Discrete Mathematics
(for Computer Science)

1) Definition: Discrete mathematics, sometimes called finite mathematics, is the study of mathematical structures that are fundamentally discrete, in the sense of not supporting or requiring the notion of continuity. Most of the objects are countable sets, such as integers.

2) Topics we will cover:

a) Logic
b) Set theory (collections of objects)
c) Number theory
d) Combinatorics
e) Probability
f) algorithmics (study of the methods of computation; computability and complexity)
(g) Graph theory

Mathematical symbols we will use:

a) Sets of numbers

$\mathbb{N}$: natural numbers

$\mathbb{Z}$: integer numbers

$\mathbb{Q}$: rational numbers

$\mathbb{R}$: real numbers

($\mathbb{C}$: complex numbers)

b) Operations on numbers:

$+$: sum

$-$: subtraction

$\times$: multiplication

$/$ (or $\div$) : division (careful on different definitions!)

$\sqrt{\ }$: square root

a^b : a to the power b

) Comparing numbers

$>$: greater than

$\geq$: greater or equal to

$<$: smaller than

$\leq$: smaller or equal to

) Approximations

$[x]$ or $\lfloor x \rfloor$: floor function

$\lceil x \rceil$: ceiling function

e) Special notations:

- Summation:

$$1 + 2 + \cdots + n = \sum_{i=1}^{n} i$$

- Products

$$1^2 \times 2^2 \times 3^2 \cdots \times n^2 = \prod_{i=1}^{n} i^2$$

- Factorial

$$1 \times 2 \times 3 \times \cdots n = n!$$

f) Special numbers

π : pi = 3.14159....

e : Euler number = 2.718....

$\sqrt{2}$: Pythagoras constant = 1.414....

g) Simple mathematical identities

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$x^{a+b} = x^a x^b$$

$$(x^a)^b = x^{ab}$$

$$\text{Log}(a \cdot b) = \text{Log}\, a + \text{Log}\, b$$

$$\text{Log}(a^b) = b\, \text{Log}\, a$$

5) Good practices in mathematics

a) Let us consider the simple problem:

solve $x^2 = 2$

It would be easy to write:

$$x^2 = 2 \Rightarrow x = \sqrt{2} \text{ or } x = -\sqrt{2}$$

BUT:

The problem is ill-defined! The text of the problem should have stated on which set to solve the equation!

Solve $x^2 = 2$ for x integer $\Rightarrow$ No solution

Solve $x^2 = 2$ for x real, positive $\Rightarrow$ $x = \sqrt{2}$

Solve $x^2 = 2$ for x real $\Rightarrow$ $x = \sqrt{2}$ or $-\sqrt{2}$

b) Check your answers!

Solve for x in $\mathbb{R}$ $\sqrt{x^2 - x - 1} = \sqrt{-x}$

Domain: $-x \geq 0 \rightarrow x \leq 0$

$x^2 - x - 1 \geq 0 \quad x \in]-\infty, \frac{1-\sqrt{5}}{2}[\cup]\frac{1+\sqrt{5}}{2}, +\infty[$

Hence: $\mathcal{D} =]-\infty, \frac{1-\sqrt{5}}{2}[$

$$\sqrt{x^2 - x - 1} = \sqrt{-x} \Rightarrow x^2 - x - 1 = -x$$

$$x^2 = 1$$

$x = 1$ ~~or~~ $x = -1$

~~Not in domain!~~

c) Make sure to not prove the obvious

Show that $\forall x$ in $\mathbb{R}$

$$(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2$$

1st approach

$$x^2 + 2x - x - 2 - x^2 + 2 = 2x + 2 - x - 2$$

$$x = x$$

But we reach $x = x$ ... which is the obvious!

I prefer that you use the following approach

Let $LHS = (x-1)(x+2) - x^2 + 2$ and $RHS = 2(x+1) - x - 2$, $\forall x \in \mathbb{R}$

Then

$$LHS = x^2 + 2x - x - 2 - x^2 + 2$$

$$= x$$

And

$$RHS = 2x + 2 - x - 2$$

$$= x$$

Therefore $LHS = RHS$ $\forall x \in \mathbb{R}$. Therefore the identity is true!

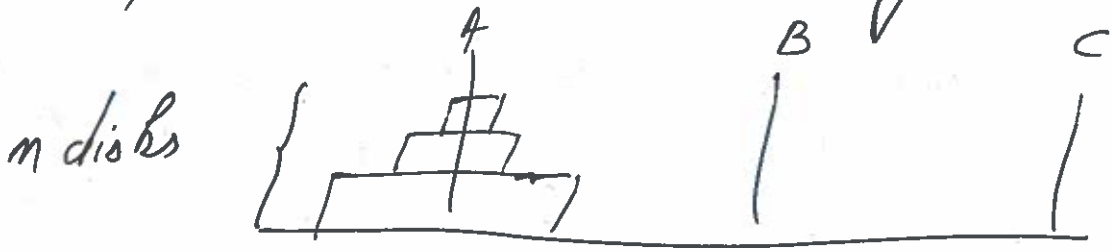
6) Examples of problems

a) Compute the sum of the first N even numbers

b) Prove that $\sqrt{2}$ is irrational

c) Prove that if n^2 is even, n is even

d) Solve the Tower of Hanoi problem



How many moves are necessary to transfer all disks from A to C, with the constraint that you cannot put a large disk on top of a small disk, and you can only move one disk at a time.