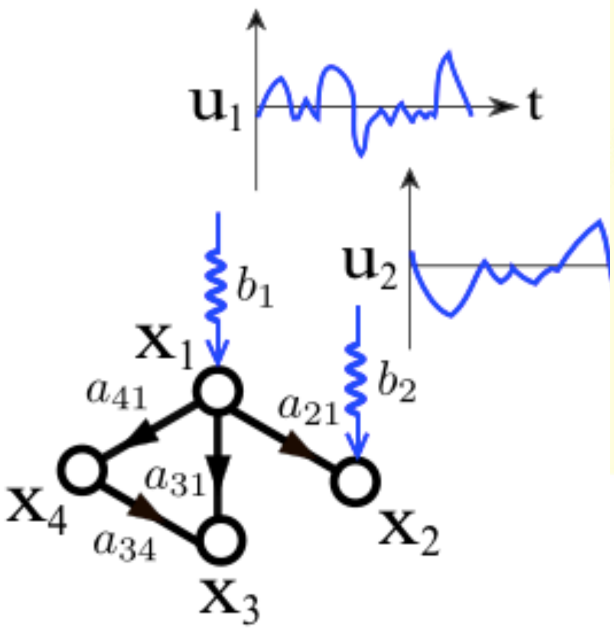
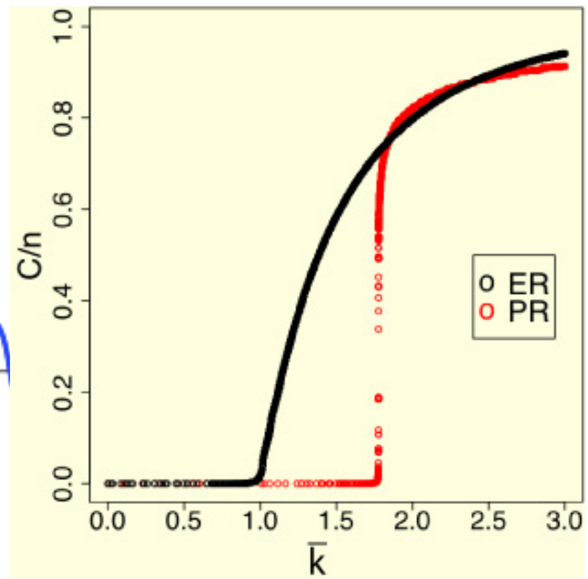
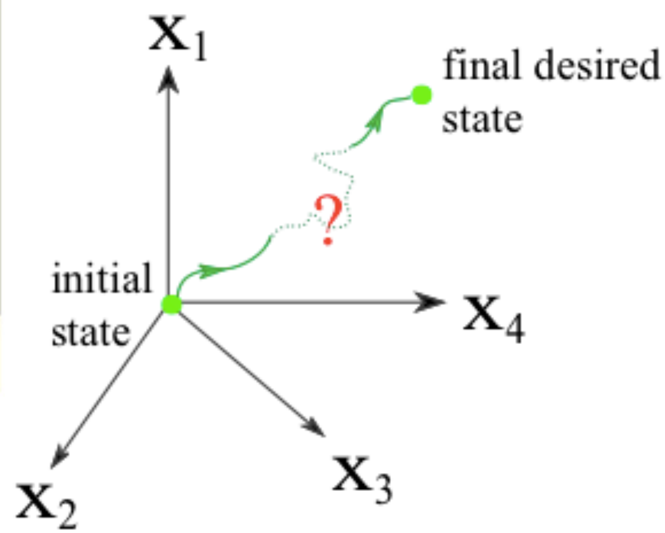
Controlling complex networks

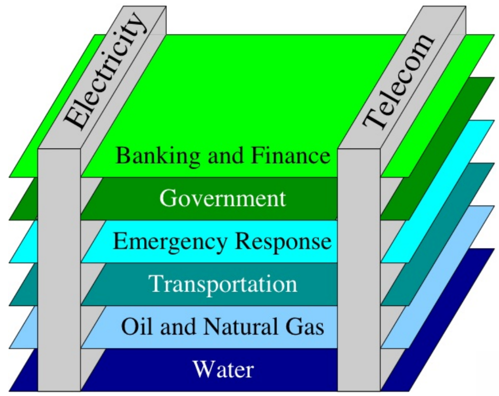




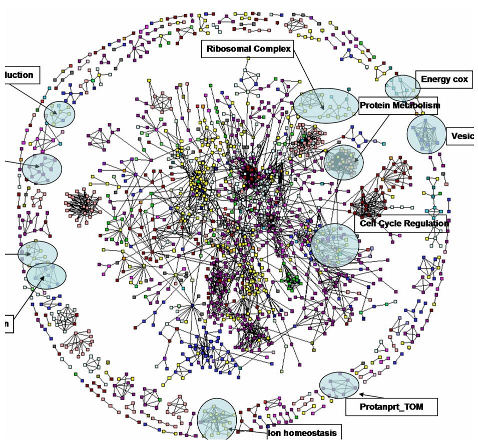


Raissa D'Souza
University of California, Davis
Dept of Mech. and Aero. Eng., Dept of CS
Complexity Sciences Center
External Professor, **Santa Fe Institute**

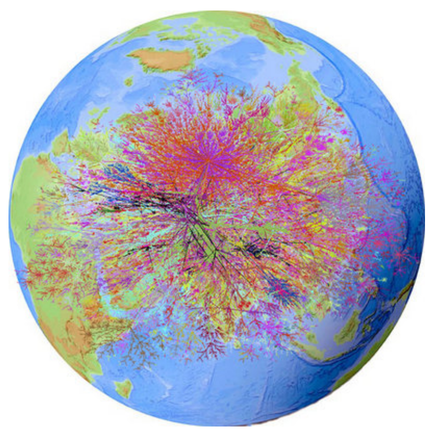
Complex, interdependent networks of modern society



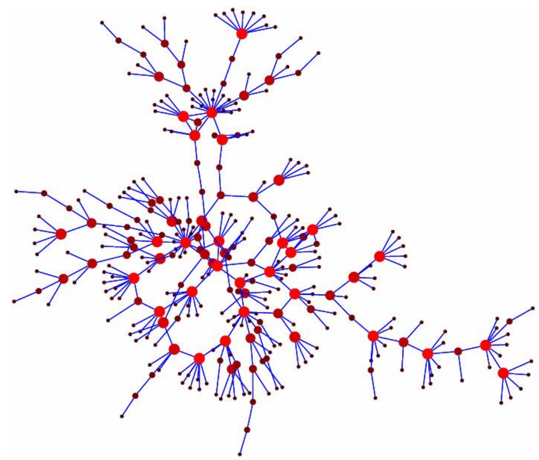
Critical Infrastructure



Biological & Ecological networks



Information and Communication technology

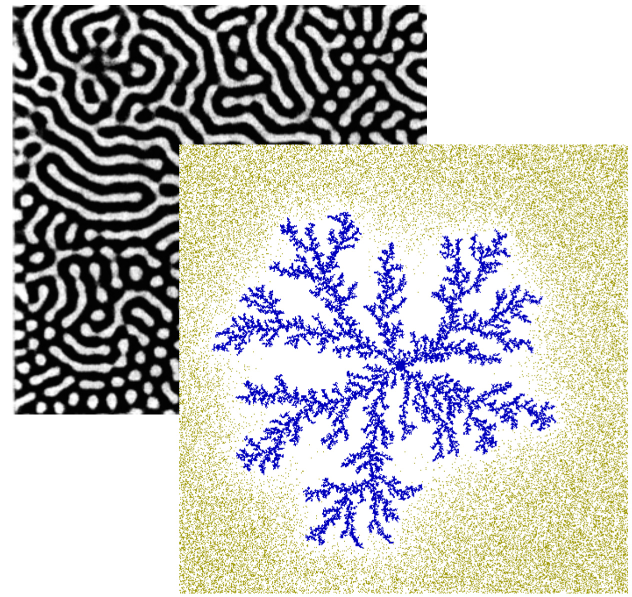


Social networks: Economics & Epidemics

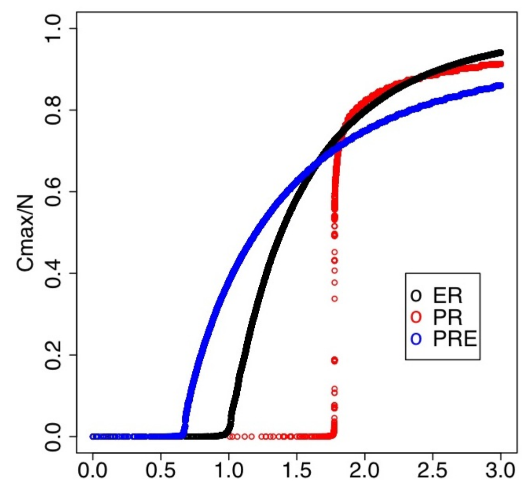
Cyber-physical, socio-technical, eco-social systems.

Each network is a complex system

Collective behaviors from simple constituent elements



- Multiple length and time scales
- Emergent behaviors
  - Self organization
    (e.g., patterns, synchronization)
  - Phase transitions
- Millions of degrees of freedom
  - Full knowledge may not be feasible.



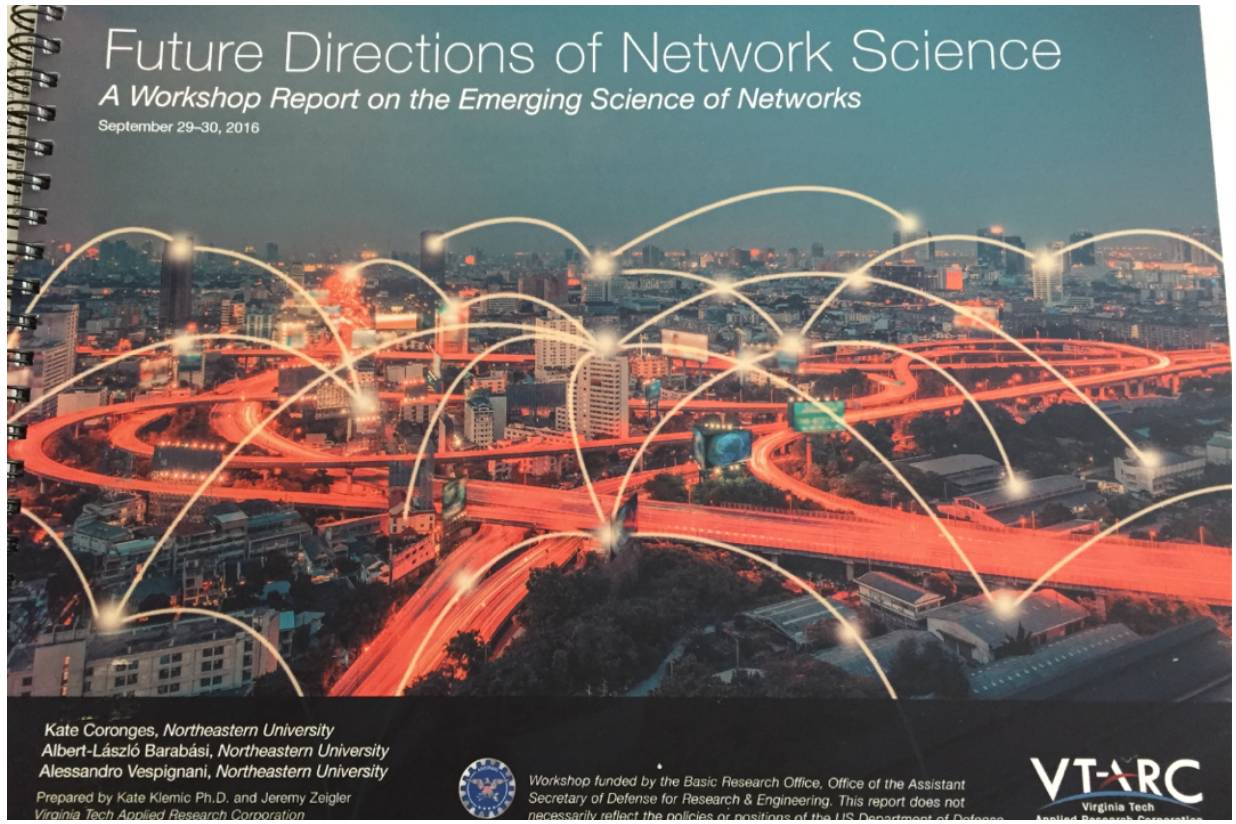
Self-organization:

Decentralized coordination and control

What do we want to "control"?

- Every degree of freedom?
- Some macroscopic property of the system?
  - E.g., do you care how many people are infected (macroscopic) or which particular people are infected (microscopic).
- Steer towards some class of behaviors? Avoid certain attractors.

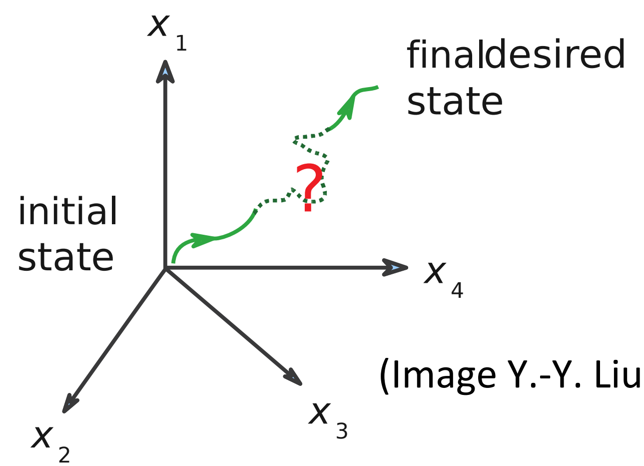
Thanks to Future Directions participants



- Patty Mabry
- Tom Valente
- Laszlo Barabasi
- Yang-Yu Liu
- Noshir Contractor
- Kate Coronges
- Ananthram Swami
- Bruce West
- Raissa D'Souza (Breakout lead)

Classes / categories of control

- Traditional mathematical control theory
  - Nodes evolving via internal nodal dynamics and interactions with other nodes
- Nonlinear dynamics:
  - Basins of attraction / control of chaos (nudging at a strategic time)
  - Control of self-organization and SOC



(Image Y.-Y. Liu)

- Social systems (nodes have "mind of their own")
- Biological systems / medicine

Traditional control theory

- Doesn't scale to modern systems with millions of degrees of freedom (may not even be able to know all the degrees of freedom, let alone their instantaneous states)

- Modern systems are: Socio-technical, cyber-physical, eco-social. Can't necessarily reduce to a set of interconnected differential equations.

