Logic 2/2

I) Conditional, biconditional

p	q	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

II) Properties

$$\neg(\neg p) \Leftrightarrow p$$

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

$$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$(a \cdot (b + c) = a \cdot b + a \cdot c$$

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

III) Two special propositions

Tautology $\equiv T$

Contradiction $\equiv F$

To show that a proposition p is a tautology, we need to show that it is always true

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

IV) Specific properties associated with conditionals

$$(p \to q) \Longleftrightarrow (\neg q \to \neg p)$$

it is false that $(p \to q) \Longleftrightarrow (q \to p)$

Example:

"If it rains, then the ground is wet"

p: it rains

q: the ground is wet

$p \to q$

$\neg p$: it does not rain

$\neg q$: the ground is dry

Proof: we want to show $(p \to q) \Leftrightarrow (\neg q \to \neg p)$

p	q	$\neg q$	$\neg p$	$p \to q$	$\neg q \to \neg p$	$q \to p$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	F
F	F	T	T	T	T	T

equivalent

Not equivalent.

Vocabulary:

Given the proposition $A = p \to q$

the proposition $B = \neg q \to \neg p$ is called the contrapositive of A

the proposition $C = q \to p$ is called the converse of A.

Another property:

$$(p \to q) \iff (\neg p \lor q)$$

p	q	$p \to q$	$\neg p$	$\neg p \lor q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

equivalent

$$(p \to q) \iff (\neg p \lor q)$$

$$\neg(p \to q) \iff \neg(\neg p \lor q)$$

$$\iff \neg(\neg p) \land \neg q$$

$$\boxed{\neg(p \to q) \iff p \land \neg q}$$

Assuming that $(p \to q)$ is false
is equivalent to assuming that
p is true AND q is false

Quantifiers

$$x + 4 = 6$$

is not a proposition for two reasons:

a) I never said what x is

→ We do this by defining a domain. D

b) Is it true for some x or for all x?

- There exist a number x in the domain $D = \mathbb{Z}$, such that $x + 4 = 6$

$$\exists\, x \in \mathbb{Z},\quad x + 4 = 6$$

- For all integer numbers x, $x+4=6$

$$\forall\, x \in \mathbb{Z},\quad x + 4 = 6$$

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$$\neg\left(\exists x \in \mathbb{Z},\ x+4=6\right)$$

is $\forall x \in \mathbb{Z},\ x+4 \neq 6$

$$\neg\left(\forall x \in \mathbb{Z},\ x+4=6\right)$$

is $\exists x \in \mathbb{Z},\ x+4 \neq 6$