

**ECS20**  
**Homework 2**  
**Due January 22, 2019**

**Exercise 1 (15 points: 5 points each for a, b, and c)**

Construct a truth table for each of these compound propositions:

a)  $(p \vee q) \rightarrow (p \oplus q)$

b)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

c)  $(p \oplus q) \rightarrow (p \oplus \neg q)$

**Exercise 2 (10 points: 5 points each for a and b)**

Construct a truth table for each of these compound propositions:

a)  $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

b)  $(p \oplus q) \wedge (p \oplus \neg q)$

**Exercise 3 (5 points)**

Show that the following proposition is a tautology:

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

**Exercise 4 (10 points)**

*The Fair Maiden Rowena wishes to wed. And her father, the Evil King Berman, has devised a way to drive off suitors. He has a little quiz for them, and here it is. It's very simple:*

Three boxes sit on a table. The first is made of gold, the second is made of silver, and the third is made of lead. Inside one of these boxes is a picture of the fair Rowena. It is the job of the White Knight to figure out – without opening them – which one has her picture.

Now, to assist him in this endeavor there is an inscription on each of the boxes. The gold box says, "Rowena's picture is in this box." The silver box says, "The picture is not in this box." The lead box says, "The picture is not in the gold box." Only one of the statements is true. Which box holds the picture?

**Exercise 5 (10 points)**

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

A says "The two of us are both knaves", and B says "A is a knave".

**Exercise 6 (10 points: 5 points each for a and b)**

Use truth tables to verify the distributive laws:

a)  $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

b)  $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

**Exercise 7 (10 points)**

Show that  $p \Leftrightarrow q$  and  $(p \wedge q) \vee (\neg p \wedge \neg q)$  are equivalent.

**Exercise 8 (10 points)**

Use either a truth table, or logical equivalences, to show the equivalence:

$$(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \rightarrow (q \wedge r)$$

**Extra Credit: (5 points)**

We are back on the island of knights and knaves (see exercise 5 above). John and Bill are residents.

John: if Bill is a knave, then I am a knight

Bill: we are different

Who is who?

**+ 3 points for turning in online!**