- But there are opportunities

Today's agenda

- Questions:
  - What does it mean to control a complex network?
  - What methods exist?
  - What are potential new directions?

- Not an exhaustive review of all literature.

- Instead an overview of the main threads of control in complex networks as I see them.

A comment: Physics meets control theory

- Control theory
  - Feedback fundamental
  - Strong controllability (every degree of freedom)
  - Symmetry is the enemy of controllability
    (no fine-grained control without symmetry breaking)
- Physics (in particular statistical physics of networks)
  - Equilibrium theory of macroscopic properties
  - Phase transitions (parametric control of macroscopic)
  - Symmetry offers a key to understanding

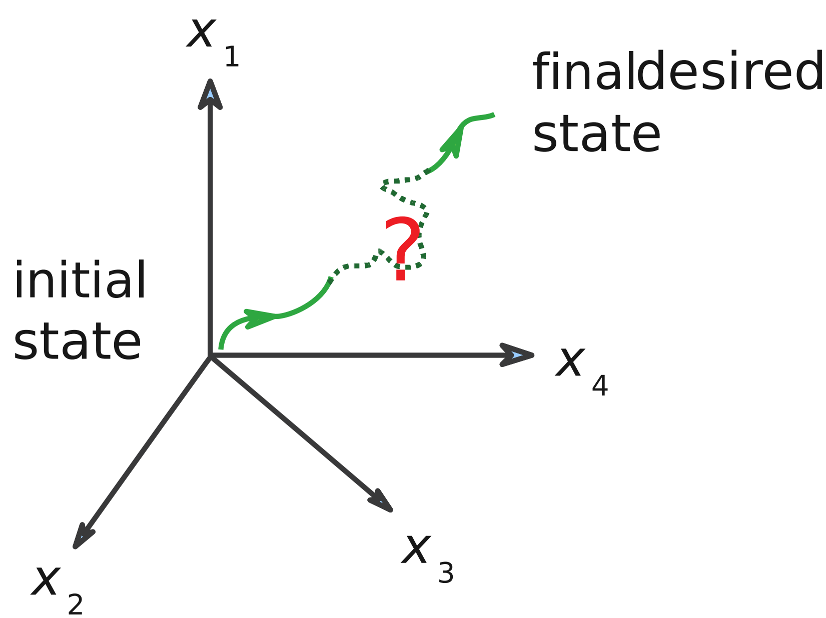
Today: Three parts

- Traditional control theory and structural control of networks
  - Emphasis on structural control of Linear Time Invariant (LTI) systems.

- Control of nonlinear dynamics

- Exerting influence in social networks

I. Traditional control theory



(Image Y.-Y. Liu)

Traditional control : Many different considerations and notions.

- Control a dynamical system with N degrees of freedom evolving in time, d**x**(t)/dt.

- Controllability
  - Can drive **x**(t) from any initial state to any final state in finite time

- Observability
  - Knowledge of a subset of **x**(t) is sufficient to guarantee full knowledge of **x**(t).

- Observability and controllability are duals!

Additional notions:

- Stabilizability: control only the amplifying nodes (dampening nodes not important)
- Detectability: all unobservable modes are asymptotically stable
- Set controllability: can bound **x**(t) to a region.

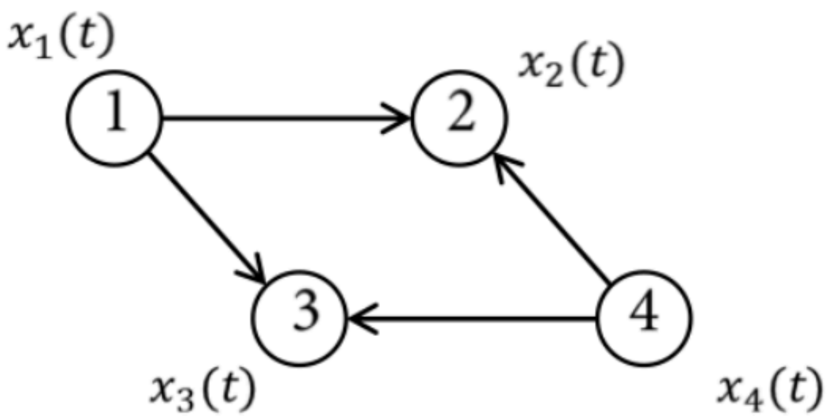
Linear time invariant (LTI) systems

Most basic starting point:
Linear Time Invariant (LTI) system

Discrete time considered here, since this is easier:

Linear time-invariant system:

$$x(t+1) = \boldsymbol{A}\, x(t)$$



$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

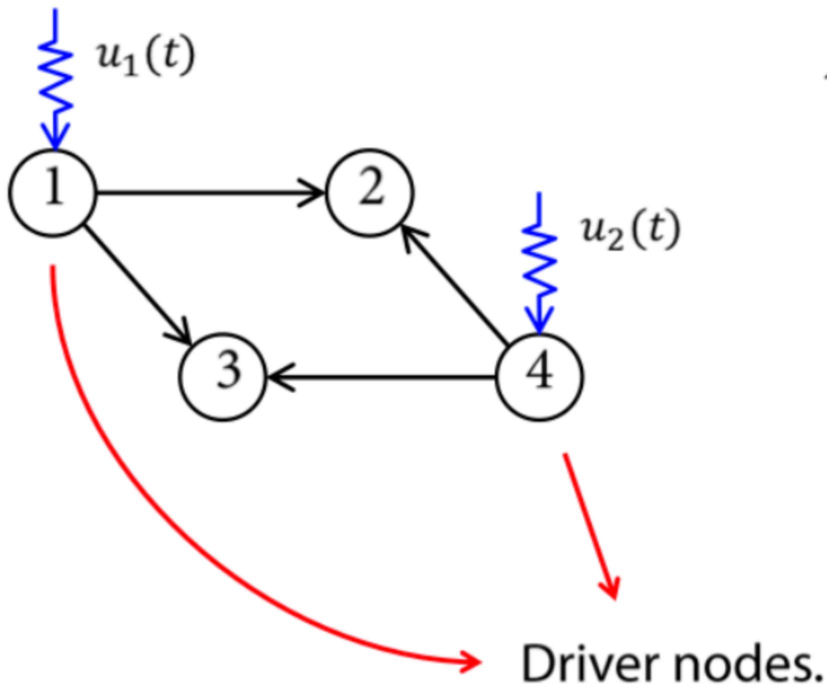
A is an NxN matrix:
It is the adjacency matrix

(Note edges are directed and weighted)

Controlling an LTI system

Linear time-invariant system:

$$x(t+1) = \boldsymbol{A}\, x(t) + \boldsymbol{B}\, u(t)$$



$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & b_2 \end{pmatrix}$$

B is a *MxN* matrix

- *N* nodes
- *M* control signals/driver nodes
- Columns are the unit vector of each driver node

We want to find N_D the minimum number of driver nodes

Controllability matrix

Linear time-invariant system: $x(t+1) = \boldsymbol{A}\,x(t) + \boldsymbol{B}\,u(t)$

$t = 0$: $x(0) = 0$

$t = 1$: $x(1) = \boldsymbol{B}u(0)$

$t = 2$: $x(2) = \boldsymbol{B}u(1) + \boldsymbol{AB}u(0)$

$\vdots$

$t = N$: $x(N) = \boldsymbol{B}u(N-1) + \boldsymbol{AB}u(N-2) + \cdots + \boldsymbol{A}^{N-1}\boldsymbol{B}u(0) = \boldsymbol{C}u$

C is a *MN x N* matrix

Controllability matrix: $\boldsymbol{C} = = [B \quad AB \quad A^2B \quad \ldots \quad A^{n-1}B]$

Kalman's controllability criteria: $\text{rank}\,\boldsymbol{C} = N$

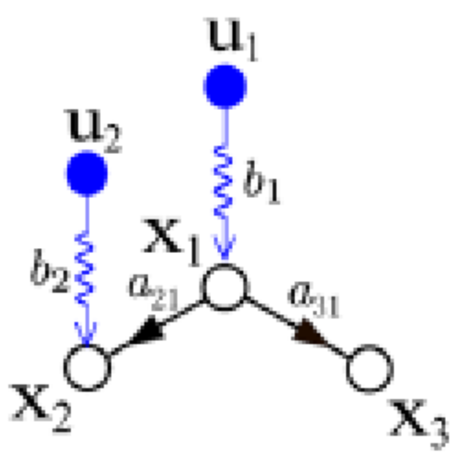
Controllable!

The matrix C is full-row rank (each row linearly independent)

Kalman, *J.S.I.A.M. Control* (1963)

The controllability matrix for a specific (A,B) example

$$\mathcal{C} = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$



$$\mathcal{C} = \begin{bmatrix} b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & a_{21}b_1 & 0 & 0 & 0 \\ 0 & 0 & a_{31}b_1 & 0 & 0 & 0 \end{bmatrix}$$

A matrix 3 x 3
B matrix is 2 x 3
C matrix is (2*3) x 3 = 6 x 3

Kalman rank condition, <u>intuition</u>

Kalman rank condition: full-row rank, rank(C) = N

- N linearly independent rows of C.
- The modes are linearly independent when input signals are constrained to only the nodes in B.

Many other controllability tests, e.g.,

- the Popov-Belevitch-Hautus theorem
- Non-singular Gramian matrix

(equivalent to Kalman)

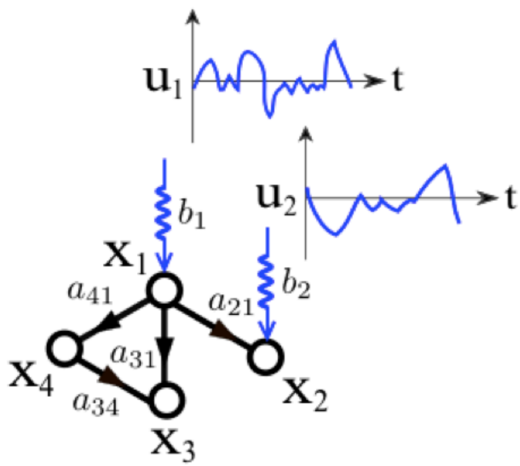
So it is "controllable" but how do we control it?!

The controllability Gramian

$$W_c(t) = \int_{t_0}^{t} e^{A(t-\tau)} BB^* e^{A^*(t-\tau)} d\tau.$$

If and only if the pair (A,B) is controllable, W_c is non-singular, and we can solve for the minimum energy control signal:

$$u(t) = -B^* e^{A^*(t_1-t)} W_c^{-1}(t_1)[e^{A(t_1-t_0)} x_0 - x_1].$$

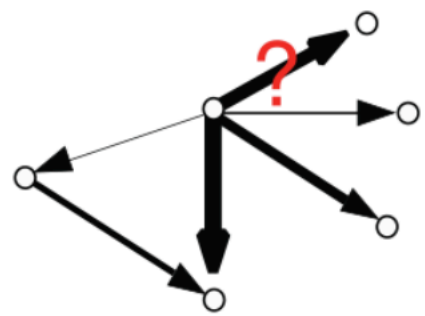


The control energy:

$$\mathcal{E}(T_f) \equiv \int_0^{T_f} \|\mathbf{u}_t\|^2 dt$$

Difficulties

1. Parameters (link weights): usually unknown.



2. If brute-force search: $(2^N\text{-}1)$ combinations. (Search all possible driver nodes)

3. Kalman's rank condition is hard to check for large system.

$$\text{rank}\,\mathbf{C} = N$$

$$\mathbf{C} = [\mathbf{B},\ \mathbf{A}\times\mathbf{B},\ \mathbf{A}^2\times\mathbf{B},\cdots,\ \mathbf{A}^{N-1}\times\mathbf{B}]\ \text{has dimension}\ N \times NM.$$

Solution: Topological (purely structural) considerations

Exploit the deep connections with linear algebra

- **Structural control** solved via **maximum matching**
  - Know which adjacency matrix elements are non-zero, but do not know the values ("weights") of the non-zero entries
  - Impact of degree distribution
  - "Control profiles": source, sinks and dilations

- "**Exact controllability**"
  - Know the exact network structure (edge weights)
  - Key consideration: the maximum multiplicity eigenvalues (trying to break up symmetries)
  - Energy of control (also, how quickly do you need it done?)

Thanks to collaborators for slides on structural control

REVIEWS OF MODERN PHYSICS, VOLUME 88, JULY–SEPTEMBER 2016

Control principles of complex systems

Yang-Yu Liu

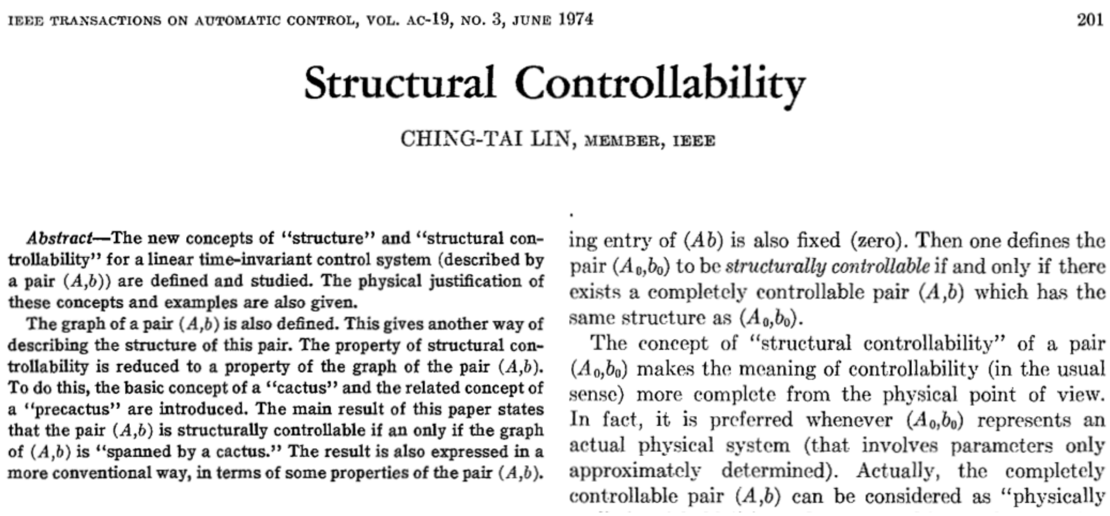
Channing Division of Network Medicine, Brigham and Women's Hospital, Harvard Medical School, Boston, Massachusetts 02115, USA and Center for Cancer Systems Biology, Dana-Farber Cancer Institute, Boston, Massachusetts 02115, USA

Albert-László Barabási

Center for Complex Network Research and Departments of Physics, Computer Science and Biology, Northeastern University, Boston, Massachusetts 02115, USA, Department of Medicine, Brigham and Women's Hospital, Harvard Medical School, Boston, Massachusetts 02115, USA, and Center for Network Science, Central European University, Budapest 1052, Hungary

(published 6 September 2016)

A solution: structural control (1974) meets maximum matching (2011)



IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-19, NO. 3, JUNE 1974 201

Structural Controllability

CHING-TAI LIN, MEMBER, IEEE

Abstract—The new concepts of "structure" and "structural controllability" for a linear time-invariant control system (described by a pair (A,b)) are defined and studied. The physical justification of these concepts and examples are also given.

