

Truth table

We want to validate the following:

Let p_1 , p_2 , and q be 3 propositions.

Show that

$$\underbrace{(p_1 \vee p_2) \rightarrow q}_{\text{LHS}} \Leftrightarrow \underbrace{(p_1 \rightarrow q) \wedge (p_2 \rightarrow q)}_{\text{RHS}}$$

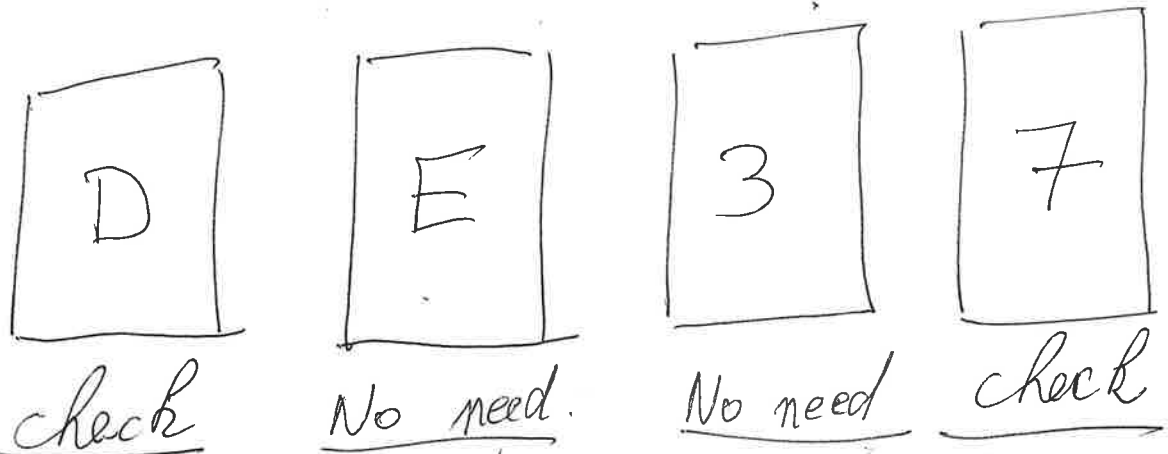
p_1	p_2	q	$p_1 \vee p_2$	LHS	$p_1 \rightarrow q$	$p_2 \rightarrow q$	RHS
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The LHS and the RHS always have the same truth values; they are logically equivalent.

Exercise 2

I have a deck of cards.
 Each card has a letter on the front and a number (between 0 and 9) on the back.

One rule: if you have a D on the front, then you have a 3 on the back.



p	q	$p \rightarrow q$	$(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$
T	T	T	$\neg q \rightarrow \neg p$: If you do not have a 3 on the back then, you do not have a D on the front.
T	F	F	
F	T	T	
F	F	T	

Inspector Craig:

① A bank in London's financial district has been robbed. The notorious criminals A, B, and C have been brought in for questioning. The following evidence was gathered:

- A is innocent
- If B is guilty then A is guilty.
- If C is innocent, then A is guilty.
- One of the three at least is guilty.

We define propositions:

- A: A is guilty
- B: B is guilty
- C: C is guilty.

Translation:

- $\neg A$ (1)
- $B \rightarrow A$ (2)
- $\neg C \rightarrow A$ (3)
- $A \vee B \vee C$ (4)

I know A is innocent (1) (4)

what about B? I ~~do~~ know

$$B \rightarrow A$$

this is logically equivalent to:

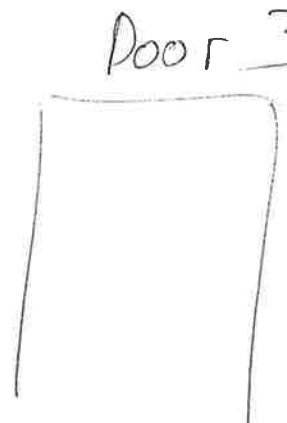
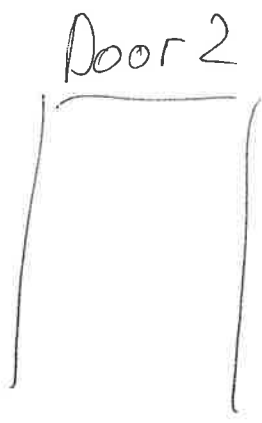
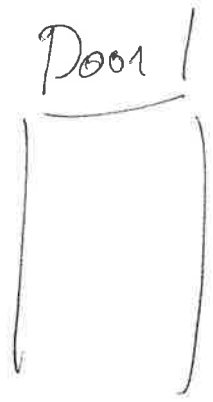
$$\neg A \rightarrow \neg B$$

we know $\neg A$ is true.

$\neg A$	$\neg B$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

therefore $\neg B$ is true: B is innocent.

Since A and B are innocent, and we know that at least one of A, B, or C is guilty, C has to be guilty.



Guard 1:
The treasure
is behind my door
↓

Guard 2:
Exactly one
of us is a liar
and the treasure
is behind my door
↓

Guard 3:
We are
all liars.

The guards are either K (Knight) or N (Knave)

