Proofs 2/21

Some elements to remember:

1) Completing the square.

$$a^2 + b^2 = a^2 + b^2 + 2ab - 2ab$$
$$= (a+b)^2 - 2ab$$

2) Inequalities

If a, b, and c are 3 real numbers

$$\left.\begin{array}{l} a > b \\ c > 0 \end{array}\right\} \rightarrow ac > bc$$

$$\left.\begin{array}{l} a > b \\ c < 0 \end{array}\right\} \rightarrow ac < bc$$

$$\begin{array}{r} a > b \\ c > d \\ \hline ac > bd \end{array}$$

3) Negation of inequalities

$$\neg(a \geqslant b) \iff a < b$$

$$\neg(a \geqslant b) \iff \neg(a \text{ greater or equal to } b)$$

$$\iff a \text{ smaller and not equal to } b)$$

$$\iff a < b$$

Proof by contradiction.

Let us assume that we want to show that a proposition A is true.
Let us assume that A is false.
hopefully we will show that we reach a contradiction.

What if $A \Leftrightarrow p \rightarrow q$

Proof by contradiction: we assume that A is false or equivalently that $\neg A$ is true.

$$\neg A \Leftrightarrow \neg(p \rightarrow q)$$
$$\Leftrightarrow \neg(\neg p \vee q)$$
$$\Leftrightarrow p \wedge \neg q$$

Assuming that $\neg A$ is true is the same thing as assuming that p is true AND q is false.

<u>Example:</u> Let n be an integer.

Show that if n^2 is even then n is even.

$\underbrace{n^2 \text{ is even}}_{p}$ then $\underbrace{n \text{ is even}}_{q}$

p: n^2 is even

? direct

q: n is even

$\neg p$: n^2 is odd

? indirect.

$\neg q$: n is odd.

Proof by contradiction.

Assumption: p is true AND $\neg q$ is true.

p is true:

n^2 is even

contradiction

AND $\neg q$ is true

n is odd

there exists an integer k such that $n = 2k+1$

$$n^2 = (2k+1)^2$$
$$= 4k^2 + 4k + 1$$

n^2 is odd $= 2\underbrace{(2k^2+2k)}_{\text{integer}} + 1$