

ECS20
Midterm2: Review problems

Exercise 1:

Construct a truth table for the proposition $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

Exercise 2:

Prove that the square of an even number is an even number using:

- (i) a direct proof
- (ii) an indirect proof
- (iii) a proof by contradiction

Exercise 3:

Prove or disprove that the product of a non zero rational number and an irrational number is irrational.

Exercise 4:

Prove that if x and y are real numbers, then $\max(x,y) + \min(x,y) = x + y$. (Hint: use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively)

Exercise 5:

Show that the product of two of the numbers $65^{1000} - 8^{2001} + 3^{177}$, $79^{1212} - 9^{2399} + 2^{2001}$, and $24^{4493} - 5^{8192} + 7^{1777}$ is nonnegative. Is your proof constructive or non constructive? (Hint: do not try to evaluate these numbers!)

Exercise 6:

Let A , B and C be sets. Show that:

- a) $(A \cup B) \subset (A \cup B \cup C)$
- b) $(A - B) - C \subset A - C$
- c) $(B - A) \cup (C - A) = (B \cup C) - A$