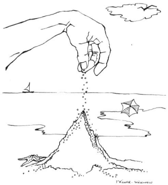
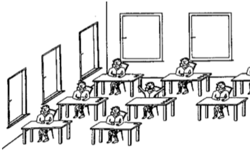
Cascades, generating functions, and electric power grid networks

Sandpile cascades: “Self-organized criticality”

The classic Bak-Tang-Wiesenfeld sandpile model:

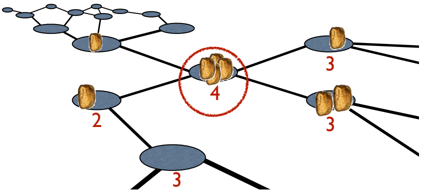


- Finite square lattice in $\mathbb{Z}^2$
- Drop grains of sand (“load”) randomly on nodes.
- Each node has a threshold for sand = coordination number.
- Load > threshold $\rightsquigarrow$ node topples = sheds sand to neighbors.
- These neighbors may topple. And their neighbors. And so on.
- Cascades of load/stress on a system.
- Open boundaries prevent inundation



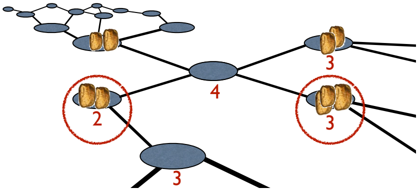
Sandpile model on networks

- Start with a network
- Drop units of load  randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors



Sandpile model on networks

- Start with a network
- Drop <u>units of load</u> randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors
- Neighbors may topple. Etc.
  Cascade (or avalanche) of topplings.



Power law tails (Universal behavior)

Double limit: $N \to \infty$; dissipation $\epsilon \to 0$

Avalance size follows power law distribution $P(s) \sim s^{-3/2}$

Power law tails seem to characterize the sizes of electrical blackouts, financial fluctuations, neuronal avalanches, earthquakes, landslides, overspill in water reservoirs, forest fires and solar flares.

Mean-field behavior is robust. (Goh et al. PRL 03, Phys. A 2004/2005, PRE 2005. PLRGs with $2 < \gamma < 3$ not mean-field.)

[1] I. Dobson, B. A. Carreras, V. E. Lynch, and D. E. Newman, Chaos **17**, 026103 (2007).
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[3] J. M. Beggs and D. Plenz, J, Neurosci. **23**, 11167 (2003).
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[6] A. Saichev and D. Sornette, Phys. Rev. E **70**, 046123 (2004).
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[9] P. Sinha-Ray and H. J. Jensen, Phys. Rev. E **62**, 3216 (2000).
[10] B. D. Malamud, G. Morein, and D. L. Turcotte, Science **281**, 1840 (1998).
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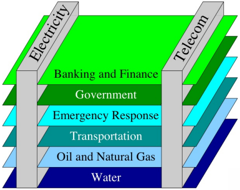
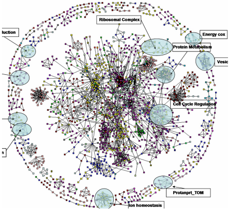
[12] M. Paczuski, S. Boettcher, and M. Baiesi, Phys. Rev. Lett. **95**, 181102 (2005).
[13] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

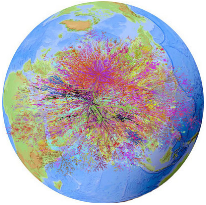
Motivation: Dynamics on interconnected networks



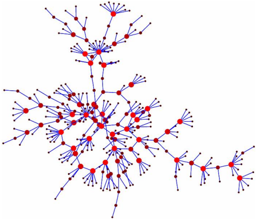
Critical Infrastructure



Biological & Ecological networks



Information and Communication technology



Social networks: Economics & Epidemics

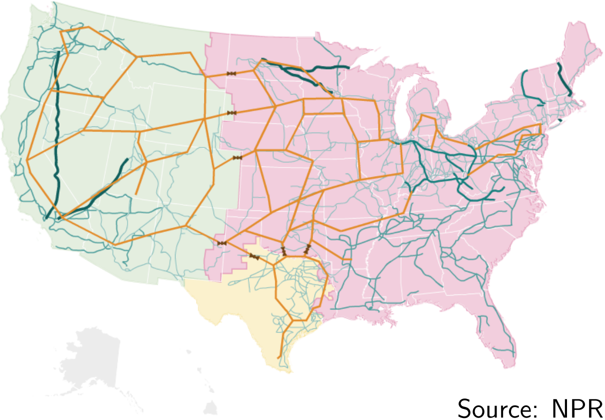
Motivation: interconnected power grids

C. Brummitt, R. M. D'Souza and E. A. Leicht *PNAS* **109** (12), 2012.

Interconnects initially built for emergency use.

