nature reviews physics
https://doi.org/10.1038/s42254-023-00566-3

Perspective

Controlling complex networks with complex nodes

Raissa M. D'Souza [1,2,3], Mario di Bernardo [4,5] & Yang-Yu Liu [6,7]

Abstract

Real-world networks often consist of millions of heterogenous elements that interact at multiple timescales and length scales. The fields of statistical physics and control theory both contribute different perspectives for understanding, modelling and controlling these systems. To address real-world systems, more interaction between these fields and integration of new paradigms such as heterogeneity and multiple levels of representation will be necessary. It may be possible to expand models from statistical physics to integrate the notion of feedback (both positive and negative) and to extend control theory formulations to more mesoscopic analysis over averages of collections of degrees of freedom. There is also the need to integrate theoretical models, machine learning and data-driven control methods. We review recent progress and identify opportunities to help advance understanding and control of real-world systems from oscillator networks and social networks to biological and technological networks.

Sections

[1]Department of Computer Science and Department of Mechanical and Aerospace Engineering, University of California, Davis, CA, USA. [2]Santa Fe Institute, Santa Fe, NM, USA. [3]Complexity Sciences Hub Vienna, Vienna, Austria. [4]Department of Electric Engineering and Information Technology, University of Naples Federico II, Naples, Italy. [5]Scuola Superiore Meridionale, Naples, Italy. [6]Channing Division of Network Medicine, Department of Medicine, Brigham and Women's Hospital and Harvard Medical School, Boston, MA, USA. [7]Center for Artificial Intelligence and Modelling, The Carl R. Woese Institute for Genomic Biology, University of Illinois at Urbana-Champaign, Urbana, IL, USA. e-mail: rmdsouza@ucdavis.edu; mario.dibernardo@unina.it; yyl@channing.harvard.edu

Introduction

Statistical physics is primarily concerned with equilibrium, macroscopic ensemble properties of a collection of elements and provides a framework for understanding and predicting the collective behaviours of massive numbers of simple, identical entities. Quintessential examples of such behaviours include the Maxwell–Boltzmann velocity distribution describing a gas of particles and the ferromagnetic phase transition when a material is cooled to the Curie temperature.

Conversely, control theory traditionally evolved as a branch of dynamical systems and engineering focused on finding methods to make a system or device behave autonomously in a desired manner robustly even in the presence of noise, delays and perturbations. It is concerned with devising feedback strategies to steer the dynamical behaviour of a system of interest towards some desired evolution, ideally via influencing relatively few microscopic degrees of freedom.

Given the massive scale of modern networks (such as the Internet or the human interactome), full knowledge of every degree of freedom and their interconnections may not be attainable – let alone exerting control over all of them. Thus, a partnership is needed between the macroscopic and the microscopic and between equilibrium and dynamical approaches to tame the behaviour of complex networks with complex nodes. We use the adjective 'complex' in the sense of complex systems, meaning potentially heterogeneous systems with nonlinear behaviours. Specifically, 'complex network' refers to the connectivity pattern between elements and 'complex node' refers to the nonlinear behaviour of individual elements.

This Perspective is organized as follows. We first provide context about the intersection of statistical physics of complex networks and control theory, of general feedback control theory and of challenges that arise from real-world networks. We then discuss existing methods and ideas from statistical physics and from control theory as applied to steering and controlling the behaviours of complex networks. We then present new approaches and modelling techniques that may prove fruitful. Finally, we conclude with a set of action items to help spark interdisciplinary progress.

Two fundamental concepts in control theory, controllability and observability, are used throughout, so we define their basic notions here. Controllability relates to the existence of control inputs able to steer a system from any initial condition to any desired terminal condition in finite time using only certain admissible manipulations. Observability is concerned with the ability to estimate the internal states of a system by measuring its inputs and outputs, typically identifying a subset of variables that carry enough information such that the whole system behaviour can be reconstructed from their measurement. Note, we use the term 'control theory' to refer to the body of work focused on the analysis and design of feedback systems to achieve a desired goal.

Background

Statistical physics and structural controllability

The late 1990s saw a rapid growth of the Internet and World Wide Web, an explosion of genomic data, the increasingly cyber-physical nature of infrastructure systems and the increasing globalization of economies. With that, a call grew for a general science of networks[1]. The tools of statistical physics, such as random graph models, generating functions and rate equations brought a wealth of understanding of the properties and behaviours of complex networks – often characterized by an underlying degree distribution that has a broad scale, spanning a few orders of magnitude. Key consequences of such a network structure are robustness to random node removal, vulnerability to targeted removal and the potential lack of an epidemic threshold. Beyond degree distribution, salient structural features of networks include small-worldness, modularity and triangular closure[2] (Fig. 1).

In 2011, an important connection between statistical physics and control of complex networks was established to analytically study the controllability of a network ensemble with linear dynamics and arbitrary degree distributions[3]. This connection was built on the framework of structural control, introduced in the 1970s[4], to solve for the controllability problem via graph-theoretical approaches in a network with linear dynamics. The problem is to determine whether control inputs exist that, when applied to some appropriately selected nodes (termed 'driver nodes'[3]), allow one to steer its dynamics from any initial condition to any desired terminal condition in finite time (in other words, rendering it controllable). Crucial insights came from mapping the problem of identifying a minimum driver node set onto a maximum matching problem on the network (Fig. 2), which was then analytically solved using the cavity method from statistical physics (discussed in-depth in the section on 'Statistical physics approaches').

In structural control, only the structure (that is, the presence or absence of a connection between elements) matters, not the weight of each connection. Traditionally, the structural control framework assumes that the nodes evolve according to a linear time-invariant dynamics: $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$ (see the section on 'New paradigms and modelling techniques' for details and extensions to nonlinear dynamics). This linearity means that the tools of linear algebra can be used to elucidate connections between network structure and controllability, including connections to phase transitions in network structure[5,6]. Also established is the controllability transition and the tradeoffs between nonlocality of the controlled state trajectories and nonlocality of control inputs[7].

Beyond structural control, much research has gone into understanding control energy[8], control profiles (based on control flow patterns)[9] and constraints from real-world systems[10]. For comprehensive reviews, see refs. 11,12. For a comprehensive primer on control theory for physicists, see ref. 13. Yet how to extend the approaches from statistical physics to dynamical, out-of-equilibrium, nonlinear systems remains an open question.

Control theory overview

In the classic control paradigm, one senses and controls the behaviour of a particular system or device of interest, such as an automobile, an aeroplane or a robot. Control design often begins with a mathematical (or computational) representation of the structure and dynamics of a system of interest (also called a plant) and consists of synthesizing a feedback control strategy that by sensing what the plant is doing computes the required inputs to drive the plant to a desired state or behaviour. Negative feedback provides the ability to stabilize behaviours with some desired performance guaranteeing robustness even in the presence of noise, delays or perturbations. In contrast, positive feedback can be used, for example, to enable bi-stability and bifurcations in a nonlinear system, allowing one to drive the system between multiple stable states.

Mathematical formulations for many classes of linear and nonlinear systems have been developed along with their control strategies[14]. Many of these approaches are distributed and/or decentralized and some use sophisticated nonlinear, adaptive, computational and time-varying approaches[15]. The focus is largely on individual systems, meaning that all the relevant degrees of freedom and their dynamics and couplings are known (Fig. 3a). This classical paradigm has been

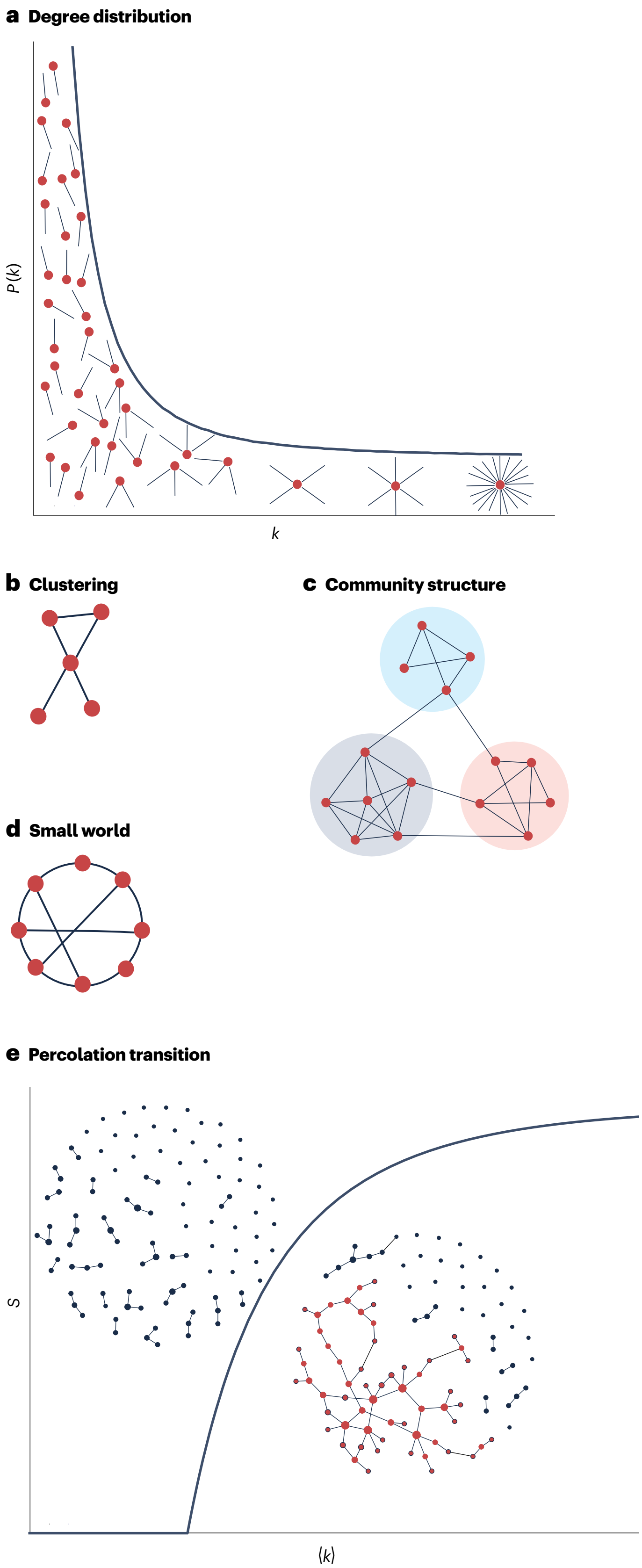

Fig. 1 | Common elements found in complex networks. **a**, Broad-scale distributions (P) of node degrees (k). **b**, Clustering with neighbours. **c**, Community structure: nodes can be allocated to groups for which the density of connections within a group is higher than the density of connections between groups. **d**, Small-world properties: most nodes are not nearest neighbours but are linked by short paths on the network. **e**, Phase transitions, such as the percolation transition in which the fractional size S of the largest connected component of the network exhibits a phase transition as the average degree increases.

consistently challenged by emerging applications that are instead characterized by large-scale collections of dynamical systems interacting with each other over a web of complex interconnections (Fig. 3b). Controlling such complex networks to orchestrate their collective behaviour is now a core problem in control theory that is the subject of much ongoing research and goes back, for example, to the pioneering work by Šiljak in the late 1970s[16].

