

Some elements to remember:

1) Completing the square.

$$\begin{aligned} a^2 + b^2 &= a^2 + b^2 + 2ab - 2ab \\ &= (a+b)^2 - 2ab \end{aligned}$$

2) Inequalities

If $a, b,$ and c are 3 real members

$$\left. \begin{array}{l} a > b \\ c > 0 \end{array} \right\} \rightarrow ac > bc$$

$$\left. \begin{array}{l} a > b \\ c < 0 \end{array} \right\} \rightarrow ac < bc$$

$$\begin{array}{r}
 a > b \\
 c > d \\
 \hline
 ac > bd
 \end{array}$$

3) Negation of inequalities

$$\neg (a \geq b) \Leftrightarrow a < b$$

$$\begin{aligned}
 \neg (a \geq b) &\Leftrightarrow \neg (a \text{ greater or equal to } b) \\
 &\Leftrightarrow \text{a smaller and not equal to } b \\
 &\Leftrightarrow a < b
 \end{aligned}$$

Proof by contradiction.

let us assume that we want to show that a proposition A is true.

Let us assume that A is false. hopefully we will show that we reach a contradiction.

What if $A \Leftrightarrow p \rightarrow q$

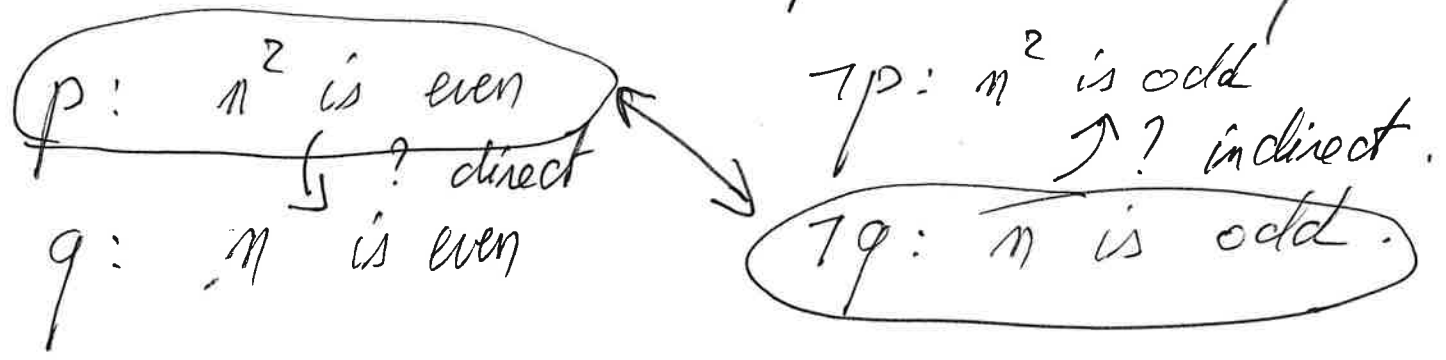
Proof by contradiction: we assume that A is false and equivalently that $\neg A$ is true.

$$\begin{aligned} \neg A &\Leftrightarrow \neg (p \rightarrow q) \\ &\Leftrightarrow \neg (\neg p \vee q) \\ &\Leftrightarrow p \wedge \neg q \end{aligned}$$

Assuming that $\neg A$ is true
 is the same thing as assuming that
 p is true AND q is false.

Example: Let n be an integer.

Show that if n^2 is even then n is even.



Proof by contradiction.

Assumption: p is true AND $\neg q$ is true.

p is true:
 n^2 is even

AND $\neg q$ is true
 n is odd

Contradiction

there exists an integer k
 such that $n = 2k + 1$
 $n^2 = (2k + 1)^2$
 $= 4k^2 + 4k + 1$
 $= 2(2k^2 + 2k) + 1$
integer

n^2 is odd