Homework 5: due 2/12/2019

ECS 20 (Winter 2019)

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Exercise 1 *(10 points for each subquestion: 20 total)*

a) Show that the following statement is true: "If there exist two integers n and m such that $2n^2 + 2n + 1 = 2m$, then $2n = 3$.

b) If x and y are rational numbers such that $x < y$, show that there exists a rational number z with $x < z < y$.

Exercise 2 *(10 points)*

Let x be a real number. Show that $\lfloor \frac{x}{3} \rfloor + \lfloor \frac{x+1}{3} \rfloor + \lfloor \frac{x+2}{3} \rfloor = \lfloor x \rfloor$.

Exercise 3 *(10 points)*

This is a generalization of exercise 2:
Let x be a real number and N an integer greater or equal to 3.
Show that $\lfloor x \rfloor = \lfloor \frac{x}{N} \rfloor + \lfloor \frac{x+1}{N} \rfloor + \ldots + \lfloor \frac{x+N-1}{N} \rfloor$.

(Hint: instead of following a proof similar to the one you used for exercise 2, define:

$$f(x) = \lfloor x \rfloor - \lfloor \frac{x}{N} \rfloor - \lfloor \frac{x+1}{N} \rfloor - \ldots - \lfloor \frac{x+N-1}{N} \rfloor$$

and show that $f(x)$ is periodic, with period 1.)

Exercise 4 *(10 points)*

Let x be a real number. Then show that $(\lceil x \rceil - x)(x - \lfloor x \rfloor) \leq \frac{1}{4}$

Exercise 5 *(10 points for each subquestion: 20 total)*

Let x be a real number. Solve the following equations:

a) $\lfloor x^2 + x - 5 \rfloor = \frac{1}{2}x$

b) $2\lfloor 4 - x \rfloor = 2x + 1$

Extra Credit *(5 points)*

Let x and y be two real numbers such that $0 < x \leq y$. We define:

a) The customized arithmetic mean m of x and y: $m = \dfrac{x+2y}{3}$

b) The customized geometric mean g of x and y: $g = x^{\frac{1}{3}} y^{\frac{2}{3}}$

c) The customized harmonic mean h of x and y: $\dfrac{3}{h} = \left(\dfrac{1}{x} + \dfrac{2}{y}\right)$

Show that:

$$x \leq h \leq g \leq m \leq y$$