From a control viewpoint, complex networks are examples of large-scale dynamical systems consisting of many continuous-time or discrete-time units interacting over a network of interconnections that can be either static or time-varying[16,17]. The key issue becomes understanding whether a given network fulfils fundamental control properties such as controllability and observability, and how to close the feedback loop across the different scales (Fig. 4) as a feedback connection needs to be established between the macroscopic behaviour of interest (such as consensus or synchronization) and the action necessary at the microscopic level to steer the resulting collective dynamics in a desired direction. Control can be achieved by controlling the nodes of the network, endowing the edges with some dynamics or communication protocol or manipulating the structure of the network itself or via a combination of these methods. One striking example is pinning control[18–21], in which controlling a relatively small fraction of the network nodes or edges is sufficient to control collective behaviour of the system towards some reference equilibrium or asymptotic trajectory[22,23]. Yet, more progress is needed to design control strategies for general systems with a broader range of nodal dynamics, as, for example, when the network structure changes in times or evolves as a function of the nodal dynamics.

Control in the context of complex networks

A challenge for control posed by real-world networks across domains is that there can be behaviours and interactions at multiple length scales and timescales, including self-organizing behaviours, which can influence each other. In some instances, only the collective behaviour is of interest (say, the overall number of infected individuals); yet, in other instances, the microscopic details may be of interest (which specific people are infected). Likewise, there can be constraints on the ability to interact with the degrees of freedom both with respect to measurement and to injecting control signals. Furthermore, it may be sufficient simply to control the system away from an undesirable state (such as system collapse) or towards an ensemble of desirable states, as opposed to controlling the system to a specific state. In-depth discussions on the challenges imposed from high-dimensionality, nonlinearity and constraints in interventions for real-world systems can be found in refs. 10,24,25.

There can be ambiguity about the network itself. In a brain network, nodes can be defined at the level of individual neurons, patches of neurons or even brain regions spanning large collections of neurons. With respect to interactions between nodes, learning the connectivity

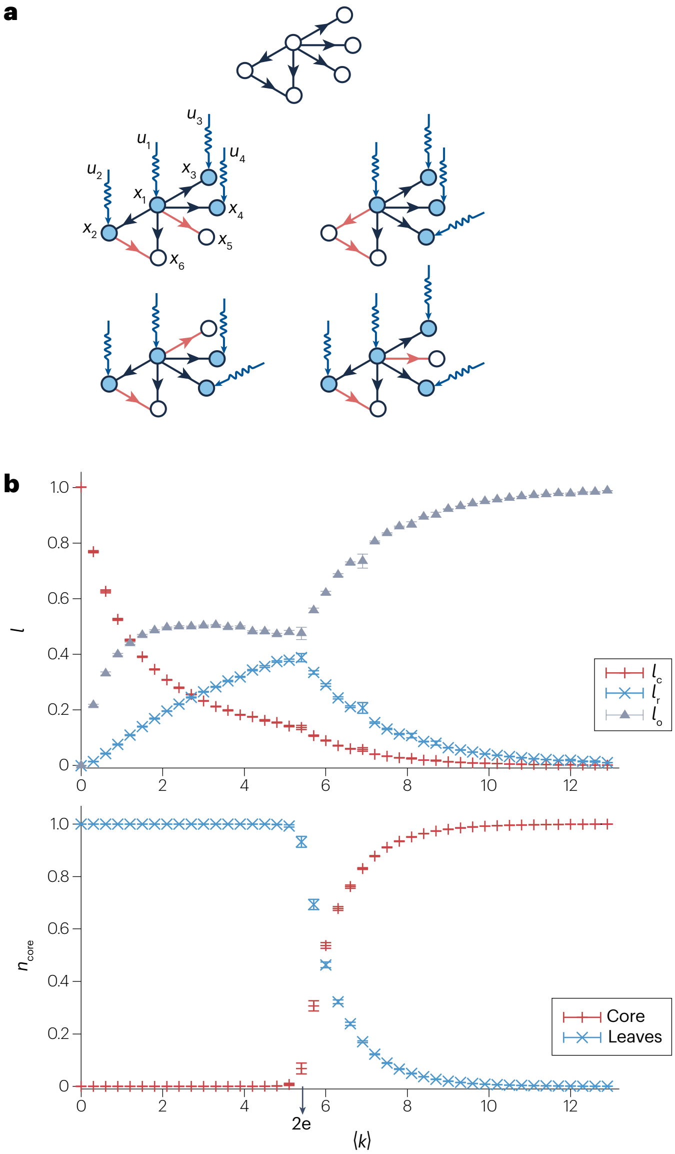

Fig. 2 | Identification of driver nodes and phase transition in the structural control framework. a, Solving the maximum matching problem on a directed network with linear nodal dynamics (of the state variables x_i's) enables identification of the minimum driver node set to ensure the structural controllability of the whole system. For a general directed network (top panel), there could be multiple maximum matchings (sets of red links). Hence, one can identify multiple minimum driver node sets (blue nodes). To each driver node, one must apply a unique control signal (u_i, blue squiggly arrows) necessary to ensure structural controllability. **b**, Control robustness and core percolation. One can quantify the robustness of control under unavoidable link failure by computing the fraction l of links that are: critical (l_c), if in its absence one must increase the number of driver nodes to maintain full control over the system (in other words, a critical link is part of all maximum matchings of the network); redundant (l_r), if it can be removed without affecting the current set of driver nodes (that is, it does not belong to any maximum matching); and ordinary (l_o), if it is neither critical nor redundant (it appears in some but not all maximum matchings). The non-monotonic behaviour of l_r as a function of mean degree $\langle k \rangle$ (upper part) is closely related to the core percolation transition on the networks (lower part), where n_{core} is the fraction of nodes in the core. The core percolation occurs where the number of distinct maximum matchings starts increasing exponentially, which renders the fraction of redundant links decreased. For directed Erdős–Rényi random networks, the core percolation occurs at $\langle k \rangle = 2e$ (lower part). Figure adapted with permission from ref. 3, Springer Nature Ltd.

pattern (that is, the 'wiring diagram') may require costly experiments and it may not be possible to learn the full connectivity pattern. Note that the presence or absence of a single edge can have pronounced consequences for properties such as betweenness centrality. Moreover, the dynamical activity taking place on the network can be equally important as the topology. For instance, in traffic networks, it is both the flows on the network and the network topology that determine congestion patterns. It is also the case that emergent long-range order can arise from the interplay of the nodal dynamics and the network structure[26,27].

To incorporate heterogeneity and multiple scales means that different parts of the system may need different types of representation. Some aspects might be best modelled with discrete time dynamics, whereas others with continuous time. Ordinary differential equations (ODEs) may best represent some aspects; partial differential equations (PDEs) may better describe other aspects. For example, in traffic management applications, ODEs can be used to describe the fluid motion of cars, whereas PDEs might better model the flow of vehicles controlled via traffic lights or other inputs that act at the agent level. How to integrate such analysis is an open question, as is the role of noise and uncertainty. Finally, application domains are as diverse as electric power grids, social networks and biological systems, each with distinct objectives and constraints, meaning that one must be careful to choose an appropriate modelling paradigm.

Key questions are what elements to measure, what elements to influence, when to influence them and how to influence them. In addition, it is crucial to develop methods to appropriately study convergence to the desired behaviour and the robustness and resilience of control strategies for complex networks.

Existing modelling paradigms

Statistical physics approaches

Notions and techniques rooted in statistical physics have been heavily leveraged to study both structural and dynamical properties of complex networks[2,28,29]. Areas of study include network growth, phase transitions and cascading failures, all of which are essential behaviours displayed by complex networks. Studies into controlling these behaviours from the statistical physics perspective are not about control in the strict sense of satisfying a controllability property but more about steering the system: for instance, steering it away from a tipping point or towards smaller failures. Direct applications of statistical physics tools to study traditional control properties of complex networks pertain primarily to controllability and observability. We discuss these topics next and provide a summary of methods in Table 1.

To study the growth of complex networks, in particular scale-free networks[30] notable for their power-law degree distribution, many analytical approaches with strong statistical physics flavour have been developed, such as continuum theory[31], the master-equation approach[32] and the rate-equation approach[33].

A core element of statistical physics is the study of phase transitions — the phenomenon of a strong change in the macroscopic behaviour of a system in response to a small change in an external control parameter at the critical point[34]. In the context of networks, an example of a phase transition is the percolation phase transition (Fig. 1e), which can be analysed using random graph models[35]. These models are based in the concept of a statistical ensemble, which is at the root of statistical physics.

A statistical ensemble of networks considers a given set of properties, such as a prescribed degree distribution. Each ensemble member is a realization of a network with a particular configuration of nodes and links and is attributed some probability (that is, statistical weight). All aspects other than the given set of properties are assumed to be completely random, thus they can be averaged over the entire ensemble by using some mean-field approaches, such as the generating function formalism[36,37] based on the branching process and the tree ansatz.

The percolation phase transition describes the sudden onset of large-scale connectivity in a network, and small interventions during the growth of connections can allow one to control the location of the critical point and can lead to explosive percolation[38,39]. For critical transitions, it has been shown that the predicted increase in fluctuations and autocorrelation times as a system reaches its 'tipping point' can serve as early warning signs[40,41].

A theoretical underpinning for the study of self-organization in statistical physics is the paradigm of self-organized criticality[42] (SOC). In SOC, the balance of competing forces, such as driving and dissipation, tunes the system to a critical point leading to cascading failures that follow a power-law distribution in size and are hence unbounded. Such cascading failures are a hallmark of complex networks such as power grids and brain networks[43]. Controlling SOC through the nature of the driving force is an important theme in the statistical physics literature[44–47], with more recent focus on 'dragon king' events[48–51].

Direct applications of statistical physics tools to controllability and observability do exist. When the control properties can be studied purely from the structure (or connectivity pattern) of the network, there are several successes.

