

Proofs 2/12

①

To prove
Show that if m is even, then m^3 is even,
when m is an integer.

P : m is even

$\neg P$ m is odd

↓ direct

↑ indirect

Q : m^3 is even

$\neg Q$ m^3 is odd.

We assume P is true:

there exists an integer k such that

$$m = 2k$$

$$m^3 = (2k)^3 = 8k^3 = 2 \underbrace{(4k^3)}_{\text{integer}}$$

m^3 is even. Q is true

Therefore $P \rightarrow Q$ is true.

Show that if m^3 is odd, then m is odd. (2)

p : m^3 is odd

$\neg p$: m^3 is even

q : m is odd

$\neg q$: m is even

Indirect proof:

I assume $\neg q$ is true.

There exists an integer k such that

$$m = 2k$$

$$m^3 = (2k)^3 = 2(\underbrace{4k^3}_{\text{integer}})$$

Therefore $\neg p$ is true.

Conclusion $\neg q \rightarrow \neg p$ is true, which is logically equivalent to $p \rightarrow q$ is true.

Example:

Show that if n is an integer, then $n(n+1)$ is even.

p : n is an integer

$\neg p$: n is not an integer

q : $n(n+1)$ is even

$\neg q$: $n(n+1)$ is odd

Analysis: We only know that

n is ~~an~~ an integer.

p : n is ~~an~~ an integer

$p \Leftrightarrow p_1 \vee p_2$

p_1 : n is even

p_2 : n is odd.

I need to prove:

$$(p_1 \vee p_2) \rightarrow q \Leftrightarrow (p_1 \rightarrow q) \wedge (p_2 \rightarrow q)$$

We want to show that

if n is even or n is odd, then $n(n+1)$ is even

p_1 : n is even

p_2 : n is odd

q : $n(n+1)$ is even

Case 1: $p_1 \rightarrow q$

We assume p_1 is true. There exists an integer k such that $n = 2k$.

$$n(n+1) = 2k(2k+1) = 2 \underbrace{(k(2k+1))}_{\text{integer}}$$

Therefore $n(n+1)$ is even: q is true.

Case 2: $p_2 \rightarrow q$

We assume p_2 is true. There exists an integer k such that $n = 2k+1$

$$\begin{aligned} n(n+1) &= (2k+1)(2k+2) \\ &= 2 \underbrace{(2k+1)(k+1)}_{\text{integer}} \end{aligned}$$

Therefore $n(n+1)$ is even: q is true.

Example:

Let m and n be 2 integers.

Show that if mn is odd, then m is odd and n is odd.

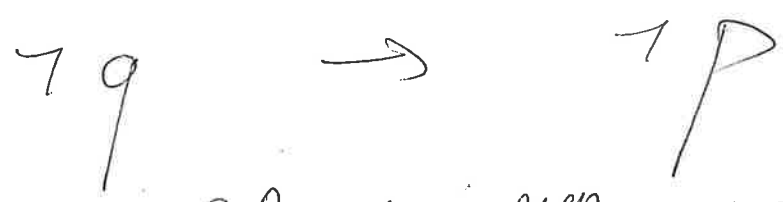
p : mn is odd

$\neg p$: mn is even

q : m is odd and n is odd

$\neg q$: m is even OR n is even.

We cannot use direct: we need to work with an indirect proof:



m is even OR n is even $\rightarrow mn$ is even.

Case 1 m is even. There exists an integer k such that $m = 2k$

$$m(n) = 2kn = 2(\underbrace{kn}_{\text{integer}})$$

mn is even: $\neg p$ is true.

Case 2 : n is even

(6)

There exists an integer k such that

$$n = 2k$$

$$n n = n \times 2k = 2 \underbrace{(nk)}_{\text{integer}}$$

$n n$ is even: $\neg p$ is true.

therefore $\neg q \rightarrow \neg p$ is true,
and consequently, $p \rightarrow q$ is true.

Exercise:

Let n be an integer. Show that if n^2 is even, then n is even.

$p : n^2 \text{ is even} \quad \neg p : n^2 \text{ is odd}$
 $q : n \text{ is even} \quad \neg q : n \text{ is odd}$

Direct proof:

We assume p is true: n^2 is even. There exist an integer k such that $n^2 = 2k$

$$n^2 + n = n(n+1)$$

Since n is an integer, $n(n+1)$ is even: there exists an integer l such that $n(n+1) = 2l$

$$2k + n = 2l$$

$$n = 2(\underbrace{l-k}_{\text{integer}})$$

n is even.

