q is true.

Exercise:

Let n be an integer. Show that if $2n^2+5n+9$ is odd, then n is even.

p: $2n^2+5n+9$ is odd $\qquad \neg p$: $2n^2+5n+9$ is even

$\downarrow$

q : n is even $\qquad \neg q$: n is odd

We assume p is true:

There exists an integer k such that

$$2n^2+5n+9 = 2k+1$$

$$5n = -2n^2+2k-8$$

$$4n+n = -2n^2+2k-8$$

$$n = -2n^2-4n+2k-8$$

$$= 2(-n^2-2n+k-4)$$

Exercise 4

Let n be an integer.

Show that if n^3 is even, then n is even.

p: n^3 is even $\qquad \neg p$: n^3 is odd

$\downarrow$? $\qquad \uparrow$?

q: n is even $\qquad \neg q$: n is odd

Directly:

I suppose p is true. There exists an integer k such that

$$n^3 = 2k$$

$$n^3 + n^2 = 2k + n^2$$

$$n^2(n+1) = 2k + n^2$$

$$n[n(n+1)] = 2k + n^2$$

$n(n+1)$ is always even.

There exists an integer ℓ such that $n(n+1) = 2\ell$

$$n \cdot 2\ell = 2k + n^2$$

$$n^2 = 2n\ell - 2k$$
$$= \underbrace{2(n\ell - k)}_{\text{integer}}$$

n^2 is even therefore n is even.

A gang of thieves has beeen harassing neighborhood merchants. The notorious criminals A, B, and C were called in. It was established that:

- C is innocent
- if A is guilty, then B is guilty
- if B is guilty, then at least one of A or C is guilty
- At least one of them is guilty.

A: A is guilty; B: B is guilty; C: C is guilty.

$\neg C$ (1)

$A \to B$ (2)

$B \to A \vee C$ (3)

$A \vee B \vee C$ (4)

Case 1 : A is innocent: $\neg A$

$$\neg A \wedge \neg C \rightarrow \neg B$$

Then $\neg B$ is true, B is innocent.

This is not possible

Case 2 : A is guilty.

Then B is guilty.

Solution:
- A, B guilty
- C innocent.