One striking example is the application of the cavity method to solve structural control problems[3]. Owing to the graphical interpretation of the structural controllability theorem[4], one can check whether a network is structurally controllable by simply inspecting its structure, avoiding expensive matrix operations that rely on detailed edge weights. In particular, one can identify a minimum set of nodes termed as driver nodes, the time-dependent control of which is sufficient to fully control the entire dynamics of the system. This identification can be achieved by mapping the structural control problem into a purely graph-theoretical problem called maximum matching[52–54]. Leveraging the cavity method[55–57] rooted in statistical physics (and its further application in solving the maximum matching problem[58]), certain control properties of a network ensemble with a prescribed degree distribution can be analytically calculated[3]. Those properties include: the size of the maximum matching, which is directly related to the minimum number of driver nodes (or control inputs) to ensure structural controllability; and the total number of distinct maximum matchings, which is directly related to the number of different control configurations and hence affects the control robustness.

Another success is in the study of observability in the electric power grid. In this system, the voltages of nodes – which can be used as state variables – can be determined using phasor measurement units (PMUs). A PMU measures the real-time voltage and line currents of its corresponding node, thus a PMU determines the state variable of not only the node it is placed on but also all its first-nearest neighbours. In this case, the observability problem can be mapped to a purely graph-theoretical problem. Indeed, the random placement of PMUs leads to a network observability transition[59], which can be analytically studied using the generating function formalism[36,37]. Moreover, the problem of identifying the minimum set of sensor nodes (that is, PMUs) in a power grid can be mapped to a classical graph-theoretical problem: the minimum dominating set problem. Despite its nondeterministic polynomial time (NP)-hard nature in general, the minimum dominating set problem can be solved by a message-passing algorithm (rooted in spin glass theory), which offers nearly optimal solutions and performs well on real-world networks[60].

The power of mapping a control problem to a purely graph-theoretical problem is also naturally an intrinsic limitation. Any control property, such as the control energy cost, that requires detailed knowledge beyond network structure will not benefit from the purely graphical interpretation and the corresponding statistical ensemble approach. Techniques in random matrix theory[61] that can directly handle edge weights of complex networks might have to be used to develop an appropriate network ensemble. In general, both the detailed structure and the dynamics matter[62].

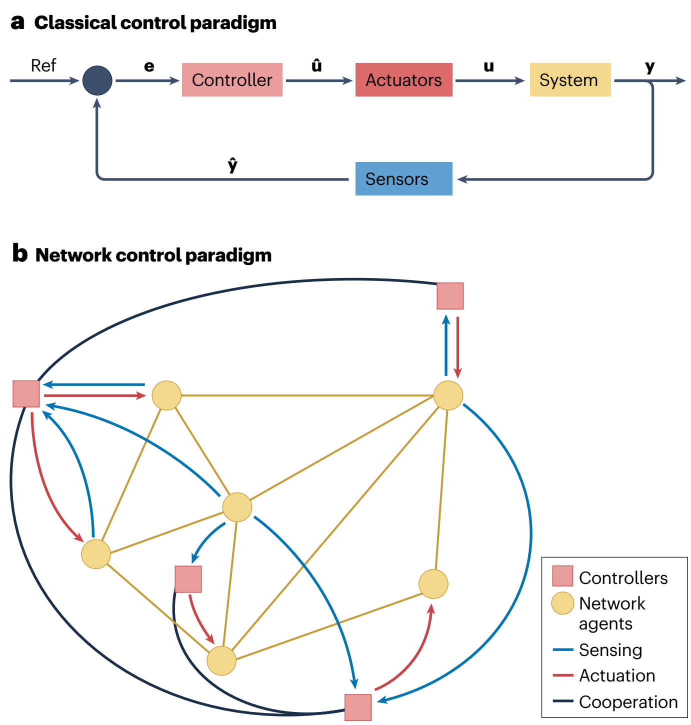

Fig. 3 | Control paradigms. a, The classic feedback control paradigm in which the output (**y**) of the system to be controlled is measured or estimated by sensors. The measured output (ŷ) is then fed back into a 'comparator' node (dark grey circle) that measures the difference between (ŷ) and the reference signal (Ref). That control error (**e**) is then fed to the controller that computes the control input (û) according to some control law. The computed input is then implemented in the actual input (**u**) to the system via a set of actuators. In this situation, all the relevant degrees of freedom and their couplings are known. **b**, A distributed and decentralized pinning control strategy. Some of the network agents (yellow circles) send information (blue arrows) about their states or outputs to controllers (pink squares). The controllers cooperate (black edges) to formulate a network control strategy and then intervene on the behaviour of a fraction of appropriately selected agents in the network (red arrows) to achieve some desired collective behaviour. Figure courtesy of Davide Salzano.

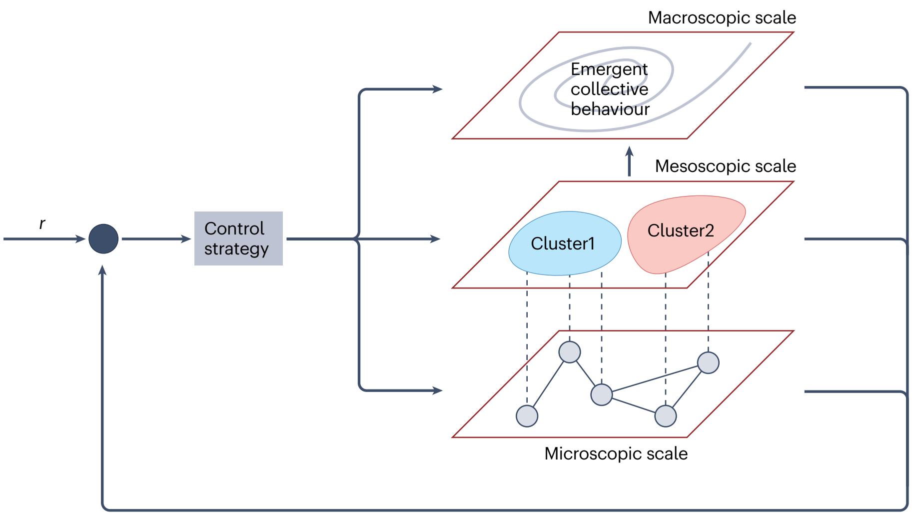

Fig. 4 | Closing the feedback loop in complex networks entails sensing, computing and actuating at different scales. Sensing and actuation can be performed at any of the scales depicted in the diagram. In this figure, we depict a centralized control strategy for simplicity; however, when dealing with network systems, the control strategy will typically be distributed and decentralized. Note r is the reference signal representing the desired behaviour of the system. Figure courtesy of Marco Coraggio.

Control theory approaches

Control theory approaches were traditionally developed for analysing and steering the behaviour of a specified system. Regardless, the problem in control can be distilled into determining what needs to be sensed, what needs to be controlled and how the information being sensed should be used to achieve the desired goal. Thus, the three key ingredients of any control design are sensing, computation and actuation[14]. Some methods are summarized in Table 2.

Typical control goals in multi-agent systems include consensus[63–71], which is the convergence of all units towards a common equilibrium point, and synchronization[72–75], which is the convergence to an asymptotic time-varying solution. They also include, among others, formation control[76–78], pattern formation[79] and coordinated motion of agents (such as flocking)[80]. The goal is often formulated in terms of performance (focused on transient properties such as settling time, rise time and overshoot, for example), stability (such as convergence to an equilibrium or a manifold in state space) and robustness to noise and external perturbations[14].

Starting from a mathematical (or data-driven) model of the system and a control goal, one can attempt to: establish controllability and observability of the system of interest; devise a control strategy and certify that the control strategy guarantees convergence and stability of the desired behaviour by means of appropriate rigorous proofs of these properties in the closed-loop network system (Fig. 5). Typically, when dealing with multi-agent systems, the focus is on devising strategies that are distributed and decentralized so that sensing, actuation and control inputs do not need to be decided in a centralized manner. Open-loop strategies, which do not rely on feedback from the sensors, are also a solution to some control problems, but typically fail to fulfil stability and performance requirements in the presence of perturbations and therefore lack robustness. Thus, we focus on closed-loop feedback strategies in this Perspective.

The controllability problem is an existence problem aimed at establishing which nodes need to be controlled to steer the collective behaviour, given the network structure, the dynamics of agents and the interaction protocol on the edges. Approaches to solve this problem in the context of complex networks include the use of structural controllability and the use of controllability Gramians[81–85], for example. Despite notable advances in the past decade, many open problems remain. Examples include understanding controllability in networks of nonlinear or time-varying systems or when the network structure evolves in time or as a function of the dynamics taking place over it (state-dependent network evolution).

The observability problem is aimed at understanding which variables carry enough information such that the whole system behaviour can be reconstructed from their measurement. Assessing observability becomes cumbersome when applied to large-scale complex networks as it entails deciding which behaviours of agents must be measured to reconstruct the overall network dynamics. Again, approaches from control such as structural observability theory have been used to this aim[82,86–88]. But many problems remain open, such as studying observability in time-varying network structures of nonlinear dynamical systems.

Controllability and observability criteria for complex networks have a twist compared with those of more traditional control theoretic approaches, in that graph-theoretical tools can be used to complement and enhance criteria on the basis of algebra or geometry. This crucial direction was first recognized in the early work by Šiljak[16] in the late 1970s and further developed in later work[82]; it can provide a viable option for dealing with large numbers of interacting dynamical variables. (We note that using graph-theoretical methods to study network problems dates back at least to the mathematical sociology community in the 1960s[89]).

If the fundamental properties of the system of interest have been analysed, a feedback control strategy (that is, a closed-loop strategy) can then be devised to achieve the control goal by exploiting the sensed information from the network and attempting to steer the system via control inputs. A fundamental issue in validating the control strategy is to analyse and prove convergence of the controlled network system starting from different initial conditions (stability) and under external perturbations (robustness). Approaches to study stability and robustness of complex networks of dynamical systems have been developed or extended from those available for homogeneous systems (for a review of some available methods, see refs. 17,21–23,90–95).

With respect to stability, approaches to study local or global stability of a given complex network system include those in which the network system is considered as a whole and its stability under perturbations is studied and those in which the aim is to prove that stability of individual agents is preserved when they are interconnected in a certain way. Examples of tools that consider the whole network system include those based on the Lyapunov direct method[90] or those based on linearization, such as the master stability function approach[96]. Other successful approaches are based on the use of incremental stability and convergence tools such as contraction theory[23,92–95] or incremental passivity[91]. These tools can be also adapted to study problems related to the concept of connective stability[16], which points to another fundamental issue touched upon earlier, namely, the role played by the underlying network structure on the dynamics taking place on it.

