Discussion 2/23

Exercise 1

Prove that $2^{1/4}$ is irrational.

P: $2^{1/4}$ is irrational.

I use a proof by contradiction to show P is true.

I assume $\neg P$ is true.

$\neg P$: $2^{1/4}$ is rational.

There exists two integers a and b, with $b \neq 0$ such that:

$$2^{1/4} = \frac{a}{b}$$

Square both items: $\left(2^{1/4}\right)^2 = \left(\frac{a}{b}\right)^2$

Using

$$(a^n)^m = a^{nm}$$

$$\left(2^{1/4}\right)^2 = 2^{\frac{1}{4} \times 2} = 2^{\frac{1}{2}} = \sqrt{2}$$

I found $\sqrt{2} = \dfrac{\overbrace{a^2}^{\text{integer}}}{\underbrace{b^2}_{\text{integer}}}$ would be rational.

However, we showed in class that $\sqrt{2}$ is irrational.

I have reached a contradiction.

Therefore p is true.

$$\cancel{2^{1/8} = \frac{a}{b} \to 2^{1/4} = \frac{a^2}{b^2}}$$

$$\cancel{\to \sqrt{2} = 2^{1/2} = \frac{a^4}{b^4}}$$

Exercise 2

Let n be an integer.

Show, using a proof by contradiction, that

if $7n^2+4$ is odd then n is odd.

The property I need to prove is an implication $p \to q$:

p: $7n^2+4$ is odd $\qquad \neg p$: $7n^2+4$ is even

q: n is odd $\qquad \neg q$: n is even.

Proof by contradiction: I assume that the property $(p \to q)$ is false.

$\neg(p \to q)$ is true.

$\neg(\neg p \vee q)$ is true.

$p \wedge \neg q$ is true.

A proof by contradiction of $p \to q$ starts by assuming that p is true AND q is false.

$7n^2+4$ is odd

n is even

there exists an integer k such that $n = 2k$

$7n^2+4 = 7 \times 4 \times k^2 + 4$

$= 2\underbrace{(7 \times 2k^2 + 2)}_{\text{integer}}$

$7n^2+4$ is even.

I have reached a contradiction.

Therefore $p \to q$ is true.

Exercise 3

Prove that there exists an integer $n > 5$ such that $2^n - 1$ is prime.

Let $P(n)$: $2^n - 1$ is prime.

I want to show is that there exists an integer $n > 5$, such that $P(n)$ is true.

Existence proof:

$n = 6$: $2^6 - 1 = 64 - 1$
$= 63$
$= 3 \times 21$

$n = 7$: $2^7 - 1 = \underbrace{127}_{\text{example}}$.

Prove or disprove that

$2^n - 1$ is not ~~prime~~ for all n integer,

$n > 5$.

Exercise: ~~Let n be an inte~~

Prove or disprove that:

For all integers a and b with $a < b$,

$$a^2 < b^2.$$

Let P: For all integers a and b with $a < b$,

$$a^2 < b^2$$

$\neg P$: there exists integers a and b with $a < b$, such that

$$a^2 \geq b^2$$

$a < b$ yes.

$a = -3$

$b = -2$

$a^2 = 9$

$b^2 = 4$

$a^2 > b^2$