The graph of a pair (A,b) is also defined. This gives another way of describing the structure of this pair. The property of structural controllability is reduced to a property of the graph of the pair (A,b). To do this, the basic concept of a "cactus" and the related concept of a "precactus" are introduced. The main result of this paper states that the pair (A,b) is structurally controllable if an only if the graph of (A,b) is "spanned by a cactus." The result is also expressed in a more conventional way, in terms of some properties of the pair (A,b).

ing entry of (Ab) is also fixed (zero). Then one defines the pair (A_0,b_0) to be *structurally controllable* if and only if there exists a completely controllable pair (A,b) which has the same structure as (A_0,b_0).

The concept of "structural controllability" of a pair (A_0,b_0) makes the meaning of controllability (in the usual sense) more complete from the physical point of view. In fact, it is preferred whenever (A_0,b_0) represents an actual physical system (that involves parameters only approximately determined). Actually, the completely controllable pair (A,b) can be considered as "physically

C-T Lin, *IEEE Trans on Automatic Control*, 1974

Y.-Y, Liu, A.L. Barabasi, J.-J. Slotine
"Controllability of complex networks"
Nature 2011.

Structured system

We treat the nonzero elements in $\boldsymbol{A}$ and $\boldsymbol{B}$ as free parameters, and we keep the zero entries fixed.

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \boldsymbol{A}^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a^*_{21} & 0 & a^*_{23} & a^*_{24} \\ a^*_{31} & 0 & 0 & a^*_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Lin, *IEEE Transactions on Automatic Control* (1974)

Structurally controllable

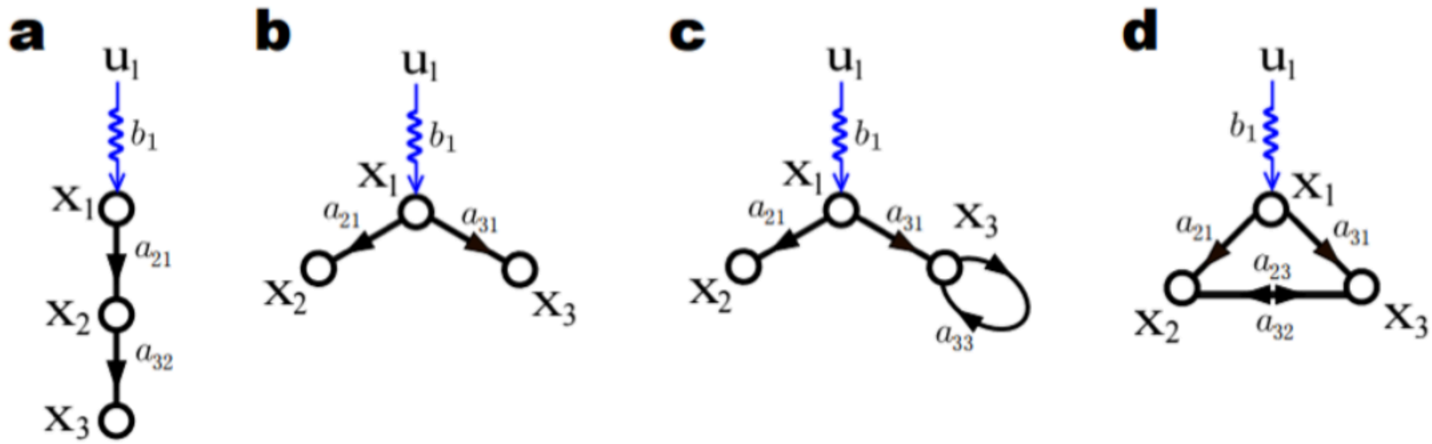
A system ($\boldsymbol{A}$,$\boldsymbol{B}$) is structurally controllable, if there exists ($\boldsymbol{A}^*$,$\boldsymbol{B}^*$) that is controllable in the original sense.

If a system is structurally controllable, then either it is controllable or it will become controllable after slight change of certain links' weights, and remains controllable for possibly large parameter variations.

Recall considering an LTI system

Lin, *IEEE Transactions on Automatic Control* (1974)

Examples

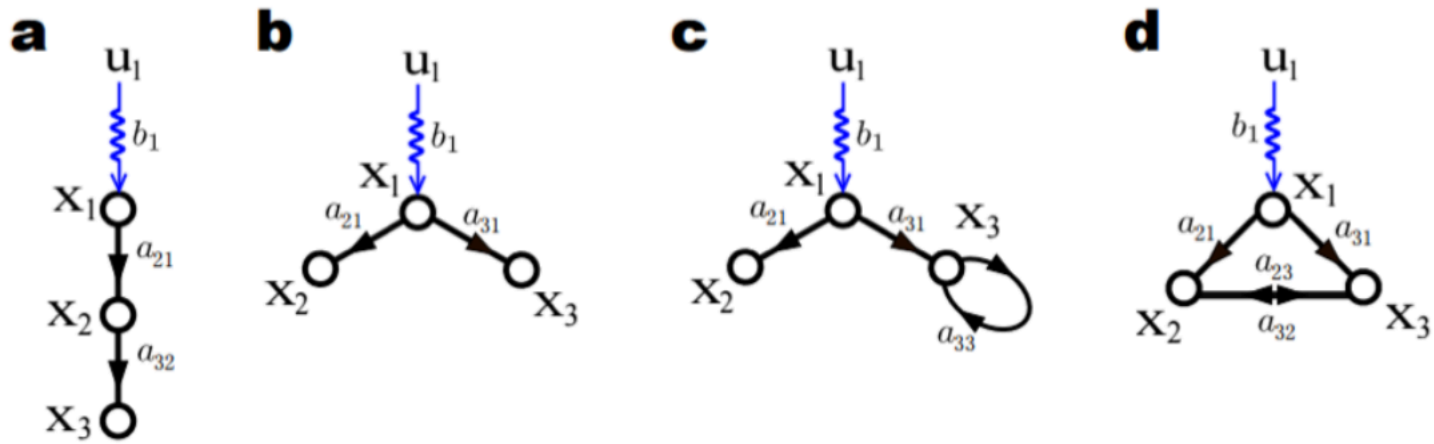


$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & a_{33} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} b_1 \\ 0 \\ 0 \end{pmatrix}$$

Examples

Loops self-regulate:



$$\mathbf{C} = [\mathbf{B},\ \mathbf{A}\cdot\mathbf{B},\ \mathbf{A}^2\cdot\mathbf{B}]$$

$$b_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & 0 & a_{32}a_{21} \end{pmatrix},\quad b_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{pmatrix},\quad b_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & a_{33}a_{31} \end{pmatrix},\quad b_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{pmatrix}$$

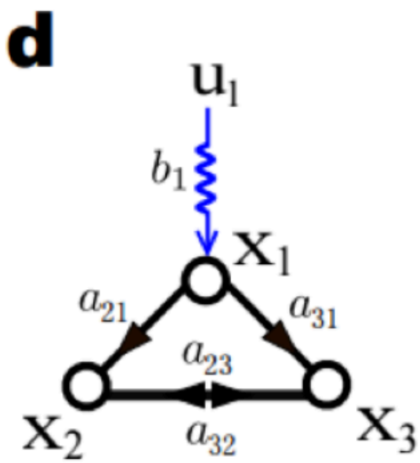
rank **C** = N=3
controllable

rank **C**=2 < N=3
uncontrollable

rank **C** = N=3
controllable

rank **C** = ?
controllable ?

Examples



$$\mathbf{C} = b_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{pmatrix}.$$

If $\begin{pmatrix} a_{21} \\ a_{31} \end{pmatrix} \propto \begin{pmatrix} a_{23}a_{31} \\ a_{32}a_{21} \end{pmatrix}$, e.g. $a_{32}a_{21}^2 = a_{23}a_{31}^2$, then rank $\mathbf{C} = 2 < N \Rightarrow$ uncontrollable!

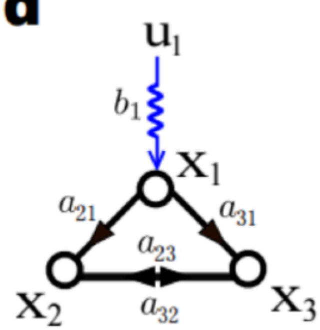
However, this case is pathological. In most cases, the system is controllable.

<u>Weak</u> versus <u>strong</u> structural control

- Strong structural control --- any assignment of the non-zero link weights yields a controllable network. (Mayeda, Yamada, SIAM 1979.)

- Weak structural control --- there exists an assignment of weights to the non-zero links that yields a controllable network.

d



$$\mathbf{C} = b_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{pmatrix}.$$

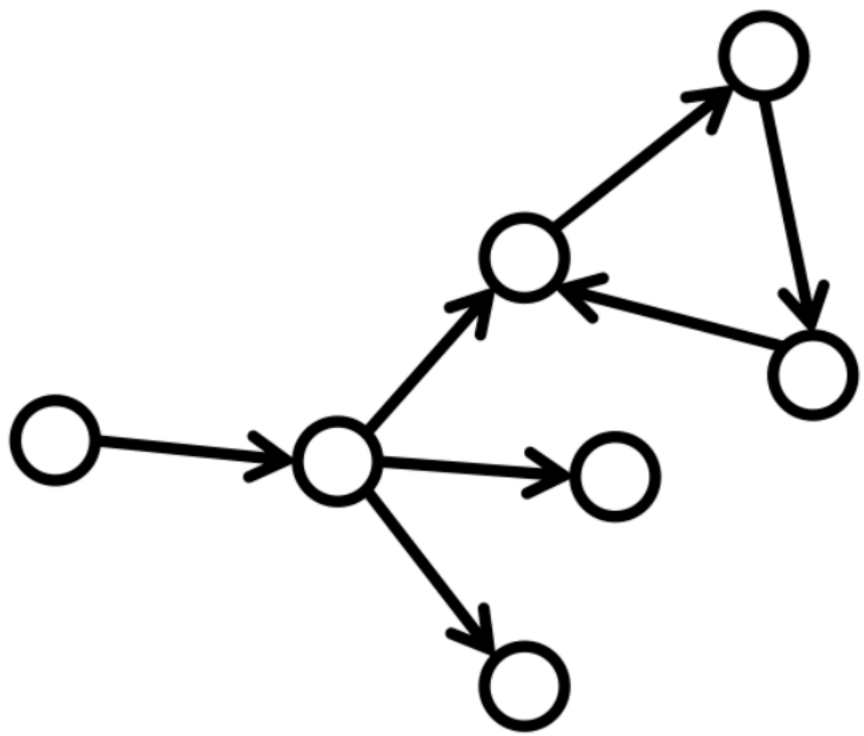
"In most cases the system is controllable"

- For uncorrelated edge weights, the set of uncontrollable weight assignments typically form a set of measure zero. (But important instances, like all equal weights!)

Why structural controllability?

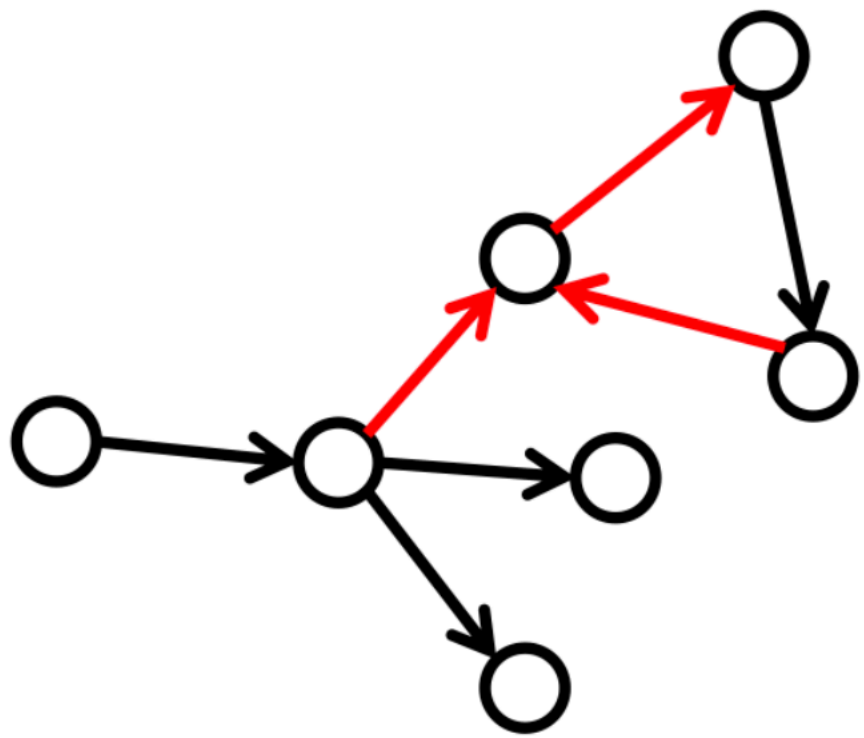
Matching in Directed Networks

Subset of links that do not share start or end points.



