*Name:*___________________________________

*ID:*______ ____________________________

ECS 20: Discrete Mathematics
Midterm
May 18, 2016

Notes:

1) You have 50 minutes, no more….
2) You can answer directly on these sheets (preferred), or on loose paper.
3) Please write your name at the top right of at least the first page!
4) There are 4 parts with a total possible number of points of 70, and one extra credit problem worth 5 points.

Part I: logic (1 question, 10 points; total 10 points)

1) A very special island is inhabited only by Knights and Knaves. Knights always tell the truth, while Knaves always lie. You meet two inhabitants: Sally and Claire. You know that one of them is the Queen of the island. Sally says, "Claire is the Queen and she is a Knave". Claire says, "Sally is not the Queen and she is a Knight'. Can you find out if Sally is a Knight or Knave? Can you find out if Claire is a Knight or Knave? Can you tell me who is the Queen? Explain your answer.

*Name:*__________________________________
*ID:*______ ____________________________

Part II: proofs and number theory (4 questions, each 10 points; total 40 points)

1) Give a direct proof, an indirect proof and a proof by contradiction of the proposition: if n^3+1 is odd, then n is even, where n is a natural number.

*Name:*____________________________________
*ID:*______ ______________________________

2) Show that for all natural numbers n >1, n^3+3n^2+2n is divisible by *2* and *3*. (*Hint: one possibility is to use Fermat's little theorem*)

3) Show that the sum of any three consecutive perfect cubes is divisible by 9 (***Note***: a perfect cube is a number that can be written in the form n^3 where *n* is an integer. The three numbers $(n-1)^3$, n^3 and $(n+1)^3$ are three consecutive perfect cubes. ***Hint***: Start by showing that $n^3 + 2n$ is a multiple of 3 (or equivalently that $n^3+2n \equiv 0\,[3]$) for all integers *n*.

*Name:*______________________________________

*ID:*_______ ________________________________

4) Evaluate the remainder of the division of 2^{473} by 13.

Part III : Set Theory and Functions (2 questions, each 10 points; total 20 points)

1) Let A and B be two sets in a domain D. Show that $\left(\overline{A}\cap B\right)\cup\left(\overline{A}\cap\overline{B}\right)\cup\left(A\cap B\right)=\overline{A}\cup B$

2) Let a and b be two strictly positive ~~real numbers~~ **integers** and let x be a real number.

Show that $$\left\lfloor \frac{\left\lfloor \frac{x}{a} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{x}{ab} \right\rfloor$$

*Name:*____________________________________
*ID:*_______ ______________________________

Extra Credit (1 question; total 5 points)

Let x be a positive real number. Solve $\lfloor x \lfloor x \rfloor \rfloor = 5$.