ECS 20: Discrete Mathematics
Midterm
February 26, 2019

Notes:
1) Midterm is open book, open notes. No computers though…
2) You have 45 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Part I: Proofs (1 question, 10 points)

Let *a and b* be two real numbers, with $a \neq 0$ and $b \neq 0$. Use a **proof by contradiction** to show that if $ab > 0$, then $\dfrac{a}{b} + \dfrac{b}{a} \geq 2$.

Part II: sets (2 questions, each 10 points; total: 20 points)

1) Let *A*, *B,* and *C* be three sets in a domain *D*. Consider the following possible two equalities, $(A \cap B) - C = (A - C) \cap (B - C)$ and $C - (A \cap B) = (C - A) \cap (C - B)$. Show that one of these equalities is always true, but the other can be false (for the latter, give an example).

2) Let A, B, and C be three sets in a domain D. We assume that $A - C \subset B$. Let x be an element of the domain D. Show that if $x \in A - B$ then $x \in C$ (*hint:* **use a proof by contradiction**)

Part III: functions (2 questions; each 10 points; total 20 points)

1) Let *n* and *m* be two ***integers***. Show that $\left\lfloor \dfrac{n+m}{2} \right\rfloor + \left\lceil \dfrac{n-m+1}{2} \right\rceil = n$

2) Let x be a real number and n a natural number. Show that $\left\lfloor \dfrac{x+1}{n} \right\rfloor = \left\lfloor \dfrac{\lfloor x \rfloor + 1}{n} \right\rfloor$

Part III: Proofs (1 questions; 10 points)

Prove that $\{p \,|\, p \text{ is a prime number}\} \cap \{k^2 - 1 \,|\, k \in N\} = \{3\}$

Part IV: extra credit (3 points)

Let x be a real number. Solve $\dfrac{x-1}{2} = \left\lfloor \dfrac{x}{2} \right\rfloor - \left\lfloor \dfrac{x+1}{2} \right\rfloor$.