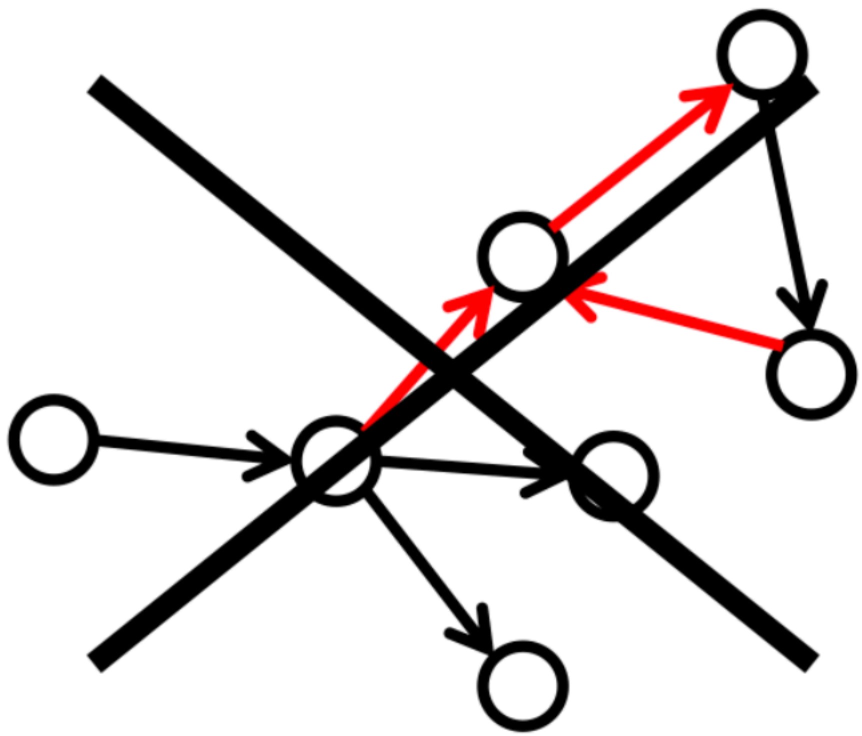
Matching in Directed Networks

Subset of links that do not share start or end points. (i.e. nodes)



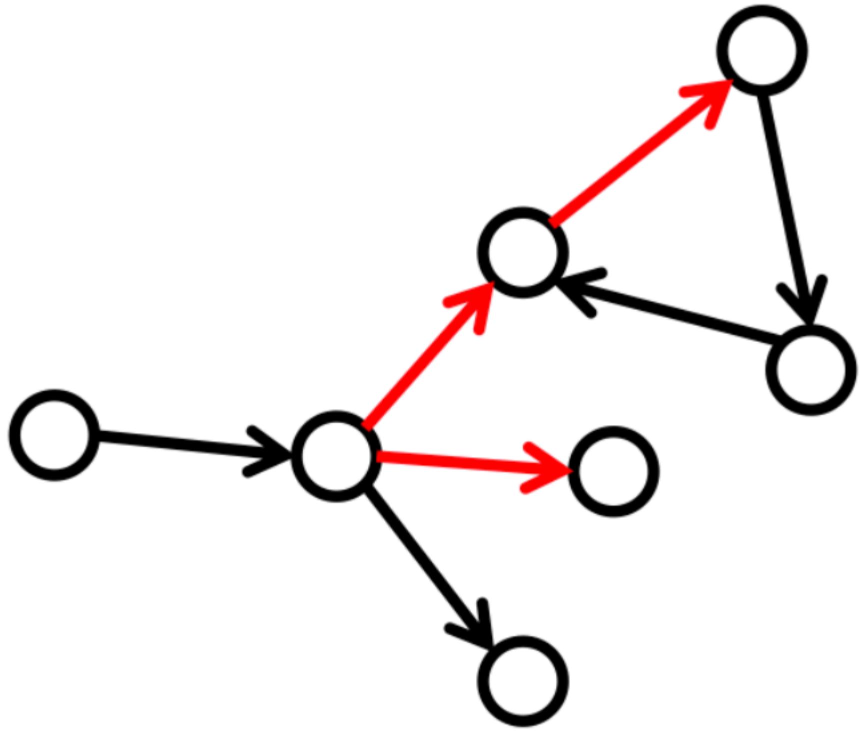
Matching in Directed Networks

Subset of links that do not share start or end points.



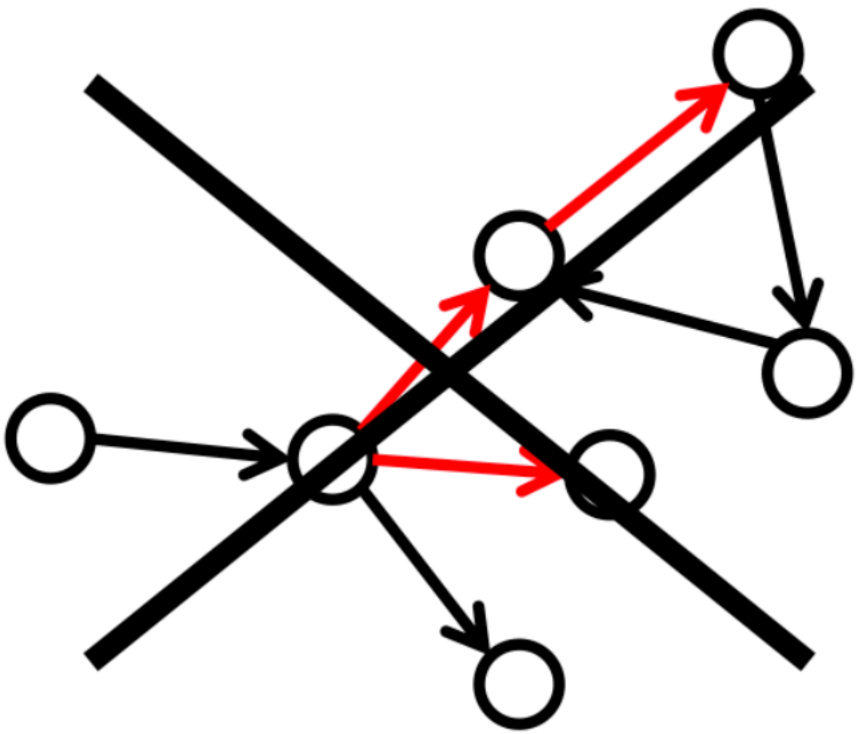
Matching in Directed Networks

Subset of links that do not share start or end points.



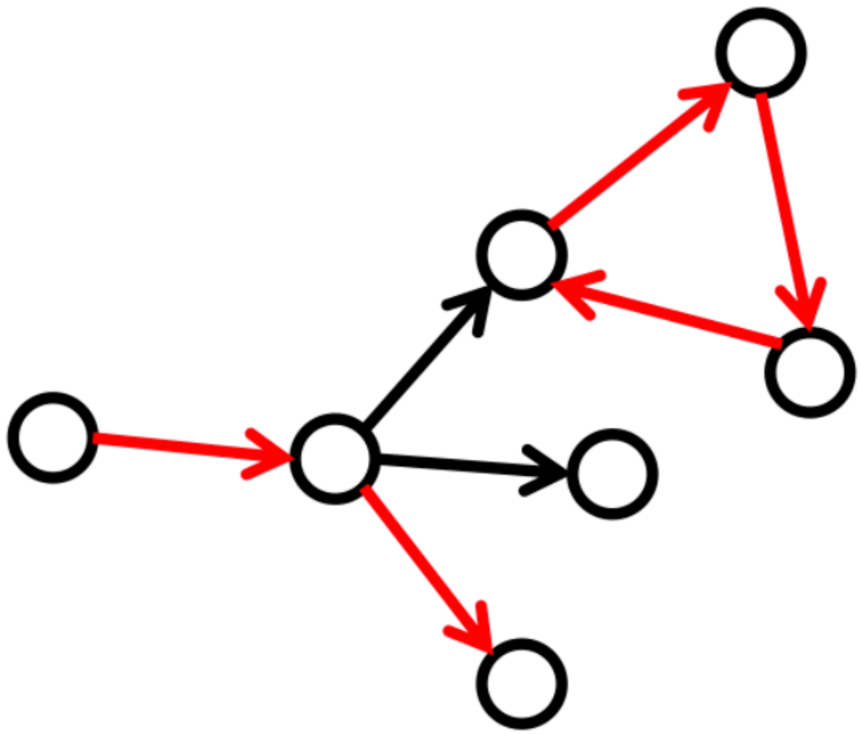
Matching in Directed Networks

Subset of links that do not share start or end points.



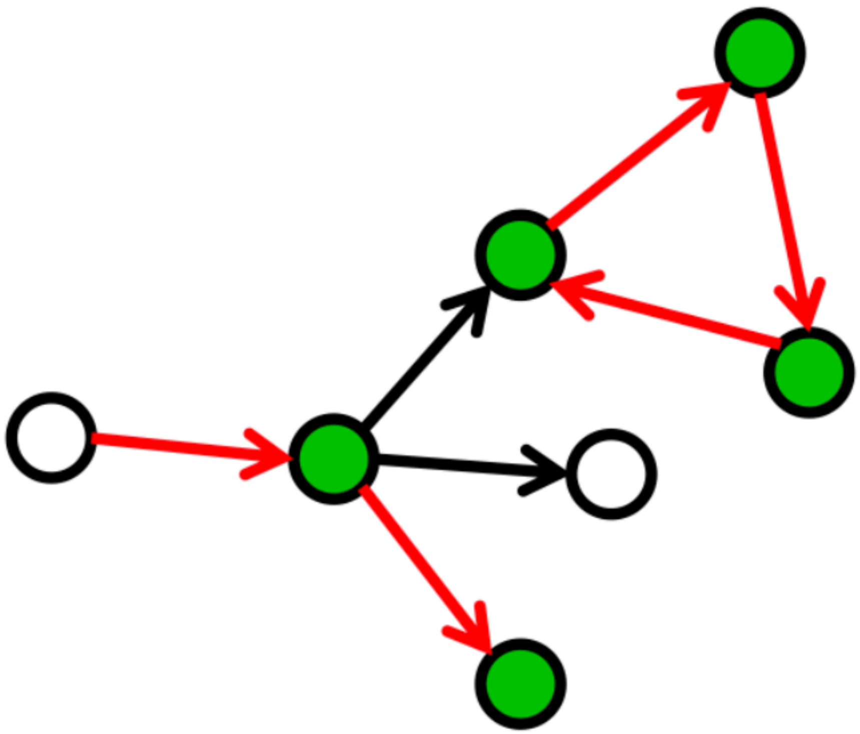
Matching in Directed Networks

Subset of links that do not share start or end points.



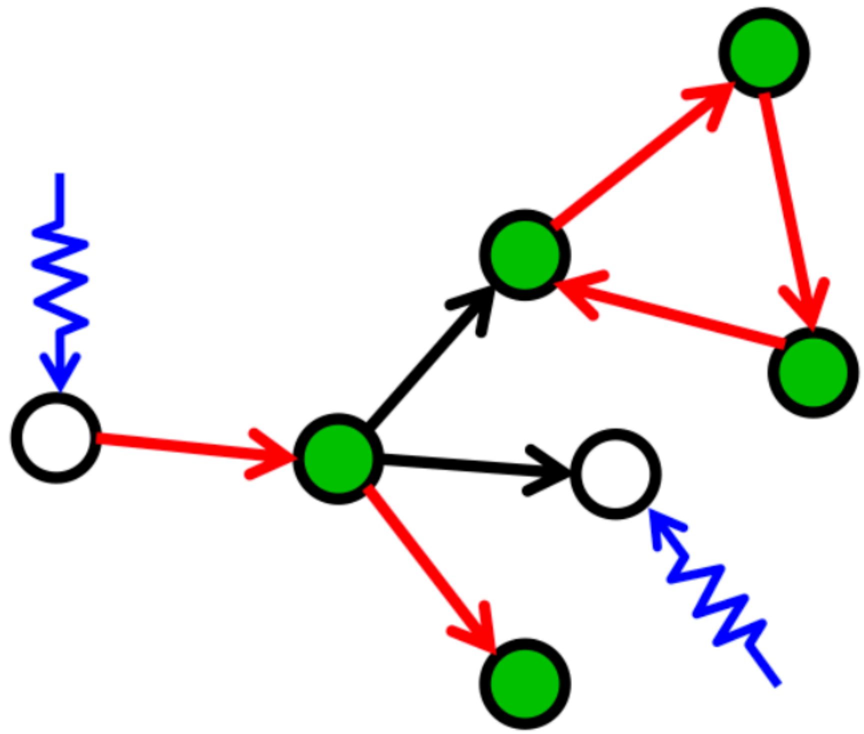
Matching in Directed Networks

Matched nodes: nodes that have links in the matching pointing at them.



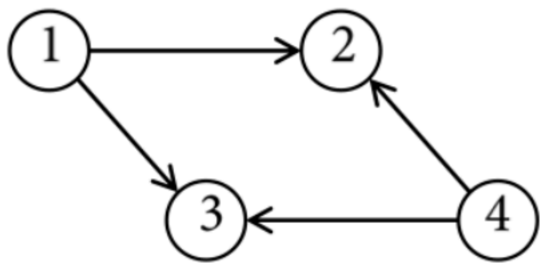
Matching in Directed Networks

The unmatched nodes are the driver nodes.



GOOD: algorithms and analytical tools!

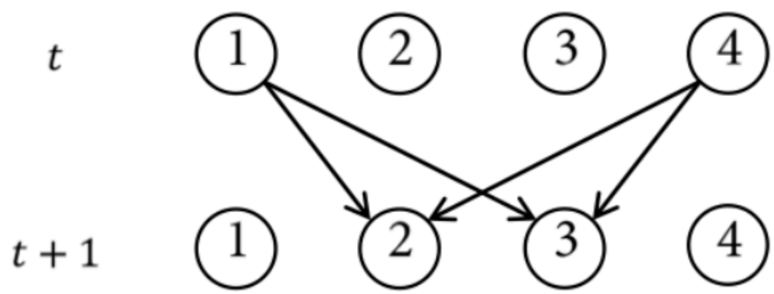
Mapping to maximum matching



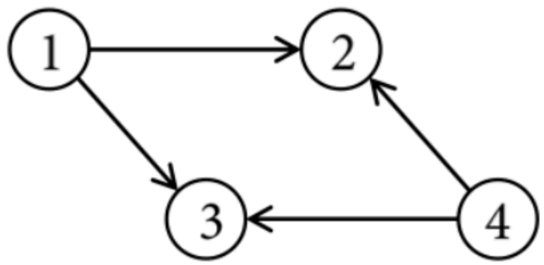
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

State of nodes at $t+1$ is completely determined by the state of neighbors at t.