Blackouts cascade from one grid to another (in a non-local manner).

How to assess impact of increasing interconnectivity?



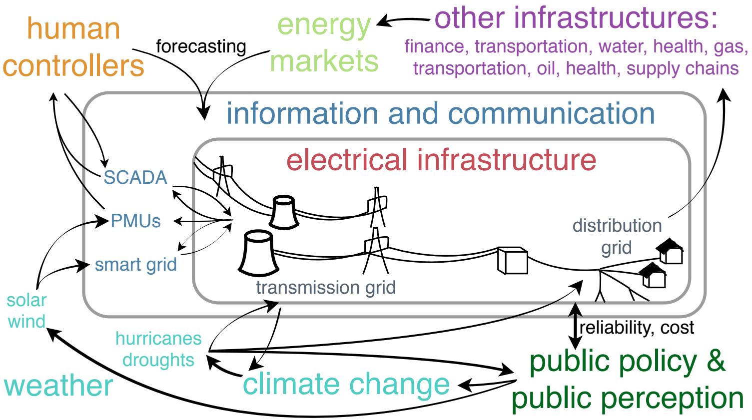
Source: NPR

Synchronization another fundamental aspect:

- Motter, A. E., Myers, S. A., Anghel, M., & Nishikawa, T., *Nature Physics*, (2013).

Real power systems: a web of feedbacks

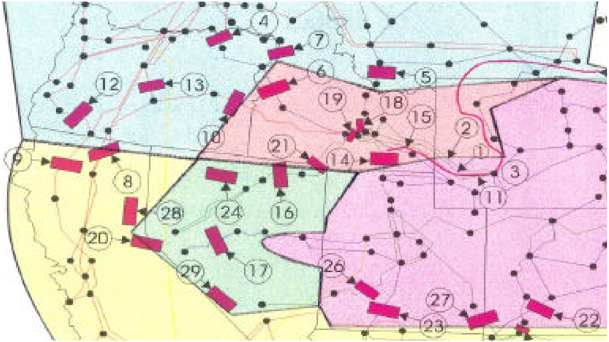
Brummitt, Hines, Dobson, Moore, R.D., *PNAS* July 23, 2013.
"Transdisciplinary electric power grid science"



Starting more simply: Stylized models of cascading transmission failure...

Real power grids – Non-local failures

(1996 Western blackout NERC report, $3 \to 4 \to 5$; $7 \to 8$, etc.)

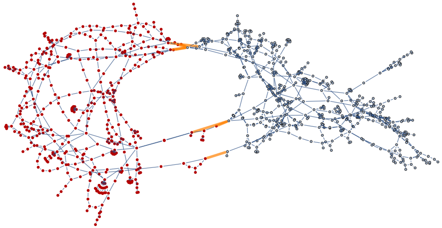


- See also Hines, Cotilla-Sanchez, Blumsack, *Chaos* 20, (2010).
  Failure of topological models to predict blackout size.
  Need Kirchoff laws! **Not epidemic spreading**.
  (Featured as *Science* Editor's Choice, 2010.)

BTW sandpile cascades on sparsely coupled networks

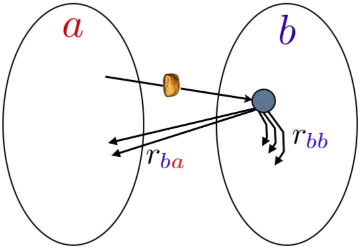
C. Brummitt, R. M. D'Souza and E. A. Leicht *PNAS* **109** (12), 2012.

Two-type network: a and b
Impact of increased a-b links.



$p_a(k_a, k_b)$, $p_b(k_a, k_b)$
(Configuration model)

Branching process treatment



$q_{ab}(r_{ba}, r_{bb}) :=$ the branch (children) distribution for an ab-*shedding*.

