Set theory 3/6

I) Definition

A set is an unordered collection of objects.

Examples of sets:

- Days of the week

$$D = \{M, T, W, Th, F, S, Su\}$$

- $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

Terminology: To write that an object x belongs to a set A, we use the symbol $\in$

$$x \in A$$

$$n \in \mathbb{N}$$

There is a special set, called the empty set, noted $\emptyset$

How to describe a set?

A set is well defined if there is a clear set of rules that defines its element.

Example: define the set of all even natural numbers that are smaller or equal to 20.

a) First possible notation:

$$S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

Roster representation of a set.

b) Second possible representation: set-builder representation.

$$S = \{ x \in \mathbb{N}, \ x \leq 20 \text{ and } x \text{ is even} \}$$

$$S = \{ x \in \mathbb{N} \ / \ x \leq 20 \wedge (\exists k \in \mathbb{N}, x = 2k) \}$$

More definitions

1) Subset

A set A is a subset of another set B if and only if every element of A is an element of B.

$$\forall x \in A, \quad x \in B$$

We write $A \subset B$

Some properties

$$A \subset A$$

$$\emptyset \subset A$$

2) Union

The union of a set A and a set B denoted $A \cup B$, is the set that contains those elements that are either in A, or in B, or in both.

$$A \cup B = \{ x \mid x \in A \vee x \in B \}$$

3) Intersection

The intersection of two sets A and B, denoted $A \cap B$, is the set that contains those elements that are ~~either~~ in A and in B.

$$A \cap B = \{ x \mid x \in A \wedge x \in B \}$$

4) Complement

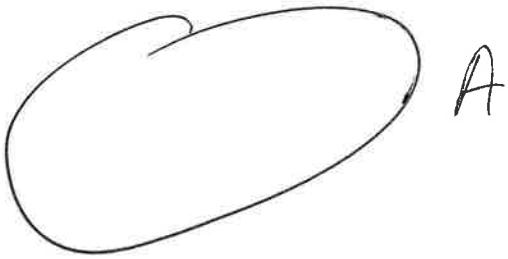
Given a domain D and a set A that belongs to D. The complement of A in D is the set of element of D that do not belong to A.

$$\overline{A} = \{ x \in D, \; x \notin A \}$$

$$= \{ x \in D, \; \neg (x \in A) \}$$

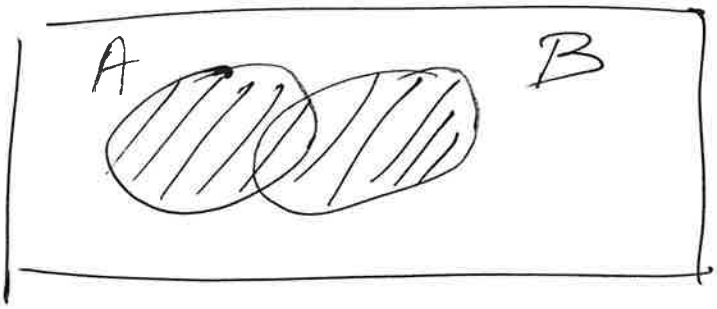
Visualization

Venn diagram: a visual for a set

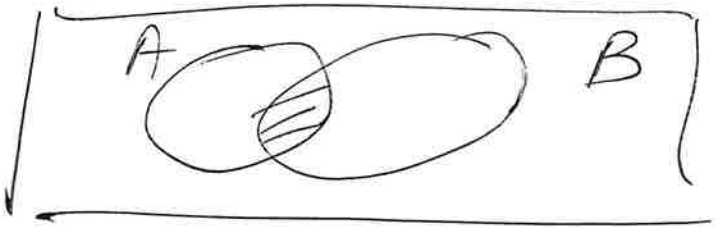


Example:

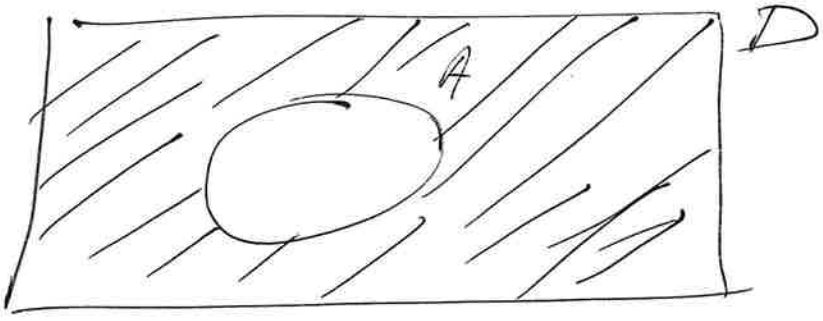
inclusion



$A \cup B$



$A \cap B$



Let A, B, and C be 3 sets in a domain D

then

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Definition Cartesian product.

Let A and B be 2 sets in a domain D. The Cartesian product of A and B, denoted $A \times B$ is the set of ordered pairs (a, b), with $a \in A$ and $b \in B$.

Example:

$$\boxed{P(x, y)} \in \underbrace{\mathbb{R} \times \mathbb{R}}_{\mathbb{R}^2}$$