Bi-partite representation



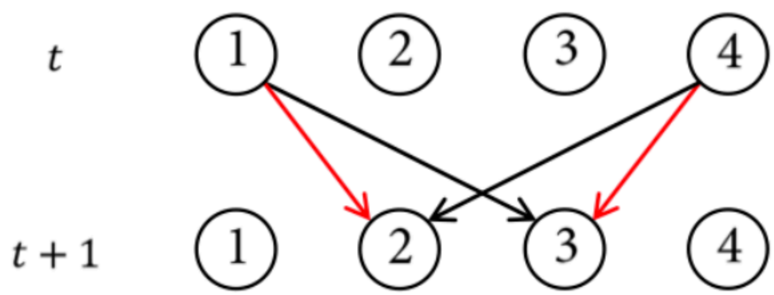
Mapping to maximum matching



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

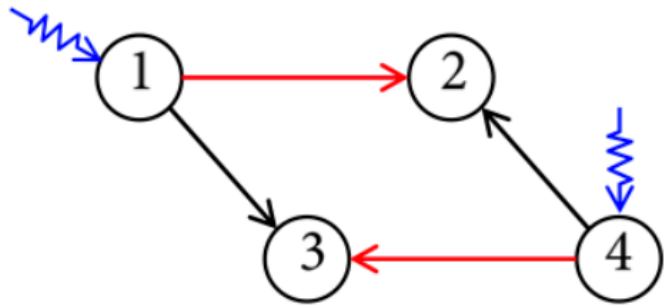
If the network is controllable at t, it is controllable at $t+1$.

(Note, we could have chosen the other matching of 1-3 and 4-2)



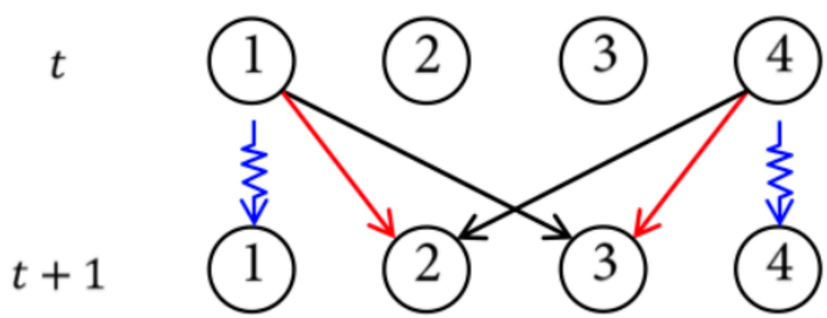
Matching: a set of links that do not share endpoints.

Mapping to maximum matching



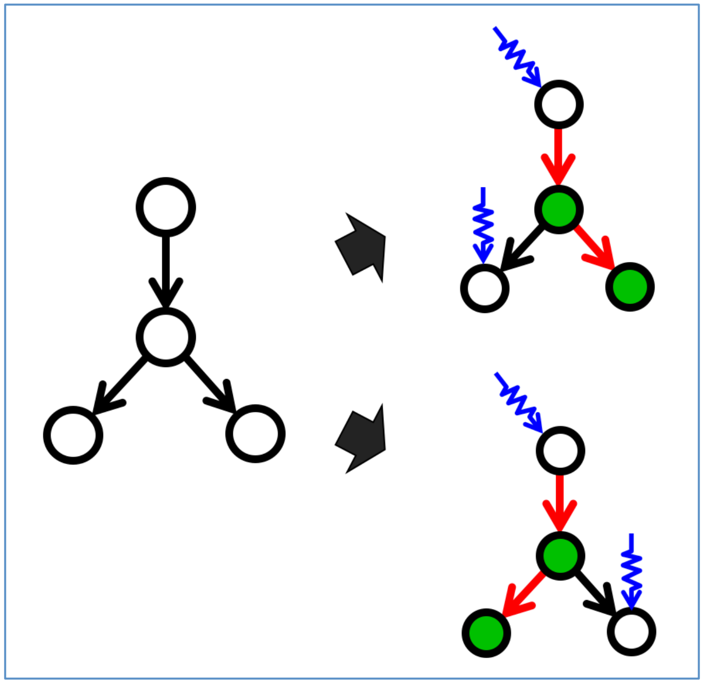
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

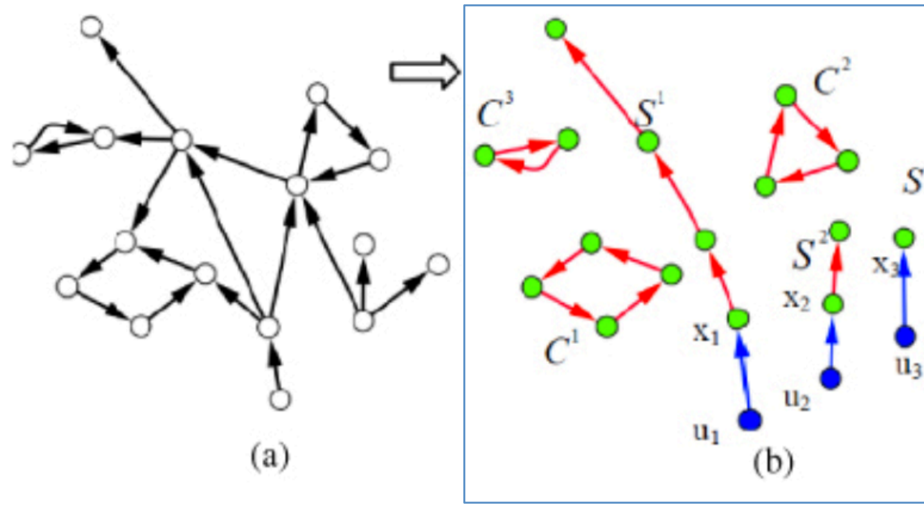
We have to control the **unmatched nodes**.



Maximum matching: finds the *value* of N_D but not the *identity* of N_D

Typically many ways to assign who are the N_D driver nodes





A matching is equivalent to finding a set of "cactus" made of stems and cycles

Vocabulary: "driver node" = control signal

"actuator" = a node driven by the control signal

(The subtlety: the same control signal can drive multiple actuators.)

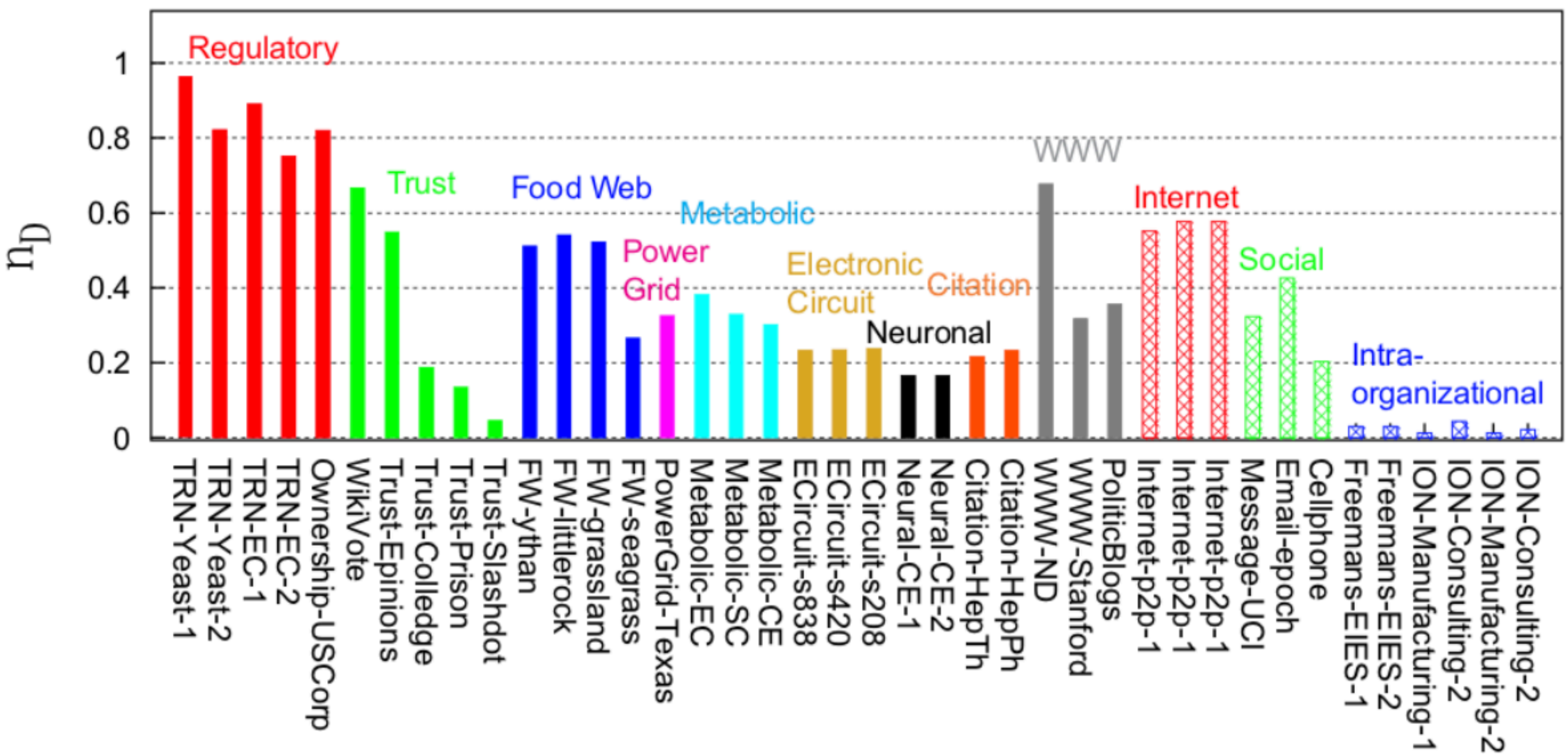
More general questions

- What type of networks are easy/hard to control?
- What properties of networks influence control?
- What characterizes driver nodes?

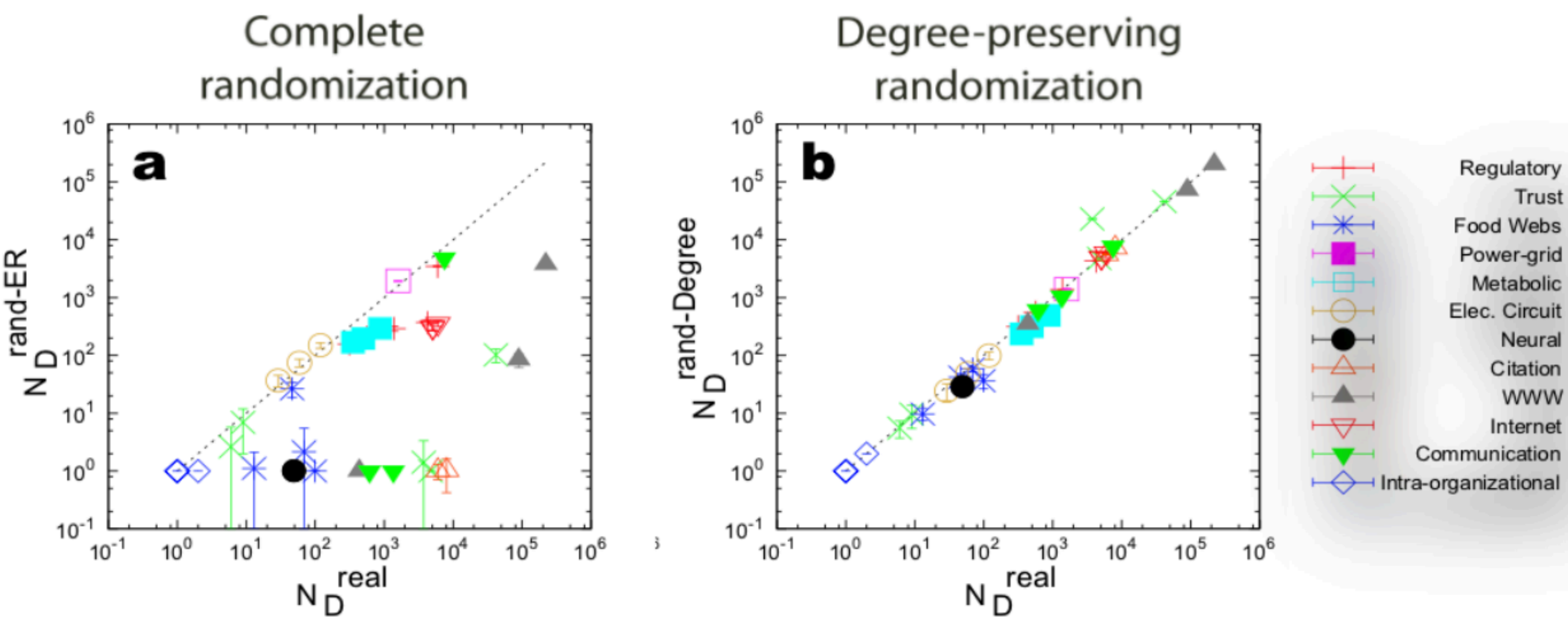
1. Systematic measurements in real networks.
2. Generate model networks with certain properties.