Overview of the calculations

From degree distribution to avalanche size distribution:

Input: degree distributions $p_a(k_a, k_b), p_b(k_a, k_b)$

$\Downarrow$ *compute*

shedding branching distributions $q_{aa}, q_{ab}, q_{ba}, q_{bb}$

$\Downarrow$ *compute*

toppling branching distributions u_a, u_b

$\Downarrow$ *plug in*

toppling branching generating functions $\mathcal{U}_a, \mathcal{U}_b$

$\Downarrow$ *plug in*

equations for avalanche size generating functions $\mathcal{S}_a, \mathcal{S}_b$

$\Downarrow$ *solve numerically, asymptotically*

Output: avalanche size distributions s_a, s_b

Shedding branch distributions q_{od}

The crux of the derivation

$q_{od}(r_{da}, r_{db}) :=$ chance a grain of sand shed from network o to d topples that node, sending r_{da}, r_{db} many grains to networks a, b.

$$q_{od}(r_{da}, r_{db}) = \underbrace{\frac{r_{do} p_d(r_{da}, r_{db})}{\langle k_{do} \rangle}}_{\mathtt{I}} \underbrace{\frac{1}{r_{da} + r_{db}}}_{\mathtt{II}} \qquad \text{for } r_{da} + r_{db} > 0.$$

- $\mathtt{I}$: chance the grain lands on a node with degree $p_d(r_{da}, r_{db})$ (Edge following: r_{do} edges leading from network o.)
- $\mathtt{II}$: "1/k assumption", sand on nodes is $\sim$ Uniform$\{0, ..., k-1\}$
- Chance of no children $= q_{od}(0,0) := 1 - \sum_{r_{da}+r_{db}>0} q_{od}(r_{da}, r_{db})$ (Probability a neighbor of any degree sheds, properly weighted.)
- Chance at least one child $= 1 - q_{od}(0,0)$.

Toppling branch distributions u_a, u_b

Key: a node topples iff it sheds at least one grain of sand.

Probability an o to d shedding leads to at least one other shedding: $1 - q_{od}(0,0)$. Probability a single shedding from an a-node yields t_a, t_b topplings:

$$u_a(t_a, t_b) = \sum_{k_a=t_a, k_b=t_b}^{\infty} p_a(k_a, k_b) Binomial[t_a; k_a, 1 - q_{aa}(0,0)] \cdot$$

$$\cdot\, Binomial[t_b; k_b, 1 - q_{ab}(0,0)].$$

(e.g., k_a neighbors, t_a of them topple, each topples with prob $1 - q_{aa}(0,0)$.)

Associated generating functions: $\mathcal{U}_a(\tau_a, \tau_b), \mathcal{U}_b(\tau_a, \tau_b)$.

Summary of distributions and their generating functions

	distribution	generating function
degree	$p_a(k_a, k_b), p_b(k_a, k_b)$	$G_a(\omega_a, \omega_b), G_b(\omega_a, \omega_b)$
shedding branch	$q_{od}(r_{da}, r_{db})$	
toppling branch	$u_a(t_a, t_b), u_b(t_a, t_b)$	$\mathcal{U}_a(\tau_a, \tau_b), \mathcal{U}_b(\tau_a, \tau_b)$
toppling size	$s_a(t_a, t_b), s_b(t_a, t_b)$	$\mathcal{S}_a(\tau_a, \tau_b), \mathcal{S}_b(\tau_a, \tau_b)$

Self-consistency equations:

$$\mathcal{S}_a = \tau_a \mathcal{U}_a(\mathcal{S}_a, \mathcal{S}_b), \tag{1}$$

$$\mathcal{S}_b = \tau_b \mathcal{U}_b(\mathcal{S}_a, \mathcal{S}_b). \tag{2}$$

Want to solve (**??**), (**??**) for $\mathcal{S}_a(\tau_a, \tau_b), \mathcal{S}_b(\tau_a, \tau_b)$.
Coefficients of $\mathcal{S}_a, \mathcal{S}_b$ = avalanche size distributions s_a, s_b.

In practice, Eqs. (**??**), (**??**) are transcendental and difficult to invert.

Basic definition of a G.F.

$G(x) = \sum_k p_k x^k$

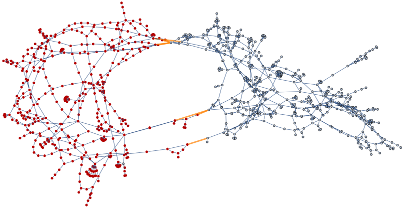
The p_k's are the probability of event of size k.

Build more complex G.F.s from the p_k's and solve for the coefficients to get the probablities of the more complex events!

Note on HW5 you will use FFT to solve for the coefficients of a simpler G.F.

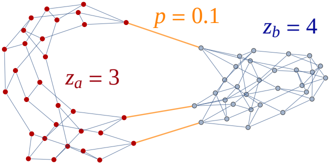
Plugging in degree distributions to the GFs

Two geographically nearby power grids in the southeastern US.