Approaches for control design abound in the literature and map onto the various areas of control theory including those based on dynamic optimization, optimal control, game theory, adaptive control, intelligent control, nonlinear control, model predictive control and robust control to name a few. Data-driven methods and control strategies based on machine learning are also increasingly being adopted to control the behaviour of complex networks. For more details, see refs. 97,98 and the discussion in the section on 'New paradigms and modelling techniques'.

Despite the advances in the field, many open challenges remain to be solved. Recent efforts in the control community have focused on the effects of noise on coordinated collective behaviour in networks, their resilience to perturbations (including structural perturbations), the development of strategies for coordination and consensus guaranteeing privacy of the nodes and the analysis and control of disturbance propagation in network systems[99–109].

Approaches from dynamical systems

Like methods from statistical physics, methods from dynamical systems also provide insights into control strategies that are typically aimed at steering and influencing the system and not on strict controllability. Many approaches directly exploit the nonlinear nature of the system. There are also data-driven methods such as system identification. We discuss these topics next.

Given the dynamical equations that model the behaviour of a system and its phase space of attractors, limit cycles and basin boundaries, one may be able to find strategic perturbations that exploit the natural trajectories to drive the system to a desirable region of phase space. Early on, this possibility was shown for chaotic attractors[110], with much follow-on work in this area of the control of chaos[111–113]. More recently, it was shown how to achieve control through a sequence of strategic kicks that, moreover, account for constraints on how the system can be perturbed[114]. Although exploiting natural trajectories in phase space is appealing, in practice, it is difficult to provide the rigorous performance guarantees and robustness to noise necessary for traditional control theory. For instance, basin boundaries can be riddled or fractal.

In related literature, there is a substantial body of work on control of chimaera states[115]. Chimaera states display surprising symmetry-breaking properties as they are defined by the coexistence of coherent and incoherent dynamics in a system of symmetrically coupled, identical oscillators[116,117]. Studies in this context include work on time-delayed feedback control[118–120], pinning control[121], periodic forcing[122], control via topology[123] or coupling modification[124] and control of chimaeras in multilayer networks[125]. A more general collection of works centred on control of self-organizing nonlinear dynamical systems is shown in ref. 126, although many open directions are yet to be explored.

Often, the equations of motion of a system are unknown. Even the state space may be unknown. But data on the system may be abundant. If the data generated by a system — its observables, that is, physical quantities that can be measured — are a function of its state, one may be able to infer the evolution of the system from time-series data. For example, many techniques for system identification or network inference have been presented in the literature (such as refs. 127–129 and references therein). In the next section, we discuss more recent approaches to this problem on the basis of operator theory and sparse identification techniques.

New paradigms and modelling techniques

Several opportunities exist to increase the applicability of the methods discussed earlier to real-world systems.

How much can one increase network complexity?

In recent years, a focus in the physics literature has been on increasing network complexity. A 'network' formally consists of a collection of

Table 1 | Notions and methods rooted in or with strong flavour of statistical physics that have been applied to study the structural, dynamical or control properties of complex networks

Notion or method	Brief description	Application
Statistical ensemble	A set of (often infinitely) many virtual copies of a system, considered together, each representing a possible configuration of the real system	The ongoing development of random graph models[2]
Generating function formalism	Mean-field approach based on the branching process and the tree ansatz	For the studies of network structure[36,37] (such as the emergence of the giant connected component), dynamics (such as cascading failures[188]) and certain control properties (such as observability transition in power grid[59]), especially for tree-like networks
Master-equation approach	Where the time evolution of a system can be described by a transition rate matrix that determines switching between states	Investigating the structure of growing networks[32]
Rate-equation approach	Useful for diverse non-equilibrium phenomena, such as aggregation, coarsening and epitaxial surface growth	Investigating the structure of growing networks[33]
Self-organized criticality	Where a system has a critical point as an attractor and thus exhibits criticality without requiring tuning of control parameters	Studying cascading failures in network systems[42–47,50]
Cavity method	Mathematical method originally developed to solve some mean-field type models in statistical physics, especially disordered systems (such as spin glasses)	Studying the control properties of complex networks in the structural control framework[3]

Table 2 | Notions and methods from control theory that have been applied to analyse and control complex networks

Notion or method	Brief description	Application
Linear and nonlinear controllability and observability criteria	Use of controllability and observability criteria from linear and nonlinear control (such as Kalman's criteria, structural controllability and observability and nonlinear controllability and observability)	Obtaining graph-theoretical criteria for controllability and observability of complex networks[81–88]
Lyapunov direct methods for stability analysis	Extensions of the Lyapunov direct method to study stability of network systems via the use of Lyapunov functions (such as connective stability or V-stability)	Studying stability of network systems, including studying global transversal stability for synchronization, nonlinear consensus problems, pinning control design and connective stability[90,91,94]
Passive and dissipative systems theory and external stability concepts as design tools for network control	Extensions of passivity and dissipativity theory to complex networks; definition and use of incremental passivity and dissipativity concepts	Studying interconnections of dynamical agents and establish their convergence properties. For example, the interconnection of passive systems is passive, and the interconnection of dissipative systems is stable under a Lyapunov diagonal stability condition[189]
Linear and nonlinear control approaches	Use of linear and nonlinear control approaches such as optimal control, adaptive control, intelligent control, robust control, nonlinear control, switched and hybrid control and proportional–integral–derivative (PID) control	Designing network control strategies based on linear and nonlinear control approaches in a range of different applications from engineering to the life sciences[21,68,71,73,74,78]
Pinning control strategies	Exerting control on a relatively small number of selected agents to steer the macroscopic behaviour of the ensemble	Designing leader–follower strategies for consensus and synchronization of complex networks[18–21]
Contraction theory and incremental stability analysis tools	Use of differential (internal or external) stability tools to investigate convergence in complex networks	Proving convergence in network systems of relevance in applications from computational neuroscience to power grids and gene regulatory networks[23,92,93,95,190,191]
Distributed and cooperative control approaches	Exploitation of distributed and cooperative control strategies to deploy sensing, actuation and control in a network system	Design of strategies for coordination, consensus and synchronization in complex networks via the design of interaction protocols among nodes, adaptation and evolution of the network edges and structure[63–80,106]. These also include game theoretical approaches[192–196]

pair-wise interactions between elements, but higher-order interactions, beyond dyadic, are often found in real-world networks. For instance, in a chemical reaction network, three reagents may be necessary for the reaction to progress; in co-author networks, a group of more than two authors is often present. The formalism of hypergraphs and simplicial complexes are being used to address this challenge[130,131]. Advances include defining their statistical ensembles[132,133] and analysis of admissible patterns of synchronization, their stability properties for both full synchronization[134–136] and cluster synchronization[137,138] and their controllability[139]. But ad hoc control strategies have not yet been fully developed.

Likewise, the paradigm of activity-driven temporal networks, which provide an instantaneous time description of the network dynamics, may prove fruitful[140,141]. In this approach, the activity potential for each node is determined from how relatively active that node is during a given time window as measured from time-resolved network data sets. The activity potential distribution function can encode the system-level dynamics.

Many real-world systems are multilayered networks. For example, individuals participate simultaneously in many distinct layers of social networks, and critical infrastructure networks often have a physical-layered or logical-layered structure. This notion underlies work on structural control of multiplex networks[142,143] and the use of graph products to capture layered critical infrastructure[144,145] or the use of multiplex control strategies[146].

Can one control non-equilibrium statistical physics models?

Statistical physics approaches tend to focus on equilibrium systems, yet there are well-known fluctuation–dissipation relations for systems that obey detailed balance. For instance, heat can be converted into work using a feedback control scheme on a double quantum dot model[147], a finding that is driving further research into feedback control and fluctuations[148]. Likewise, there are several classic models of driven, far-from-equilibrium systems, such as the SOC model[42] (described in the section on 'Statistical physics approaches'), the Kardar–Parisi–Zhang (KPZ) equation[149] and the asymmetric simple exclusion process (ASEP) model[150]. Although these models have many universal behaviours, which are behaviours that are governed by general attributes such as underlying symmetries independent of dynamical details of the system, it may be possible to use feedback to influence the driving and thus control the behaviours.

How far can one go beyond linear models in the structural control framework?

A fundamental limitation in the classical framework of structural control[4] is that it relies on linear time-invariant dynamics, $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$, where the elements in A and B are either fixed zeros or independent free parameters. This framework is based on the notion of structural controllability for linear systems. In particular, the system (A, B) is structurally controllable if one can set the non-zero elements in A and B to certain values such that the resulting system is controllable in the usual sense, that is, it satisfies Kalman's criterion of controllability: $\mathrm{rank}[B, AB, \ldots, A^{N-1}B] = N$.

More recently, a structural control framework based on the notion of structural accessibility for nonlinear systems has been developed[151,152]. This framework is applicable to general nonlinear systems $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{g}(\mathbf{x}(t))\,\mathbf{u}(t)$ with very mild assumptions on the dynamics (that $\mathbf{f}(\mathbf{x}(t))$ and $\mathbf{g}(\mathbf{x}(t))$ are meromorphic functions). The notion of structural accessibility can be considered as a nonlinear generalization of structural controllability in linear systems[4]. Surprisingly, structural accessibility and structural controllability have almost the same graph-theoretical conditions. The key difference is that 'self-loops' (corresponding to intrinsic nodal dynamics) can be used to satisfy the graph-theoretical conditions for structural controllability, but not for structural accessibility. This structural accessibility framework has been applied to ecological and biochemical systems to identify driver nodes from their underlying network structure[151].

How to deal with very large complex multi-agent systems?

Another pressing open problem in control is that of taming the dynamics of complex systems in the limit that the number of agents becomes very large or, more precisely, when the system exhibits emergent properties that become invariant with respect to the number of agents. In this context, the problem becomes that of finding a macroscopic description of the observables of interest one wishes to control and defining suitable control strategies to change them. Doing so entails the problem of closing the loop across the macroscopic scale of the variables of interest and the microscopic agent level at which control can be typically exerted. For extremely complex and large networks, achieving any control objective beyond identifying driver nodes is a challenging task, even for networks of linear dynamical systems.

An advance called the continuification (or continuation) method[153,154] involves turning the microscopic problem described by a large set of ODEs into a PDE describing the observables of interest at the macroscopic level (continuification stage). A macroscopic control action is then designed using techniques for the control of PDEs[155,156], and the resulting control law is ultimately discretized so that it can be deployed back at the microscopic agent level[157]. In this approach, the challenge becomes that of finding methods to continuify a given problem of interest and to discretize appropriately, to obtain a meaningful distributed control strategy at the microscopic level. A major hurdle is that typically when the control law resulting from the PDE is discretized then all or most of the agents will be affected by the control input, contrary to the goal of controlling a network through interacting with only a relatively small fraction of controller or driver nodes, such as in the spirit of pinning control.