The fraction of driver nodes $n_D = N_D / N$

n_D for real networks

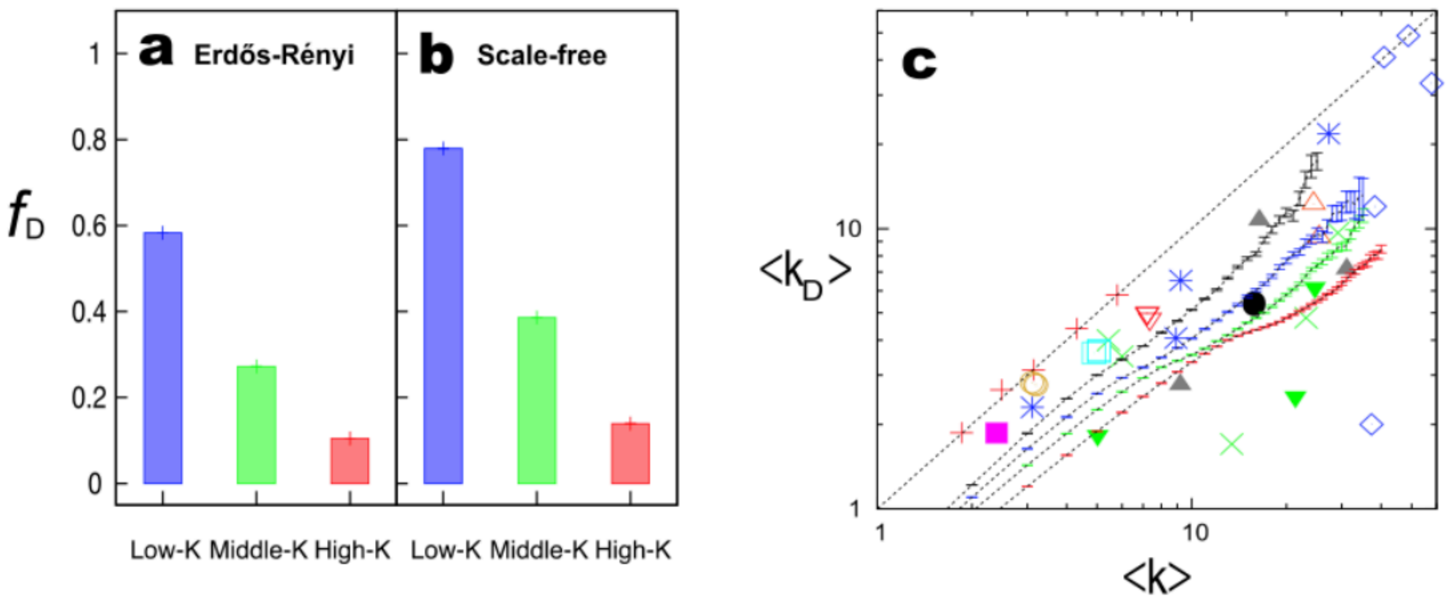


Randomization of real networks



Controllability is determined by the degree sequence.
But how?

Heterogeneous networks are harder to control.



Broad-scale degree distributions have N_D much larger then narrow-scale network of same size N.

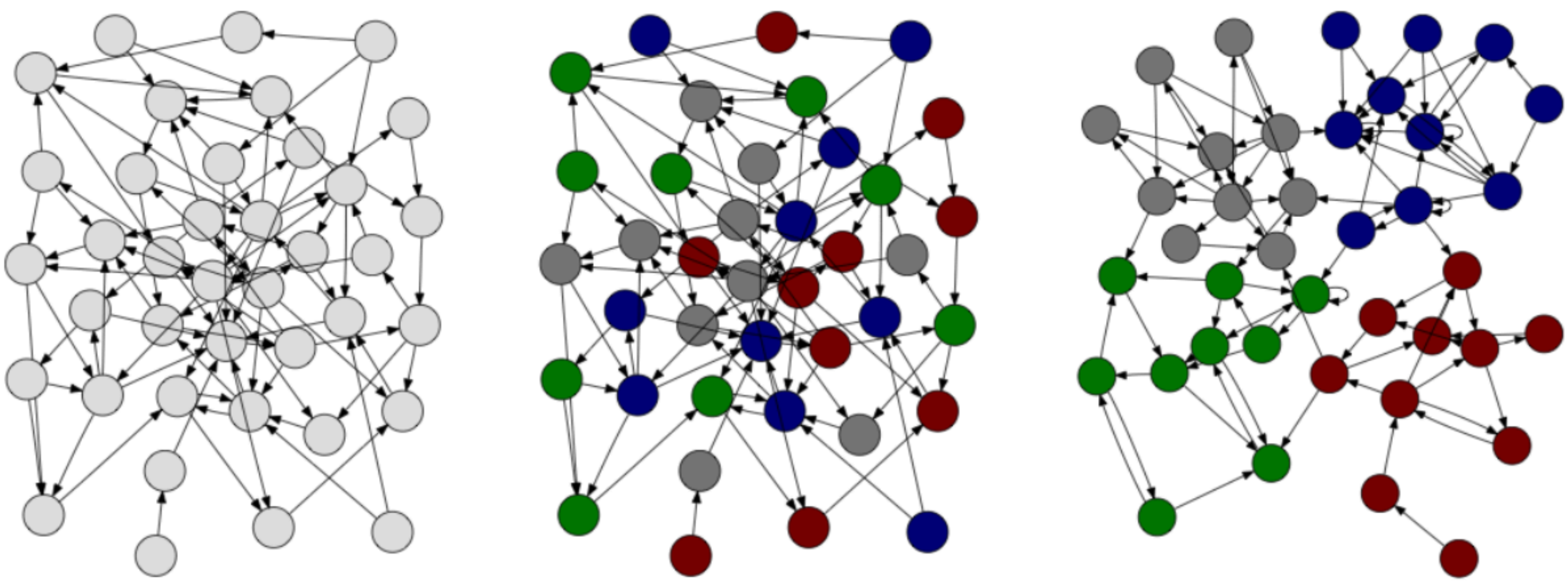
Driver nodes tend to avoid hubs.

(Hubs have so many links, they are almost always in the matching)

Does modularity (community structure) matter?

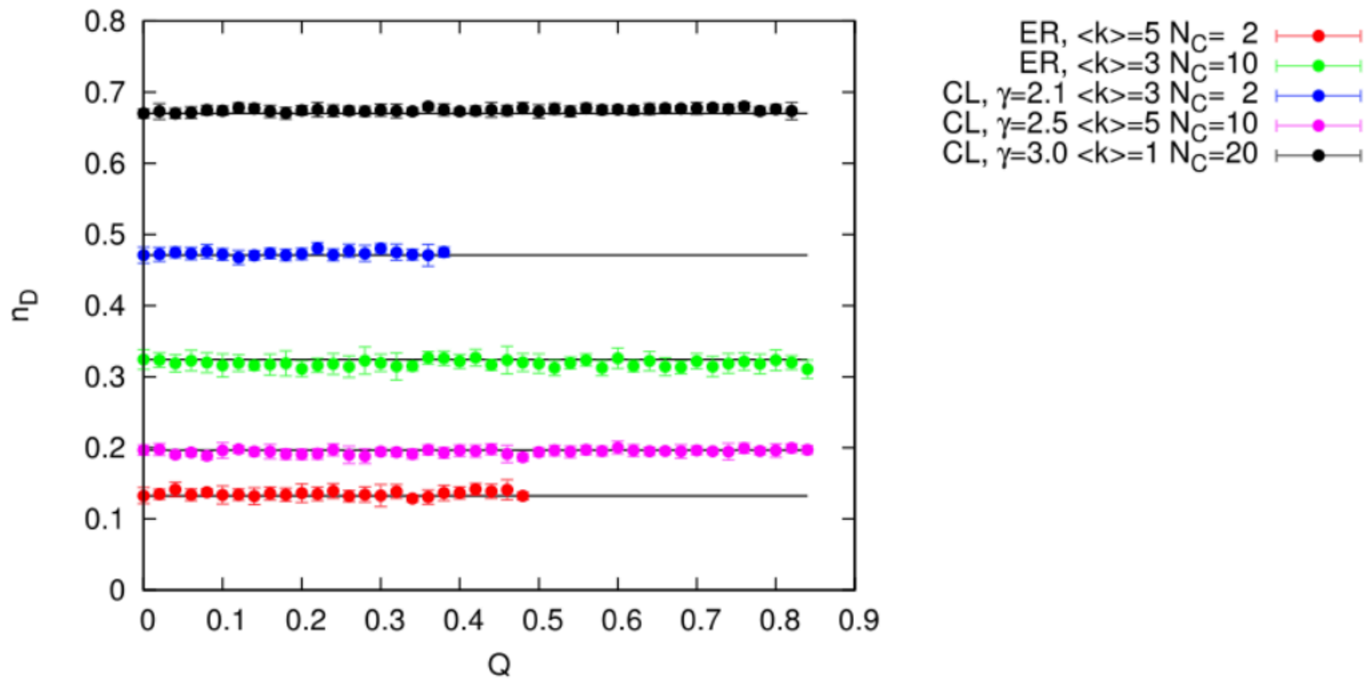
Modularity

$$Q = \frac{1}{E}\sum_{ij}\left[A_{ij} - \frac{k_i^{in}k_j^{out}}{E}\right]\delta_{c_i,c_j}$$



Community structure

$$Q = \frac{1}{E} \sum_{ij} \left[A_{ij} - \frac{k_i^{in} k_j^{out}}{E} \right] \delta_{c_i, c_j}$$



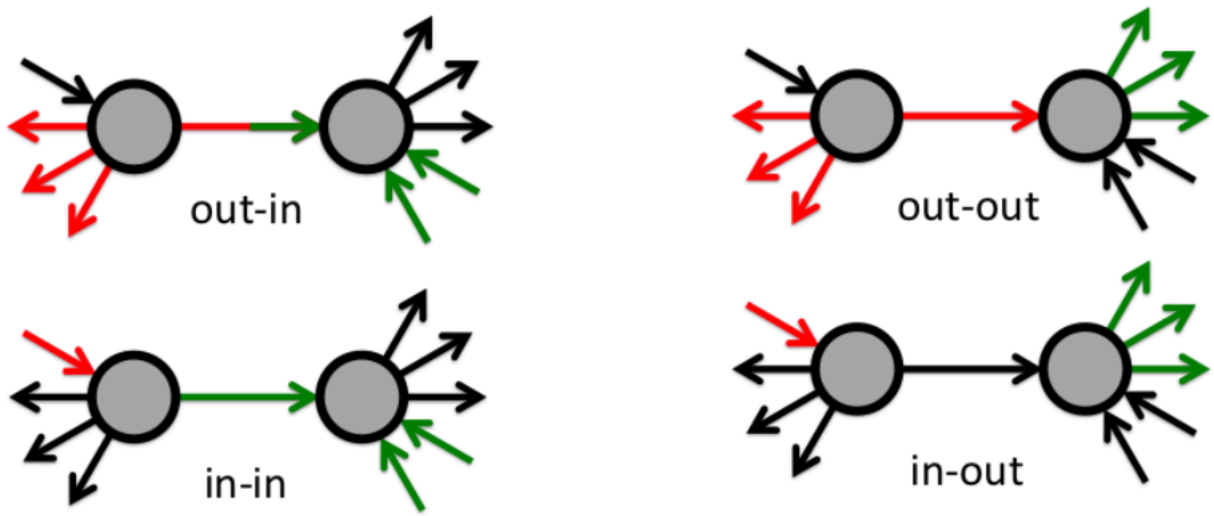
No effect found.

Modularity (community structure) does not matter!

Do correlations matter?

Degree-degree correlations

- Correlations between the degrees of connected node pairs.
- Directed networks have 4 types of degree correlations:

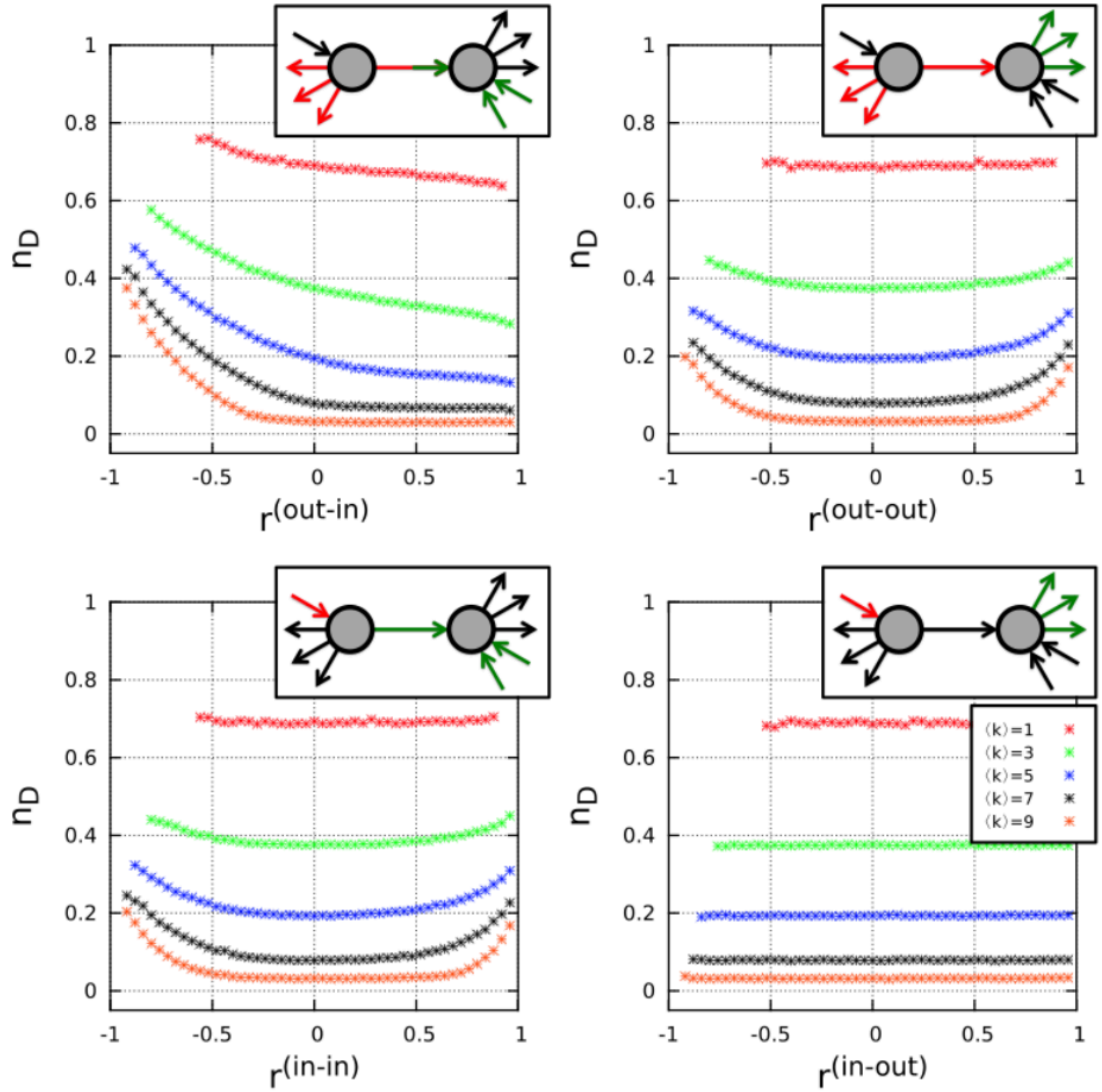


- Quantified by their Pearson coefficient:

$$r_{(\alpha,\beta)} = \frac{\frac{1}{E}\sum_e \left(k_e^{(\alpha)} - \overline{k^{(\alpha)}}\right)\left(j_e^{(\beta)} - \overline{j^{(\beta)}}\right)}{\sigma^{(\alpha)}\sigma^{(\beta)}} \qquad \alpha, \beta \in \{\text{in}, \text{out}\}$$

Simulations

SF, γ=2.5

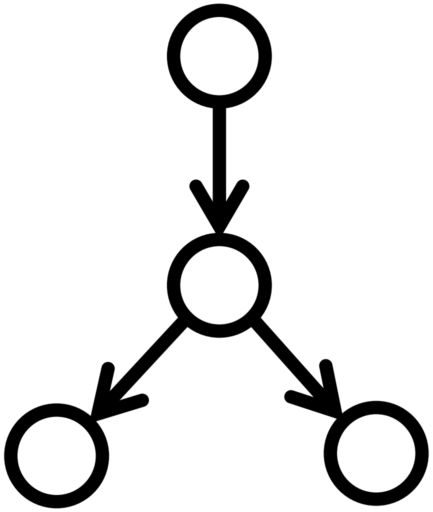


Yes, degree-degree correlations matter!