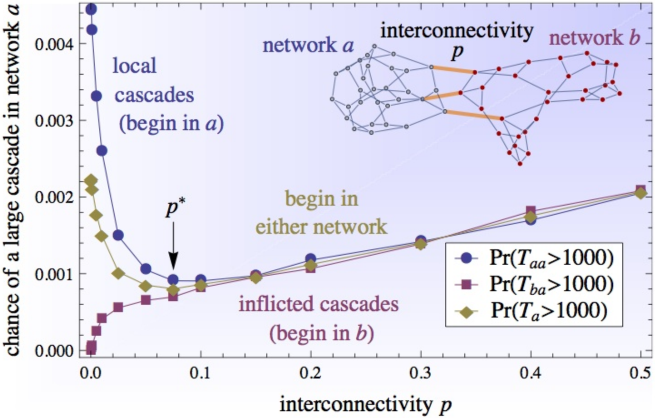
	Grid c	Grid d
# nodes	439	504
$\langle k_{int} \rangle$	2.4	2.9
$\langle k_{ext} \rangle$	0.02	0.01
clustering	0.01	0.08

Idealization, random regular graphs:



$$\mathcal{U}_a(\tau_a, \tau_b) = \frac{(p - p\tau_a + (z_a + 1)(\tau_a + z_a - 1))^{z_a}(1 + p(\tau_b - 1) + z_b)}{(z_a + 1)^{z_a} z_a^{z_a}(z_b + 1)}$$

Main findings: For an individual network, optimal p^*



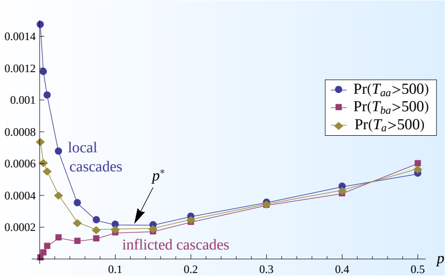
- (Blue curve) self-inflicted cascades (second network is reservoir).
- (Red curve) inflicted from the second network
- (Gold curve) Neglecting the origin of the cascade

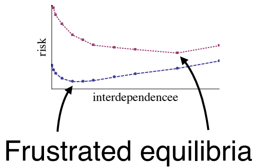
Effects balance at a stable critical point, $p^* \approx 0.1$.

Main findings: Increased systemic risk

- **More interconnections fuel larger system-wide cascades.**

– Each new interconnection adds capacity and load to the system (Here capacity is a node's degree, interconnections increase degree)



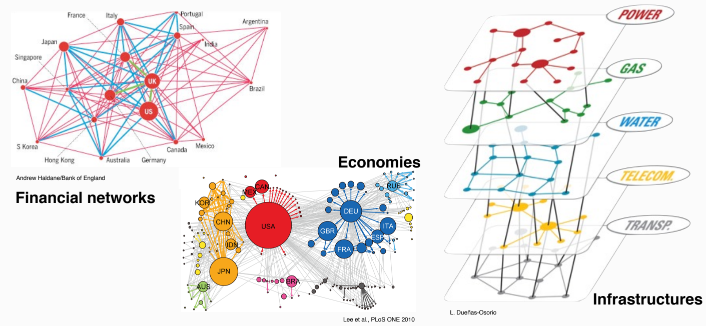


– So an individual operator adding edges to achieve p^* may inadvertantly cause larger global cascades.

- **Suppressing largest cascades amplifies small and intermediate ones**

Optimal interdependence

"Some networking is good. Too much is overwhelming."

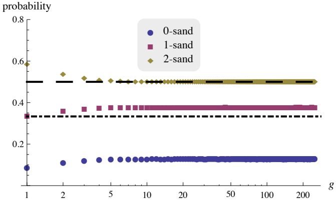


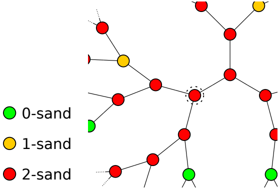
- **Financial markets**: Battiston, et al., *J. of Econ. Dyn. & Control* 36 (2012).
- **Synchronization**: Hunt., Korniss, Szymanski. *PRL*, 2010.
- **"Islanding"** in power grids: Andersson, et al. *IEEE Trans. Power Systems*, 2005.
- **"Islanding"** among traders: Saavedra, Hagerty, Uzzi, *PNAS*, 2011, *PLoS ONE* 2011.

More realism in BTW network cascades

- SOC equilibrium "$1/k$ assumption": Each node of degree k has uniform probability to have between 0 and $k-1$ grains of sand in steady-state. (Perfectily fine for an "annealed" network.)