Another framework is based on the graphon control of large-scale networks of linear systems[158]. Graphons are the limits of converging graph sequences, which form a natural, non-parametric method to model and estimate extremely large networks[159]. Graphon theory has emerged as a subfield of graph theory and spurred wide interest owing to its connections with statistical physics, extremal combinatorics and non-parametric statistical analysis on networks[160–162]. The graphon-based strategy for controlling complex and large network systems consists of three steps. One first identifies the graphon limit of a sequence S of finite network systems as the number of nodes goes to infinity. One then solves the corresponding control problem in this limit. Finally, one can generate control laws for any system along the sequence S of finite network systems by approximating the control law for the limit system. This strategy has been used on large-scale complex networks to solve the state-to-state control problem and the linear quadratic regulator problem.

As a statistical framework for network games and intervention, the notion of graphon games (in which a continuum of heterogeneous agents interact according to a graphon) has also been proposed[163]. This framework is another interesting example of using graphon theory to study interventions on large networks. How to leverage graphon theory to control arbitrarily large networks with general nonlinear dynamics remains an outstanding problem.

As research moves towards larger networks, an increasingly important problem is to investigate the possibility of controlling a complex network of interest by controlling and/or observing the mesoscopic scale. Such a mesoscale could be at the level of communities or clusters of nodes or edges. Doing so is an open problem that deserves further attention and requires a proper definition of what an appropriate mesoscopic level description is from a control viewpoint.

Can one use data to construct effective equations of motion?

Beyond the well-established approach of system identification, there are alternative approaches to reconstructing the effective equations of motion.

One such approach is the Koopman operator method. It is a linear transformation on a vector space of observables where the evolution can be written as a linear expansion in terms of eigenfunctions of an operator known as the Koopman operator. Doing so provides a linear rule of evolution, but an infinite dimensional space of observables. Instabilities can be associated with modes that have positive eigenvalues and one can even identify the role of individual nodes in escalating the instabilities by their relative amplitudes in the associated eigenvectors. The usefulness of the Koopman operator for the analysis of dynamical systems is well established[164,165]. It also has applications to nonlinear flows[166]. More recent highlights apply the approach to optimal controllers[167,168] and feedback control[169–171]. A useful primer is ref. 172, and two recent comprehensive treatments are refs. 173,174.

A different data-driven approach relies on the assumption that, despite being high dimensional, the dynamics are primarily influenced by only a few main variables so that the equations are sparse in the space of possible functions. Sparsity-promoting techniques and machine learning can be used in combination on noisy measurement data to identify governing equations, a technique known as SINDY (sparse identification of nonlinear dynamics)[175]. SINDY has been extended to include the effects of actuation and it has been shown how to enhance performance of model predictive control based on limited, noisy data[176].

More recently, dimension reduction techniques have been used on high-dimensional time-series data to map it onto a low-dimensional subspace, with SINDY then used to determine the reduced dynamics.

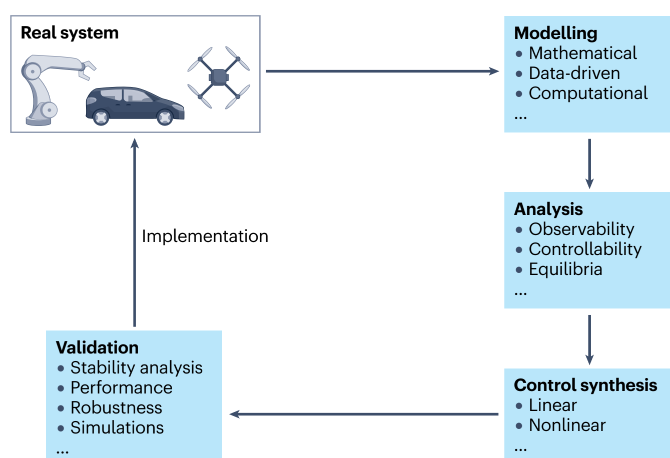

Fig. 5 | Main stages of a classical closed-loop controller design. Starting from the real system, a model is first obtained whose properties in the absence of control are analysed. A control strategy is then designed to fulfil the desired specifications and then must be validated before being implemented. Often, this design approach requires several iterations before an accurate controller is achieved. Figure courtesy of Gian Carlo Maffettone.

If the resulting phase space consists of a few fixed points, the system can be tuned to induce desired instabilities and attractors, implementing feedforward control of high-dimensional, nonlinear, network systems[177].

How can machine learning and data-driven control methods be used to tame complexity?

As the complexity of problems of interest in applications increases alongside the available computational power, control strategies for complex networks based on machine learning and data-driven methods are becoming increasingly popular in different areas of science and technology. A notable example is that of prototypical designs of interconnected autonomous vehicles. Companies such as Google Waymo (https://waymo.com) and others have proposed the use of deep learning to design self-driving cars or to achieve autonomous vehicle platooning, such as truck platooning (https://highways.dot.gov/research/laboratories/saxton-transportation-operations-laboratory/Truck-Platooning). Another example is that of autonomous robots and swarm robotics, in which computational techniques based on machine learning are also being used increasingly often[178]. Methods for data-driven control of networks under different scenarios have been proposed, as discussed in the previous sections, but a framework for their use in more general settings is still lacking.

Nevertheless, it is becoming increasingly clear that data-driven and learning-based methods[179,180] might be the only option when the problem is too hard to solve analytically, such as when a mathematical model cannot be derived or the task to be solved is too complex. One such scenario is when the goal is to achieve control by adapting dynamically in time the structure of temporal networks in response to changes in the dynamics and therefore the states of the agents interconnected by it[140,141] (see ref. 181 for a simpler illustrative example). Solving this problem might be extremely important in real applications in which the goal is to endow the network with the ability to rewire its structure to maintain its desired function even in the presence of faults or perturbations. Examples could include the case of self-organizing power grids able to island themselves to prevent faults or overcurrents, or the case of groups of autonomous vehicles or robots changing the structure of their interconnections to better perform obstacle avoidance or complex manoeuvres.

Action items

A multidisciplinary and interdisciplinary research effort is required to advance the current state of the art and address problems in which complexity is not only tamed but also exploited to achieve better control performance and to solve complex tasks. The aim should be twofold. First, to bridge the gap between disciplines and extend the use of techniques such as mean-field approaches to the control of complex networks[182], taking into account realistic constraints and the need to achieve feedback strategies that guarantee desired stability, performance and robustness properties. Second, to identify a set of paradigmatic problems or benchmark case studies that could be used to validate and contrast different approaches to control complex systems. Doing so is particularly important as applications that arise in a multitude of different domains and techniques developed in specific areas could be abstracted to solve more general problems. An example is the technique of phase response curves to analyse the dynamics of nonlinear oscillators (such as neurons) that were recently proposed as a tool to achieve control of more general classes of nonlinear systems (see ref. 183 and references therein).

To move this field forward and to facilitate collaboration across disciplines, we call for community efforts. One action item could be to initiate a series of challenges to benchmark methods in solving fundamental problems in controlling complex systems. In the field of computer science, holding challenges has become a tradition. There is a long list of successful challenges, including the Microsoft Imagine Cup, Google AI Challenge, ImageNet Challenge and Netflix Prize. It can be argued that some of the challenges (such as the ImageNet Challenge) catalysed the boom of artificial intelligence that the world is experiencing today. Likewise, in the field of systems biology and translational medicine, there has been an excellent paradigm of running challenges: the so-called DREAM challenges, which provide high-quality benchmark biomedicine data sets, invite participants to propose solutions, foster collaboration and build communities in the process. Network control researchers can certainly learn from those existing challenge platforms in other fields to further advance our field, so that the 'wisdom of the crowd' provides the greatest impact on science.

Owing to the multidisciplinary nature of controlling complex systems, challenges do not have to focus on purely theoretical problems but can be applied or even translational. For example, there was an attempt to perform structural controllability analysis of a directed human protein interaction network to identify disease genes and drug targets[184]. Further studies on this topic are warranted. Moreover, there are many potential applications of control theory in designing ways to better manipulate the human gut microbiome — the inner ecosystem in humans consisting of trillions of microorganisms interacting with each other in a complicated way[185]. One very practical control problem in this area is to design a well-defined consortium of live microorganisms (often referred to as probiotic cocktails, bugs-as-drugs or live biotherapeutic products) to decolonize certain pathogens and prevent future infection[186]. Another possibly interesting arena for benchmarking methods would be protection and control of microgrids[187] (that is, a local electrical grid with defined electrical boundaries, acting as a single and controllable entity).

The ultimate goal is to combine tools and techniques from different areas of science and technology to address the crucial problem of closing the control loop across different scales, to orchestrate the collective behaviour of large-scale, complex systems. Solving this problem has potential to have enormous impact in a plethora of different applications across domains.

Published online: 24 March 2023

References

1. National Research Council. *Network Science* (The National Academies Press, 2005).
2. Newman, M. E. J. *Networks: An Introduction* (Oxford Univ. Press, 2018).
3. Liu, Y.-Y., Slotine, J.-J. & Barabási, A.-L. Controllability of complex networks. *Nature* **473**, 167–173 (2011).
4. Lin, C.-T. Structural controllability. *IEEE Trans. Autom. Control.* **19**, 201 (1974).
5. Liu, Y. Y., Csoka, E., Zhou, H. & Posfai, M. Core percolation on complex networks. *Phys. Rev. Lett.* **109**, 205703 (2012).
6. Jia, T. et al. Emergence of bimodality in controlling complex networks. *Nat. Commun.* **4**, 2002 (2013).
7. Sun, J. & Motter, A. E. Controllability transition and nonlocality in network control. *Phys. Rev. Lett.* **110**, 208701 (2013).
8. Yan, G., Ren, J., Lai, Y.-C., Lai, C.-H. & Li, B. Controlling complex networks — how much energy is needed? *Phys. Rev. Lett.* **108**, 218703 (2012).
9. Ruths, J. & Ruths, D. Control profiles of complex networks. *Science* **343**, 1373–1376 (2014).
10. Motter, A. E. Networkcontrology. *Chaos* **25**, 097621 (2015).
11. Liu, Y.-Y. & Barabási, A.-L. Control principles of complex systems. *Rev. Mod. Phys.* **88**, 053006 (2016).
12. Xiang, L., Chen, F., Ren, W. & Chen, G. Advances in network controllability. *IEEE Circuits Syst. Mag.* **19**, 8–32 (2019).