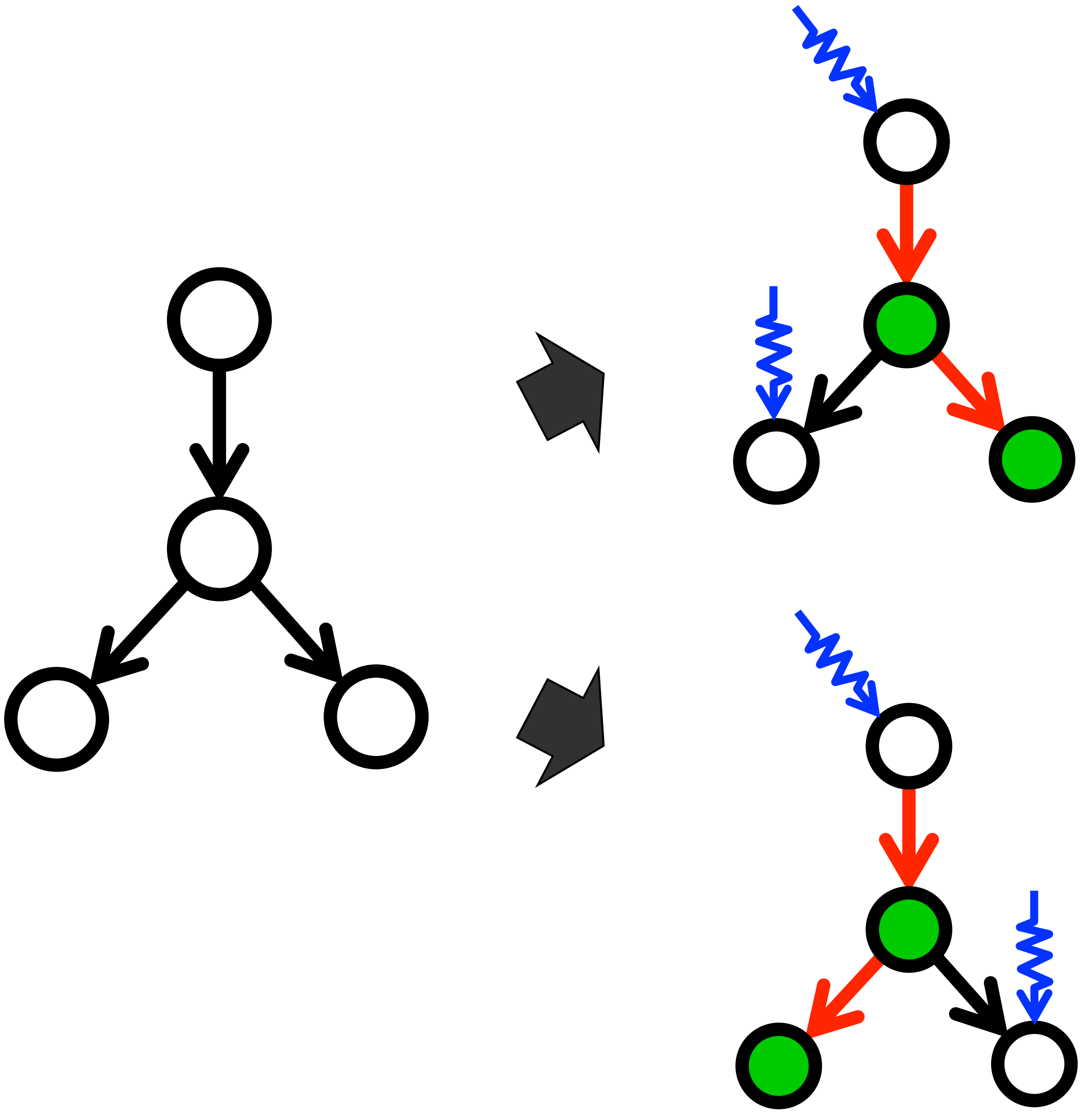
- Pósfai, M., Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2013). Effect of correlations on network controllability. *Scientific Reports*, *3*, 1067.

Beyond driver nodes

- Control node categories

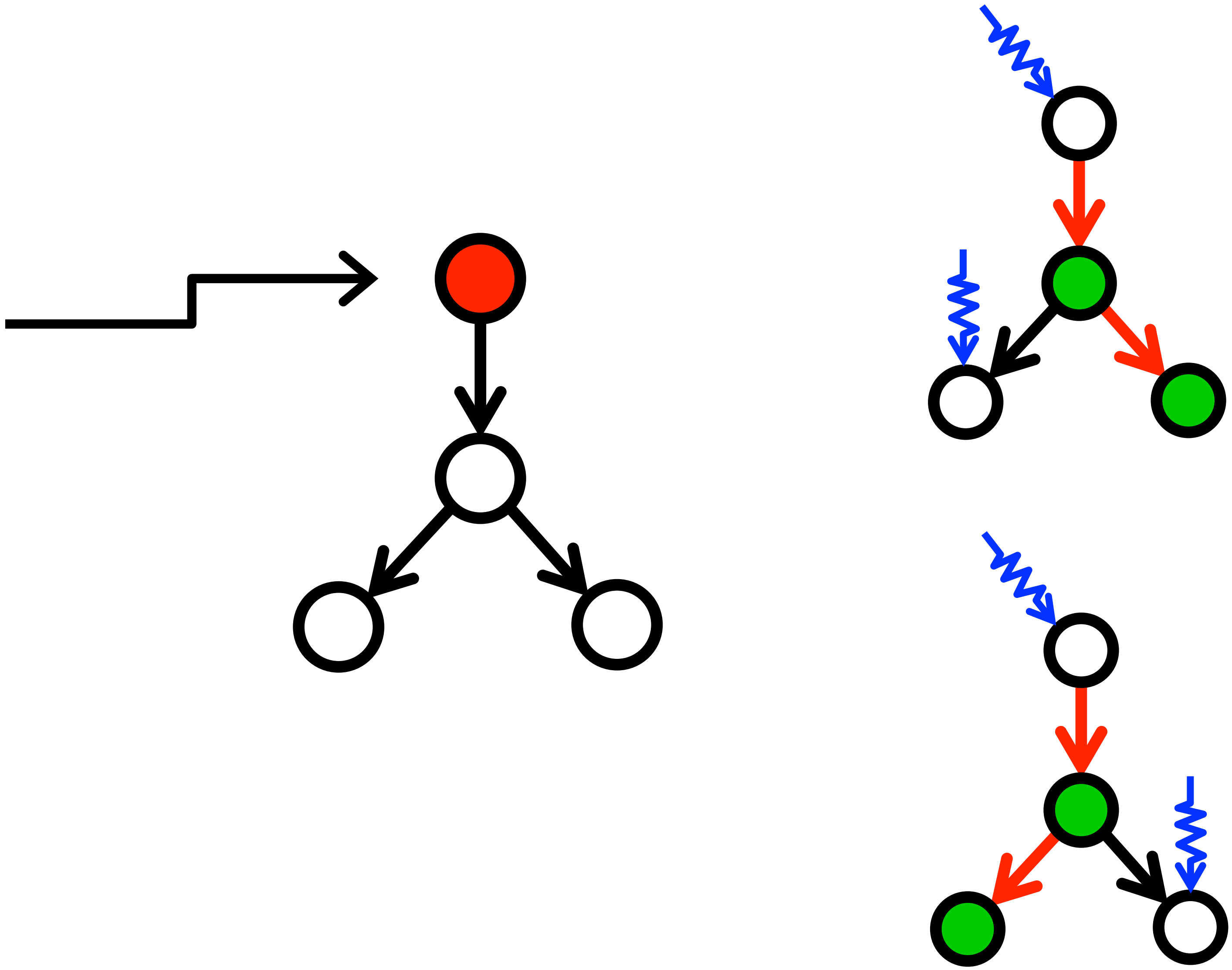


Control Node Categories



Control Node Categories

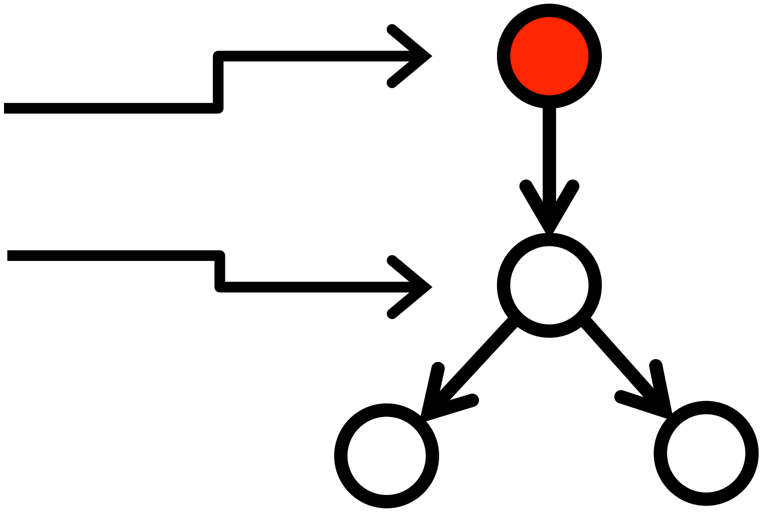
Critical
Always driver.

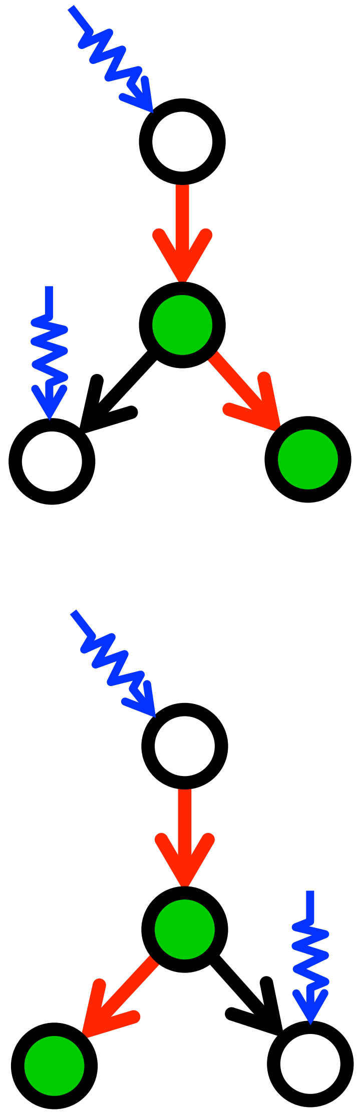


Control Node Categories

Critical
Always driver.

Redundant
Never driver.



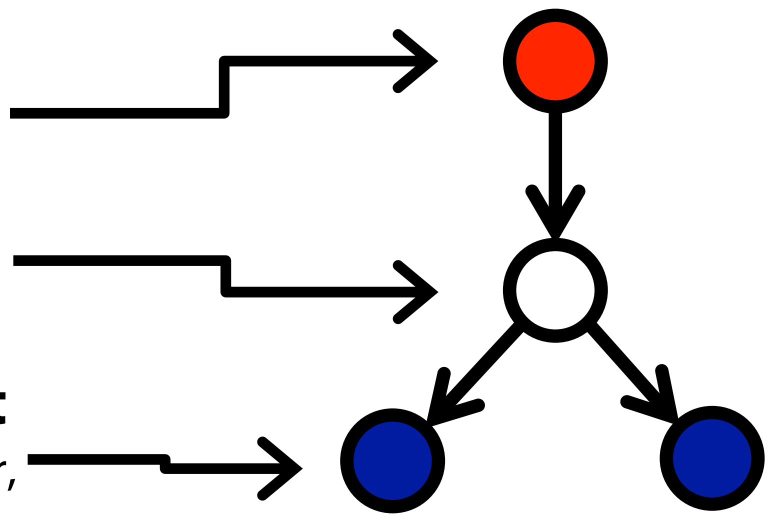


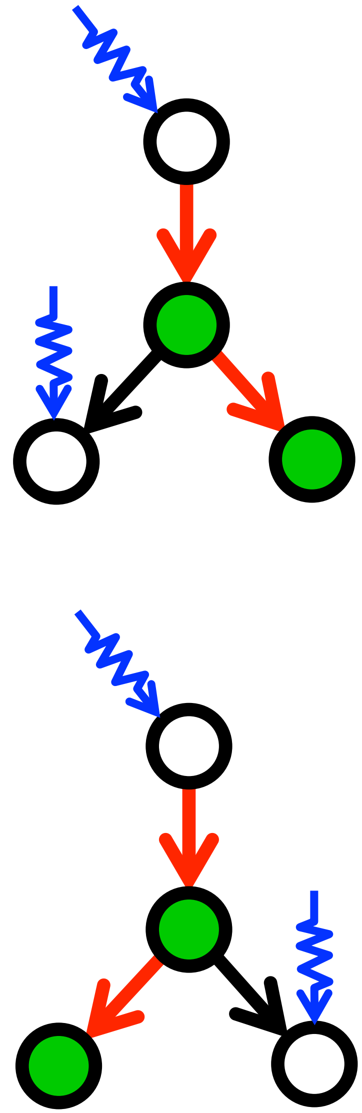
Control Node Categories

Critical
Always driver.