- But, by inspection, a node that just toppled has 0 grains w.h.p., (i.e., $1/k$ overestimates topplings)

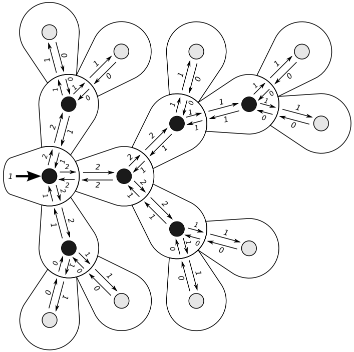


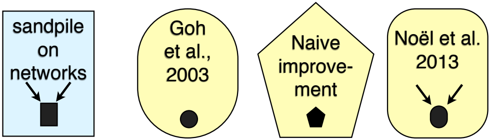


- **Put in the correct microscopic probabilities into generating functions and loose power law tail!**

P.-A. Noël, C. D. Brummitt, R.D.
Phys. Rev. Lett. **111** 0780701, Aug 12 2013.

"Controlling self-organizing dynamics on networks using models that self-organize".



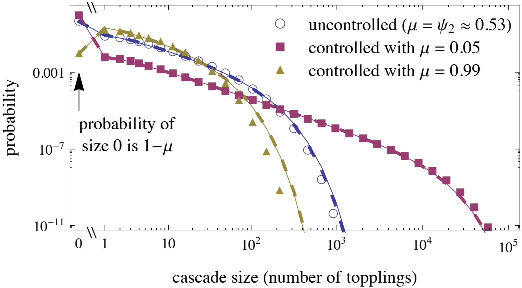


	R_0 = 1 by design	R_0 < 1	R_0 = 1 by self-organization
macroscopically accurate	yes (by universality?)	no	yes
microscopically accurate	no	yes	yes

Controlling the BTW model away from the SOC state

Noël, Brummitt, R.D., Phys. Rev. Lett. **111** *0780701, 2013*

Control parameter μ:
probability grain lands on a node at threshold*



- Avoid cascades, $\mu = 0.05 \rightarrow$ larger cascades when they do occur.

- Ignite cascades, $\mu = 0.99 \rightarrow$ smaller cascades, but more frequent.

Becomes more costly to find an eligible grain as $\mu \rightarrow 0$.

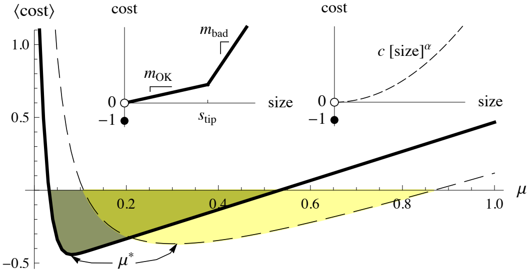
* *Others have examined: topology interventions, increasing ϵ, altering cascade mechanism, etc.*

Accounting for costs: **Optimal control levels**

Too much control can be detrimental.

Accounting for costs with larger events more costly.

Optimal μ^*



- Frequently triggering cascades mitigates large events but sacrifices short-term profit.

- Avoiding cascades maximizes short-term profit but suffers from rare, massive events.

Physics

Physics **6**, 90 (2013)

Viewpoint

Getting Out of Control

Nuno A. M. Araújo
Computational Physics for Engineering Materials, ETH Zurich, Wolfgang-Pauli-Strasse 27, CH-8093 Zurich, Switzerland

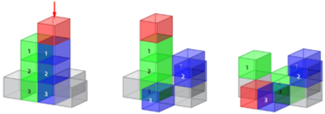
Published August 12, 2013

Complex systems—like sandpiles prone to avalanches—may become uncontrollable if too much effort is put into controlling them.

Subject Areas: **Complex Systems, Statistical Physics**

A Viewpoint on:
Controlling Self-Organizing Dynamics on Networks Using Models that Self-Organize
Pierre-André Noël, Charles D. Brummitt, and Raissa M. D'Souza
Phys. Rev. Lett. ***111****, 2013* – Published August 12, 2013

While driving along a desert highway, we can easily predict the consequences of turning the wheel and changing lanes. However, in heavy traffic this is not the case. Traffic dynamics is complex and the response to any individual change depends on how the other drivers accommodate it [1]. This type of cooperative dynamics in complex systems makes controlling them a scientific and technological challenge [2]. Now, writing in *Physical Re-*



Popular Press

- "When Networks Network", *Science News*, Sept 22, 2012
- "The mathematics of averting the next big network failure", Wired, Mar 19, 2013.