13. Bechhoefer, J. *Control Theory for Physicists* (Cambridge Univ. Press, 2021).
14. Åström, K. J. & Murray, R. M. *Feedback Systems: An Introduction for Scientists and Engineers* 2nd edn (Princeton Univ. Press, 2021).
15. Khalil, H. K. *Nonlinear Systems* (Prentice Hall, 2002).
16. Siljak, D. D. *Large-scale Dynamic Systems: Stability and Structure* (North-Holland, 1978).
17. Bullo, F. *Lectures on Network Systems* 1.6 edn (Kindle Direct Publishing, 2022).
18. Li, X., Wang, X. & Chen, G. Pinning a complex dynamical network to its equilibrium. *IEEE Trans. Circuits Syst.* **51**, 2074–2087 (2004).
19. Wang, X. & Chen, G. Pinning control of scale-free dynamical networks. *Phys. A Stat. Mech. Appl.* **310**, 521–531 (2002).
20. Sorrentino, F., di Bernardo, M., Garofalo, F. & Chen, G. Controllability of complex networks via pinning. *Phys. Rev. E* **75**, 046103 (2007).
21. Su, H. & Wang, X. *Pinning Control of Complex Networked Systems* 1st edn (Springer, 2013).
22. Moreau, L. Stability of multiagent systems with time-dependent communication links. *IEEE Trans. Autom. Control* **50**, 169–182 (2005).
23. Cisneros-Velarde, P., Jafarpour, S. & Bullo, F. Contraction theory for dynamical systems on Hilbert spaces. *IEEE Trans. Autom. Control* **67**, 6710–6715 (2021).
24. Murray, R. M. *Control in an Information Rich World* (Society for Industrial and Applied Mathematics, 2003).
25. Lamnabhi-Lagarrigue, F. et al. Systems and control for the future of humanity, research agenda: current and future roles, impact and grand challenges. *Annu. Rev. Control* **43**, 1–64 (2017).
26. Matheny Matthew, H. et al. Exotic states in a simple network of nanoelectromechanical oscillators. *Science* **363**, eaav7932 (2019).
27. Salova, A. & D'Souza, R. M. Decoupled synchronized states in networks of linearly coupled limit cycle oscillators. *Phys. Rev. Res.* **2**, 043261 (2020).
28. Albert, R. & Barabási, A.-L. Statistical mechanics of complex networks. *Rev. Mod. Phys.* **74**, 47–97 (2002).
29. Dorogovtsev, S. N., Goltsev, A. V. & Mendes, J. F. F. Critical phenomena in complex networks. *Rev. Mod. Phys.* **80**, 1275 (2008).
30. Barabási, A.-L. & Albert, R. Emergence of scaling in random networks. *Science* **286**, 509–512 (1999).
31. Albert, R. & Barabási, A.-L. Topology of evolving networks: local events and universality. *Phys. Rev. Lett.* **85**, 5234–5237 (2000).
32. Dorogovtsev, S. N., Mendes, J. F. F. & Samukhin, A. N. Structure of growing networks with preferential linking. *Phys. Rev. Lett.* **85**, 4633–4636 (2000).
33. Krapivsky, P. L., Redner, S. & Leyvraz, F. Connectivity of growing random networks. *Phys. Rev. Lett.* **85**, 4629–4632 (2000).
34. Stanley, H. E. *Introduction to Phase Transitions and Critical Phenomena* (Oxford Univ. Press, 1971).
35. Erdős, P. & Rényi, A. On the evolution of random graphs. *Publ. Math. Inst. Hungarian Acad. Sci.* **5**, 17–61 (1960).
36. Newman, M. E. J., Strogatz, S. H. & Watts, D. J. Random graphs with arbitrary degree distributions and their applications. *Phys. Rev. E* **64**, 026118 (2001).
37. Callaway, D. S., Newman, M. E. J., Strogatz, S. H. & Watts, D. J. Network robustness and fragility: percolation on random graphs. *Phys. Rev. Lett.* **85**, 5468–5471 (2000).
38. Achlioptas, D., D'Souza, R. M. & Spencer, J. Explosive percolation in random networks. *Science* **323**, 1453–1455 (2009).
39. D'Souza, R. M., Gómez-Gardeñes, J., Nagler, J. & Arenas, A. Explosive phenomena in complex networks. *Adv. Phys.* **68**, 123–223 (2019).
40. Scheffer, M. et al. Early-warning signals for critical transitions. *Nature* **461**, 53–59 (2009).
41. Boettiger, C. & Hastings, A. Quantifying limits to detection of early warning for critical transitions. *J. R. Soc. Interface* **9**, 2527–2539 (2012).
42. Bak, P., Tang, C. & Wiesenfeld, K. Self-organized criticality: an explanation of the 1/*f* noise. *Phys. Rev. Lett.* **59**, 381–384 (1987).
43. D'Souza, R. M. Curtailing cascading failures. *Science* **358**, 860–861 (2017).
44. Cajueiro, D. O. & Andrade, R. F. Controlling self-organized criticality in sandpile models. *Phys. Rev. E* **81**, 015102 (2010).
45. Cajueiro, D. O. & Andrade, R. F. Dynamical programming approach for controlling the directed Abelian Dhar–Ramaswamy model. *Phys. Rev. E* **82**, 031108 (2010).
46. Noël, P.-A., Brummitt, C. D. & D'Souza, R. M. Controlling self-organizing dynamics on networks using models that self-organize. *Phys. Rev. Lett.* **111**, 078701 (2013).
47. Qi, J. & Pfenninger, S. Controlling the self-organizing dynamics in a sandpile model on complex networks by failure tolerance. *EPL* **111**, 38006 (2015).
48. Sornette, D. Dragon-kings, black swans and the prediction of crises. CCSS Working Paper No. CCSS-09-005 (2009).
49. Cavalcante, H. L., Oria, M., Sornette, D., Ott, E. & Gauthier, D. J. Predictability and suppression of extreme events in a chaotic system. *Phys. Rev. Lett.* **111**, 198701 (2013).
50. Lin, Y., Burghardt, K., Rohden, M., Noël, P.-A. & D'Souza, R. M. Self-organization of dragon king failures. *Phys. Rev. E* **98**, 022127 (2018).
51. Mikaberidze, G. & D'Souza, R. M. Sandpile cascades on oscillator networks: the BTW model meets Kuramoto. *Chaos* **32**, 053121 (2022).
52. Yamada, T. & Foulds, L. R. A graph-theoretic approach to investigate structural and qualitative properties of systems: a survey. *Networks* **20**, 427 (1990).
53. Commault, C., Dion, J. M. & Van Der Woude, J. W. Characterization of generic properties of linear structured systems for efficient computations. *Kybernetika* **38**, 503–520 (2002).
54. Murota, K. *Matrices and Matroids for Systems Analysis* (Springer, 2009).
55. Mézard, M. & Parisi, G. The Bethe lattice spin glass revisited. *Eur. Phys. J. B* **20**, 217 (2001).
56. Mézard, M. & Parisi, G. The cavity method at zero temperature. *J. Stat. Phys.* https://doi.org/10.1023/A:1022221005097 (2003).
57. Mezard, M., Parisi, G. & Virasoro, M. *Spin Glass Theory and Beyond* Vol. 9 (World Scientific, 1986).
58. Zdeborová, L. & Mézard, M. The number of matchings in random graphs. *J. Stat. Mech. Theory Exp.* **05**, P05003 (2006).
59. Yang, Y., Wang, J. & Motter, A. Network observability transitions. *Phys. Rev. Lett.* **109**, 258701 (2012).
60. Zhao, J.-H, Habibulla, Y. & Zhou, H.-J. Statistical mechanics of the minimum dominating set problem. *J. Stat. Phys.* **159**, 1154–1174 (2015).
61. Wigner, E. P. Random matrices in physics. *SIAM Rev.* **9**, 1–23 (1967).
62. Gates, A. J. & Rocha, L. M. Control of complex networks requires both structure and dynamics. *Sci. Rep.* **6**, 24456 (2016).
63. Olfati-Saber, R. & Murray, R. M. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Autom. Control.* **49**, 1520–1533 (2004).
64. Paley, D. A., Leonard, N. E., Sepulchre, R., Grunbaum, D. & Parrish, J. K. Oscillator models and collective motion. *IEEE Control. Syst. Mag.* **27**, 89–105 (2007).
65. Jadbabaie, A., Lin, J. & Morse, A. S. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans. Autom. Control.* **48**, 988–1001 (2003).
66. Tanner, H. G., Jadbabaie, A. & Pappas, G. J. Flocking in fixed and switching networks. *IEEE Trans. Autom. Control.* **52**, 863–868 (2007).
67. Leonard, N. E. & Fiorelli, E. *Proc. 40th IEEE Conference on Decision and Control* (Cat. No.01CH37228) (IEEE, 2001).
68. Olfati-Saber, R. Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Trans. Autom. Control* **51**, 401–420 (2006).
69. Olfati-Saber, R., Fax, J. A. & Murray, R. M. Consensus and cooperation in networked multi-agent systems. *Proc. IEEE* **95**, 215–233 (2007).
70. Wei, R. & Beard, R. W. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Trans. Autom. Control* **50**, 655–661 (2005).
71. Sepulchre, R. Consensus on nonlinear spaces. *Annu. Rev. Control* **35**, 56–64 (2011).
72. DeLellis, P., diBernardo, M. & Garofalo, F. Novel decentralized adaptive strategies for the synchronization of complex networks. *Automatica* **45**, 1312–1318 (2009).
73. Dorfler, F., Chertkov, M. & Bullo, F. Synchronization in complex oscillator networks and smart grids. *Proc. Natl Acad. Sci. USA* **110**, 2005–2010 (2013).
74. Scardovi, L. & Sepulchre, R. *2008 47th IEEE Conference on Decision and Control* 546–551 (IEEE, 2008).
75. Wieland, P., Sepulchre, R. & Allgöwer, F. An internal model principle is necessary and sufficient for linear output synchronization. *Automatica* **47**, 1068–1074 (2011).
76. Andrea, R. D. & Dullerud, G. E. Distributed control design for spatially interconnected systems. *IEEE Trans. Autom. Control* **48**, 1478–1495 (2003).
77. Bullo, F., Cortés, J. & Martínez, S. *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms* (Princeton Univ. Press, 2009).
78. Shamma, J. S. (ed.) *Cooperative Control of Distributed Multi-agent Systems* (John Wiley & Sons, Ltd, 2007).
79. Oh, K.-K., Park, M.-C. & Ahn, H.-S. A survey of multi-agent formation control. *Automatica* **53**, 424–440 (2015).
80. Kumar, V., Leonard, N. & Morse, A. S. *Cooperative Control* (Springer Berlin, 2005).
81. Porfiri, M. & di Bernardo, M. Criteria for global pinning-controllability of complex networks. *Automatica* **44**, 3100–3106 (2008).
82. Mesbahi, M. & Egerstedt, M. *Graph Theoretic Methods in Multiagent Networks* (Princeton Univ. Press, 2010).
83. Mesbahi, M. On state-dependent dynamic graphs and their controllability properties. *IEEE Trans. Autom. Control* **50**, 387–392 (2005).
84. Rahmani, A., Ji, M., Mesbahi, M. & Egerstedt, M. Controllability of multi-agent systems from a graph-theoretic perspective. *SIAM J. Control Optim.* **48**, 162–186 (2009).
85. Pasqualetti, F., Zampieri, S. & Bullo, F. Controllability metrics, limitations and algorithms for complex networks. *IEEE Trans. Control Netw. Syst.* **1**, 40–52 (2014).
86. Bianchin, G., Frasca, P., Gasparri, A. & Pasqualetti, F. The observability radius of networks. *IEEE Trans. Autom. Control* **62**, 3006–3013 (2017).
87. Liu, Y.-Y., Slotine, J.-J. & Barabási, A.-L. Observability of complex systems. *Proc. Natl Acad. Sci. USA* **110**, 2460–2465 (2013).
88. Sundaram, S. & Hadjicostis, C. N. Structural controllability and observability of linear systems over finite fields with applications to multi-agent systems. *IEEE Trans. Autom. Control* **58**, 60–73 (2013).
89. Harary, F., Norman, R. Z. & Cartwright, D. *Structural Models: An Introduction to the Theory of Directed Graphs* (John Wiley & Sons, 1965).
90. Xiang, J. & Chen, G. On the V-stability of complex dynamical networks. *Automatica* **43**, 1049–1057 (2007).
91. Arcak, M. Passivity as a design tool for group coordination. *IEEE Trans. Autom. Control* **52**, 1380–1390 (2007).
92. Slotine, J.-J. E. & Wang, W. in *Cooperative Control: A Post-Workshop Volume 2003 Block Island Workshop on Cooperative Control* (eds Kumar, V., Leonard, N. & Morse, A. S.) 207–228 (Springer, 2005).
93. Forni, F. & Sepulchre, R. A differential Lyapunov framework for contraction analysis. *IEEE Trans. Autom. Control* **59**, 614–628 (2014).
94. Moylan, P. & Hill, D. Stability criteria for large-scale systems. *IEEE Trans. Autom. Control* **23**, 143–149 (1978).
95. di Bernardo, M., Fiore, D., Russo, G. & Scafuti, F. in *Complex Systems and Networks: Dynamics, Controls and Applications* (eds Lü, J., Yu, X., Chen, G. & Yu, W.) 313–339 (Springer, 2016).

96. Pecora, L. M. & Carroll, T. L. Master stability functions for synchronized coupled systems. *Phys. Rev. Lett.* **80**, 2109–2112 (1998).
97. Baggio, G., Bassett, D. S. & Pasqualetti, F. Data-driven control of complex networks. *Nat. Commun.* **12**, 1429 (2021).
98. Nguyen, T. T., Nguyen, N. D. & Nahavandi, S. Deep reinforcement learning for multiagent systems: a review of challenges, solutions, and applications. *IEEE Trans. Cybern.* **50**, 3826–3839 (2020).
99. Battistelli, G. & Chisci, L. Kullback–Leibler average, consensus on probability densities, and distributed state estimation with guaranteed stability. *Automatica* **50**, 707–718 (2014).
100. Dibaji, S. M., Ishii, H. & Tempo, R. Resilient randomized quantized consensus. *IEEE Trans. Autom. Control* **63**, 2508–2522 (2018).
101. Fiore, D. & Russo, G. Resilient consensus for multi-agent systems subject to differential privacy requirements. *Automatica* **106**, 18–26 (2019).
102. Nozari, E., Tallapragada, P. & Cortés, J. Differentially private distributed convex optimization via functional perturbation. *IEEE Trans. Control Netw. Syst.* **5**, 395–408 (2018).
103. Mo, Y. & Murray, R. M. Privacy preserving average consensus. *IEEE Trans. Autom. Control* **62**, 753–765 (2017).
104. Xie, S., Russo, G. & Middleton, R. H. Scalability in nonlinear network systems affected by delays and disturbances. *IEEE Trans. Control Netw. Syst.* **8**, 1128–1138 (2021).
105. Stüdli, S., Seron, M. M. & Middleton, R. H. From vehicular platoons to general networked systems: string stability and related concepts. *Annu. Rev. Control* **44**, 157–172 (2017).
106. Russo, G., Wirth, F. & Shorten, R. On synchronization in continuous-time networks of nonlinear nodes with state-dependent and degenerate noise diffusion. *IEEE Trans. Autom. Control* **64**, 389–395 (2019).
107. Li, T., Wu, F. & Zhang, J. Multi-agent consensus with relative-state-dependent measurement noises. *IEEE Trans. Autom. Control* **59**, 2463–2468 (2014).
108. Burbano-L, D. A., Russo, G. & Bernardo, M. D. Pinning controllability of complex network systems with noise. *IEEE Trans. Control Netw. Syst.* **6**, 874–883 (2019).
109. Della Rossa, F. & De Lellis, P. Synchronization and pinning control of stochastic coevolving networks. *Annu. Rev. Control* **53**, 147–160 (2022).
110. Ott, E., Grebogi, C. & Yorke, J. A. Controlling chaos. *Phys. Rev. Lett.* **64**, 1196–1199 (1990).
111. Ditto, W. L., Rauseo, S. N. & Spano, M. L. Experimental control of chaos. *Phys. Rev. Lett.* **65**, 3211–3214 (1990).
112. Shinbrot, T., Grebogi, C., Ott, E. & Yorke, J. A. Using small perturbations to control chaos. *Nature* **363**, 411 (1993).
113. Boccaletti, S., Grebogi, C., Lai, Y.-C., Mancini, H. & Maza, D. The control of chaos: theory and applications. *Phys. Rep.* **329**, 103–197 (2000).
114. Cornelius, S. P., Kath, W. L. & Motter, A. E. Realistic control of network dynamics. *Nat. Commun.* **4**, 1942 (2013).
115. Bick, C. & Martens, E. A. Controlling chimeras. *New J. Phys.* **17**, 033030 (2015).
116. Kuramoto, Y. & Davaasambuu, B. Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. *Nonlin. Phenom. Complex Syst.* **5**, 380–385 (2002).
117. Abrams, D. M. & Strogatz, S. H. Chimera states for coupled oscillators. *Phys. Rev. Lett.* **93**, 174102 (2004).
118. Sieber, J., Omel'chenko, O. E. & Wolfrum, M. Controlling unstable chaos: stabilizing chimera states by feedback. *Phys. Rev. Lett.* **112**, 054102 (2014).
119. Gjurchinovski, A., Scholl, E. & Zakharova, A. Control of amplitude chimeras by time delay in oscillator networks. *Phys. Rev. E* **95**, 042218 (2017).
120. Zakharova, A., Semenova, N., Anishchenko, V. & Schöll, E. Time-delayed feedback control of coherence resonance chimeras. *Chaos* **27**, 114320 (2017).
121. Gambuzza, L. V. & Frasca, M. Pinning control of chimera states. *Phys. Rev. E* **94**, 022306 (2016).
122. Semenov, V., Zakharova, A., Maistrenko, Y. & Schöll, E. Delayed-feedback chimera states: forced multiclusters and stochastic resonance. *EPL* **115**, 10005 (2016).
123. Bera, B. K., Majhi, S., Ghosh, D. & Perc, M. Chimera states: effects of different coupling topologies. *EPL* **118**, 10001 (2017).
124. Ruzzene, G., Omelchenko, I., Schöll, E., Zakharova, A. & Andrzejak, R. G. Controlling chimera states via minimal coupling modification. *Chaos* **29**, 051103 (2019).
125. Omelchenko, I., Hülser, T., Zakharova, A. & Schöll, E. Control of chimera states in multilayer networks. *Front. Appl. Math. Stat.* **4**, 00067 (2019).
126. Schöll, E., Klapp, S. H. L. & Hövel, P. *Control of Self-organizing Nonlinear Systems* (Springer, 2016).
127. Ljung, L. *System Identification: Theory for User* (Prentice Hall, 1999).
128. Xue, Y. & Bogdan, P. Reconstructing missing complex networks against adversarial interventions. *Nat. Commun.* **10**, 1738 (2019).
129. Timme, M. Revealing network connectivity from response dynamics. *Phys. Rev. Lett.* **98**, 224101 (2007).
130. Battiston, F. et al. The physics of higher-order interactions in complex systems. *Nat. Phys.* **17**, 1093–1098 (2021).
131. Bianconi, G. *Higher-Order Networks* (Cambridge Univ. Press, 2021).
132. Ghoshal, G., Zlatic, V., Caldarelli, G. & Newman, M. E. Random hypergraphs and their applications. *Phys. Rev. E* **79**, 066118 (2009).
133. Courtney, O. T. & Bianconi, G. Generalized network structures: the configuration model and the canonical ensemble of simplicial complexes. *Phys. Rev. E* **93**, 062311 (2016).
134. Lucas, M., Cencetti, G. & Battiston, F. Multiorder Laplacian for synchronization in higher-order networks. *Phys. Rev. Res.* **2**, 033410 (2020).
135. Gambuzza, L. V. et al. Stability of synchronization in simplicial complexes. *Nat. Commun.* **12**, 1255 (2021).
136. Ferraz de Arruda, G., Tizzani, M. & Moreno, Y. Phase transitions and stability of dynamical processes on hypergraphs. *Commun. Phys.* **4**, 24 (2021).
137. Zhang, Y., Latora, V. & Motter, A. E. Unified treatment of synchronization patterns in generalized networks with higher-order, multilayer, and temporal interactions. *Commun. Phys.* **4**, 195 (2021).
138. Salova, A. & D'Souza, R. M. Cluster synchronization on hypergraphs. Preprint at https://doi.org/10.48550/arXiv.2101.05464 (2021).
139. Chen, C., Surana, A., Bloch, A. M. & Rajapakse, I. Controllability of hypergraphs. *IEEE Trans. Netw. Sci. Eng.* **8**, 1646–1657 (2021).
140. Perra, N., Goncalves, B., Pastor-Satorras, R. & Vespignani, A. Activity driven modeling of time varying networks. *Sci. Rep.* **2**, 469 (2012).
141. Liu, S., Perra, N., Karsai, M. & Vespignani, A. Controlling contagion processes in activity driven networks. *Phys. Rev. Lett.* **112**, 118702 (2014).
142. Posfai, M., Gao, J., Cornelius, S. P., Barabasi, A. L. & D'Souza, R. M. Controllability of multiplex, multi-time-scale networks. *Phys. Rev. E* **94**, 032316 (2016).
143. Menichetti, G., Dall'Asta, L. & Bianconi, G. Control of multilayer networks. *Sci. Rep.* **6**, 20706 (2016).
144. González, A. D., Chapman, A., Dueñas-Osorio, L., Mesbahi, M. & D'Souza, R. M. Efficient infrastructure restoration strategies using the recovery operator. *Comput.-Aided Civ. Infrastruct. Eng.* **32**, 991–1006 (2017).
145. Chapman, A., González, A. D., Mesbahi, M., Dueñas-Osorio, L. & D'Souza, R. M. *2017 IEEE 56th Annual Conference on Decision and Control* (*CDC*) 493–498 (IEEE, 2017).
146. Burbano, D. & di Bernardo, M. Multiplex PI control for consensus in networks of heterogeneous linear agents. *Automatica* **67**, 310–320 (2016).
147. Annby-Andersson, B., Samuelsson, P., Maisi, V. F. & Potts, P. P. Maxwell's demon in a double quantum dot with continuous charge detection. *Phys. Rev. B* **101**, 165404 (2020).
148. Bhattacharyya, D. & Jarzynski, C. From a feedback-controlled demon to an information ratchet in a double quantum dot. *Phys. Rev. E* **106**, 064101 (2022).
149. Kardar, M., Parisi, G. & Zhang, Y. C. Dynamic scaling of growing interfaces. *Phys. Rev. Lett.* **56**, 889–892 (1986).
150. Spitzer, F. Interaction of Markov processes. *Adv. Math.* **5**, 246–290 (1970).
151. Angulo, M. T., Moog, C. H. & Liu, Y.-Y. A theoretical framework for controlling complex microbial communities. *Nat. Commun.* **10**, 1045 (2019).
152. Angulo, M. T., Aparicio, A. & Moog, C. H. Structural accessibility and structural observability of nonlinear networked systems. *IEEE Trans. Netw. Sci. Eng.* **7**, 1656–1666 (2020).
153. Nikitin, D., Wit, C. C. D. & Frasca, P. A continuation method for large-scale modeling and control: from ODEs to PDE, a round trip. *IEEE Trans. Autom. Control* **67**, 5118–5133 (2021).
154. Nikitin, D. *Scalable Large-scale Control of Network Aggregates* (Université Grenoble Alpes, 2021).
155. Krstic, M. & Smyshlyaev, A. *Boundary Control of PDEs* (SIAM Press, 2008).
156. Smyshlyaev, A. & Krstic, M. *Adaptive Control of Parabolic PDEs* (Princeton Univ. Press, 2010).
157. Maffettone, G., Boldini, A., di Bernardo, M. & Porfiri, M. Continuification control of large-scale multiagent systems in a ring. *IEEE Control Syst. Lett.* **7**, 841–846 (2023).
158. Gao, S. & Caines, P. E. Graphon control of large-scale networks of linear systems. *IEEE Trans. Autom. Control* **65**, 4090–4105 (2020).
159. Borgs, C. & Chayes, J. *Proceedings of the 2017 ACM Conference on Economics and Computation* 665–672 (Association for Computing Machinery, 2017).
160. Lovász, L. *Large Networks and Graph Limits* Vol. 60 (American Mathematical Society, 2012).
161. Borgs, C., Chayes, J. T., Lovász, L., Sós, V. T. & Vesztergombi, K. Convergent sequences of dense graphs I: subgraph frequencies, metric properties and testing. *Adv. Math.* **219**, 1801–1851 (2008).
162. Lovász, L. & Szegedy, B. Limits of dense graph sequences. *J. Comb. Theory Ser. B* **96**, 933–957 (2006).
163. Parise, F. & Ozdaglar, A. Graphon games: a statistical framework for network games and interventions. *Econometrica* **91**, 191–225 (2023).
164. Mezić, I. & Banaszuk, A. Comparison of systems with complex behavior. *Phys. D Nonlin. Phenom.* **197**, 101–133 (2004).
165. Mezić, I. Spectral properties of dynamical systems, model reduction and decompositions. *Nonlin. Dyn.* **41**, 309–325 (2005).
166. Rowley, C. W., MeziĆ, I., Bagheri, S., Schlatter, P. & Henningson, D. S. Spectral analysis of nonlinear flows. *J. Fluid Mech.* **641**, 115–127 (2009).
167. Brunton, S. L., Brunton, B. W., Proctor, J. L. & Kutz, J. N. Koopman invariant subspaces and finite linear representations of nonlinear dynamical systems for control. *PLoS ONE* **11**, e0150171 (2016).
168. Kaiser, E., Kutz, J. N. & Brunton, S. L. Data-driven discovery of Koopman eigenfunctions for control. *Mach. Learn. Sci. Technol.* **2**, 035023 (2021).
169. Arbabi, H., Korda, M. & Mezić, I. *2018 IEEE Conference on Decision and Control* (*CDC*) 6409–6414 (IEEE, 2018).
170. Peitz, S. Controlling nonlinear PDEs using low-dimensional bilinear approximations obtained from data. Preprint at https://doi.org/10.48550/arXiv.1801.06419 (2018).
171. Peitz, S. & Klus, S. Koopman operator-based model reduction for switched-system control of PDEs. *Automatica* **106**, 184–191 (2019).
172. Arbabi, H. *Koopman Spectral Analysis and Study of Mixing in Incompressible Flows.* PhD thesis, University of California (2017).
173. Brunton, S. L., Budišić, M., Kaiser, E. & Kutz, J. N. Modern Koopman theory for dynamical systems. *SIAM Rev.* **64**, 229–340 (2022).
174. Mauroy, A., Mezić, I. & Susuki, Y. *The Koopman Operator in Systems and Control* (Springer, 2020).

175. Brunton, S. L., Proctor, J. L. & Kutz, J. N. Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proc. Natl Acad. Sci. USA* **113**, 3932–3937 (2016).
176. Kaiser, E., Kutz, J. N. & Brunton, S. L. Sparse identification of nonlinear dynamics for model predictive control in the low-data limit. *Proc. R. Soc. A Math. Phys. Eng. Sci.* **474**, 0335 (2018).
177. Morrison, M. & Kutz, J. N. Nonlinear control of networked dynamical systems. *IEEE Trans. Netw. Sci. Eng.* **8**, 174–189 (2021).
178. Hüttenrauch, M., Šošić, A. & Neumann, G. Deep reinforcement learning for swarm systems. *J. Mach. Learn. Res.* **20**, 1966–1996 (2019).
179. Garrabé, É. & Russo, G. Probabilistic design of optimal sequential decision-making algorithms in learning and control. *Annu. Rev. Control.* **54**, 81–102 (2022).
180. Hewing, L., Wabersich, K. P., Menner, M. & Zeilinger, M. N. Learning-based model predictive control: toward safe learning in control. *Annu. Rev. Control Robot. Auton. Syst.* **3**, 269–296 (2020).
181. Kempton, L. C., Herrmann, G. & di Bernardo, M. Distributed optimisation and control of graph Laplacian eigenvalues for robust consensus via an adaptive multilayer strategy. *Int. J. Robust. Nonlin. Control* **27**, 1499–1525 (2017).
182. Fornasier, M. & Solombrino, F. Mean-field optimal control. *ESAIM: COCV* **20**, 1123–1152 (2014).
183. Sepulchre, R. Spiking control systems. *Proc. IEEE* **110**, 577–589 (2022).
184. Vinayagam, A. et al. Controllability analysis of the directed human protein interaction network identifies disease genes and drug targets. *Proc. Natl Acad. Sci. USA* **113**, 4976–4981 (2016).
185. Gilbert, J. A. et al. Current understanding of the human microbiome. *Nat. Med.* **24**, 392–400 (2018).
186. Xiao, Y., Angulo, M. T., Lao, S., Weiss, S. T. & Liu, Y.-Y. An ecological framework to understand the efficacy of fecal microbiota transplantation. *Nat. Commun.* **11**, 3329 (2020).
187. Altin, N. & Eyimaya, S. E. *2021 10th International Conference on Renewable Energy Research and Application* (*ICRERA*) 412–417 (IEEE, 2021).
188. Watts, D. J. A simple model of global cascades on random networks. *Proc. Natl Acad. Sci. USA* **99**, 5766–5771 (2002).
189. Arcak, M., Meissen, C. & Packard, A. *Networks of Dissipative Systems: Compositional Certification of Stability, Performance, and Safety* (Springer, 2016).
190. Bullo, F. *Contraction Theory for Dynamical Systems* 1.0 edn (Kindle Direct Publishing, 2022).
191. Lohmiller, W. & Slotine, J.-J. E. On contraction analysis for nonlinear systems. *Automatica* **34**, 683–696 (1998).
192. Marden, J. R. & Shamma, J. S. Game theory and control. *Annu. Rev. Control Robot. Auton. Syst.* **1**, 105–134 (2018).
193. Lewis, F. L., Zhang, H., Hengster-Movric, K. & Das, A. *Cooperative Control of Multi-Agent Systems* (Springer, 2014).
194. Gadjov, D. & Pavel, L. A passivity-based approach to nash equilibrium seeking over networks. *IEEE Trans. Autom. Control* **64**, 1077–1092 (2019).
195. Semsar-Kazerooni, E. & Khorasani, K. Multi-agent team cooperation: a game theory approach. *Automatica* **45**, 2205–2213 (2009).
196. Gharesifard, B. & Cortés, J. Distributed convergence to Nash equilibria in two-network zero-sum games. *Automatica* **49**, 1683–1692 (2013).

Acknowledgements
The authors thank all the participants of the Satellite Symposium on 'Controlling Complex Networks' held at the International School and Conference on Network Science (NetSci) 2014, 2015, 2016, 2017, 2018, 2019 and 2021 and all the participants in the focus group on control held at the Future Directions in Network Science meeting in 2016 for very stimulating discussions that have helped shape our understanding. The authors also thank M. Coraggio, G. Maffettone and D. Salzano from the University of Naples Federico II, Italy and the Scuola Superiore Meridionale, Naples for their help in generating Figs. 3–5. The authors also thank A. Aparicio, C. Chen and X.-W. Wang and G. Mikaberidze for their comments on the manuscript.

Author contributions
All authors contributed to all aspects of this work.

Competing interests
The authors declare no competing interests.

Additional information
Peer review information *Nature Reviews Physics* thanks Guanrong Chen, Francesco Bullo and the other, anonymous, reviewer(s) for their contribution to the peer review of this work.