Redundant
Never driver.

Intermittent
Sometimes driver,
sometimes not.



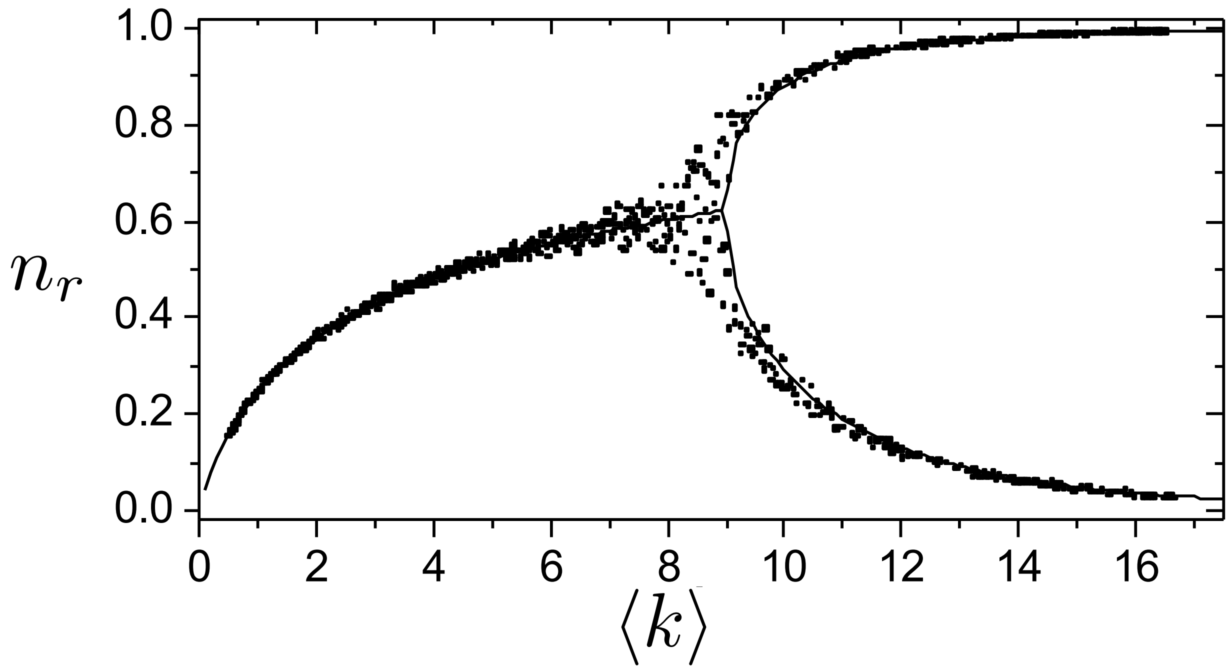


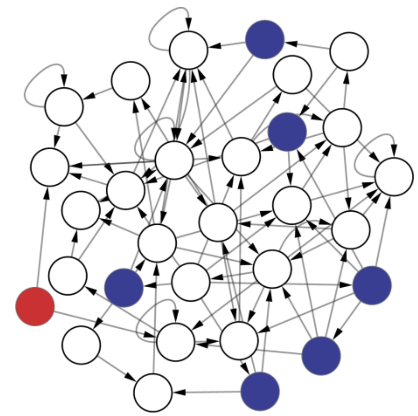
A node is critical iff it has no incoming links.

Redundant Nodes

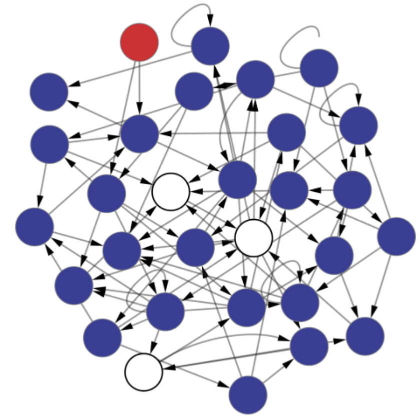
Emergence of bi-modality

SF network with increasing average degree.





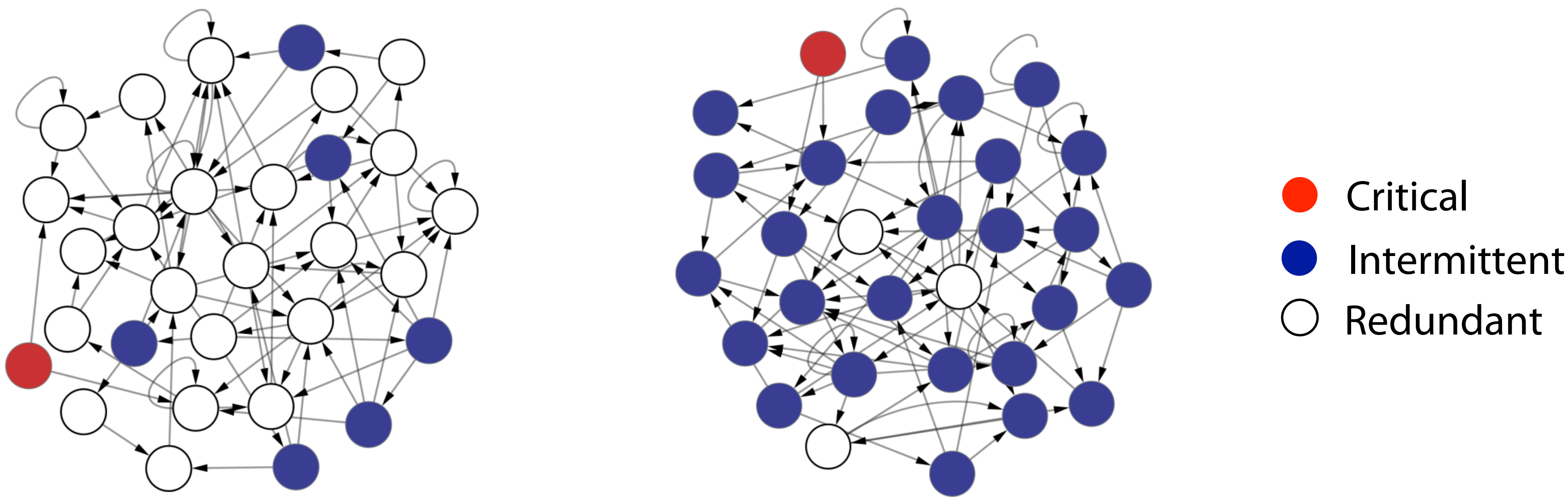
Centralized control.



Distributed control.

Jia, T., Liu, Y. Y., Csóka, E., Pósfai, M., Slotine, J. J., & Barabási, A. L. (2013). "Emergence of bimodality in controlling complex networks". *Nature Comm., 4*.

Puzzle



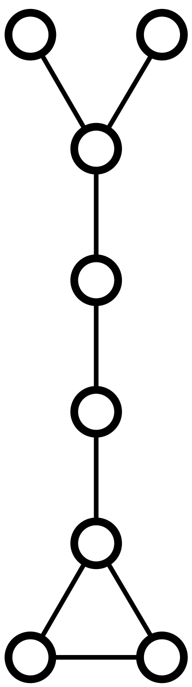
Centralized control. **Distributed control.**

How to explain this behavior? Answer: **Core Percolation**

- Liu, Y. Y., Csóka, E., Zhou, H., & Pósfai, M. (2012). "Core percolation on complex networks". *Phys. Rev. Lett.*, *109* (20), 205703.
- Jia, T., & Pósfai, M. (2014). Connecting core percolation and controllability of complex networks. *Scientific reports*, *4*, 5379.

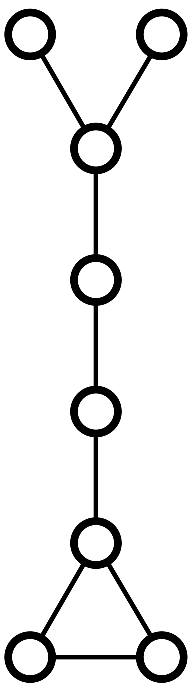
Greedy Leaf Removal

1

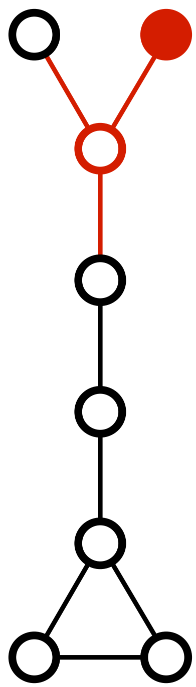


Greedy Leaf Removal

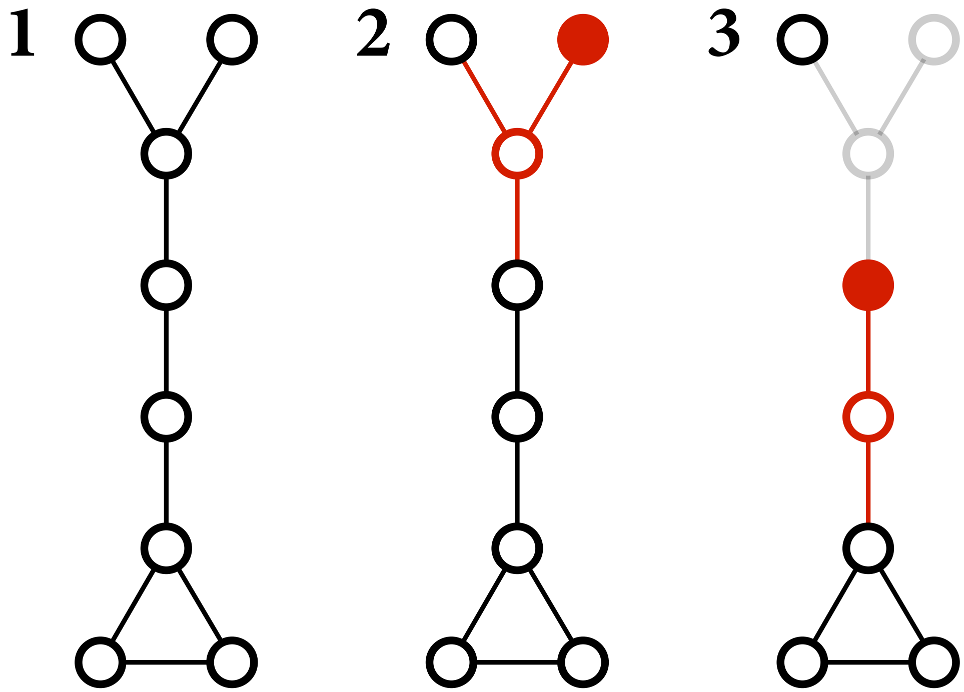
1



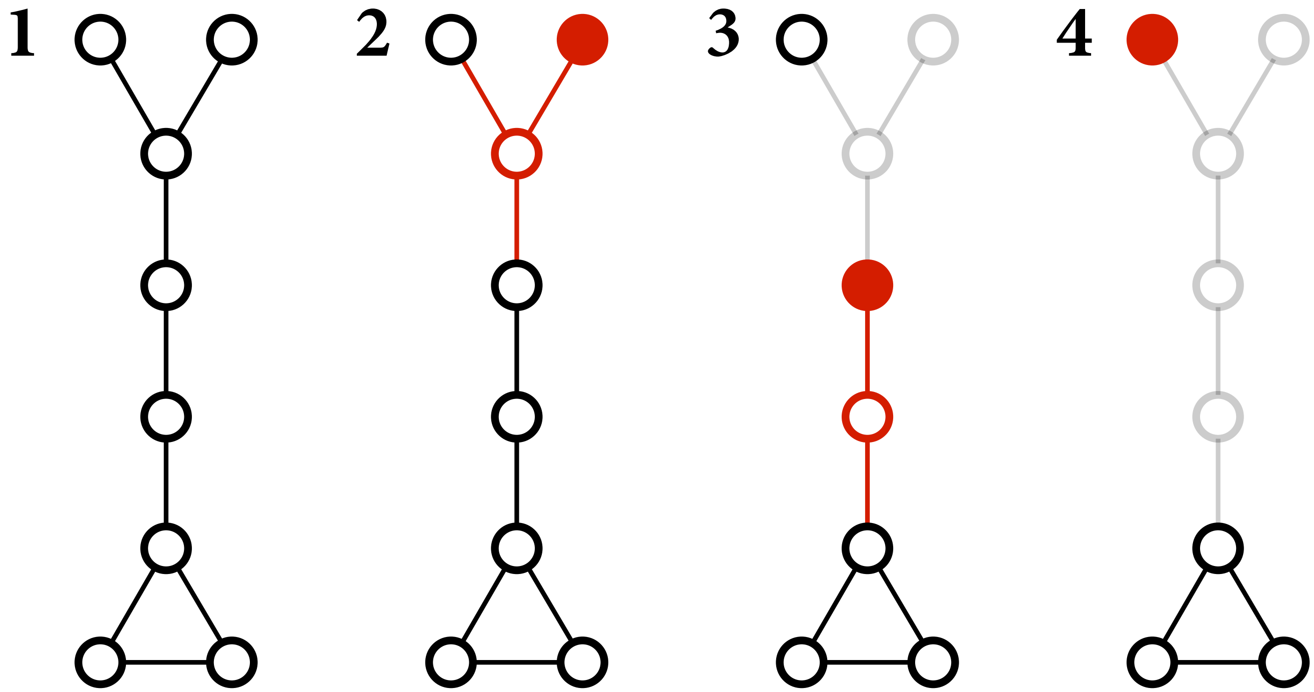
2



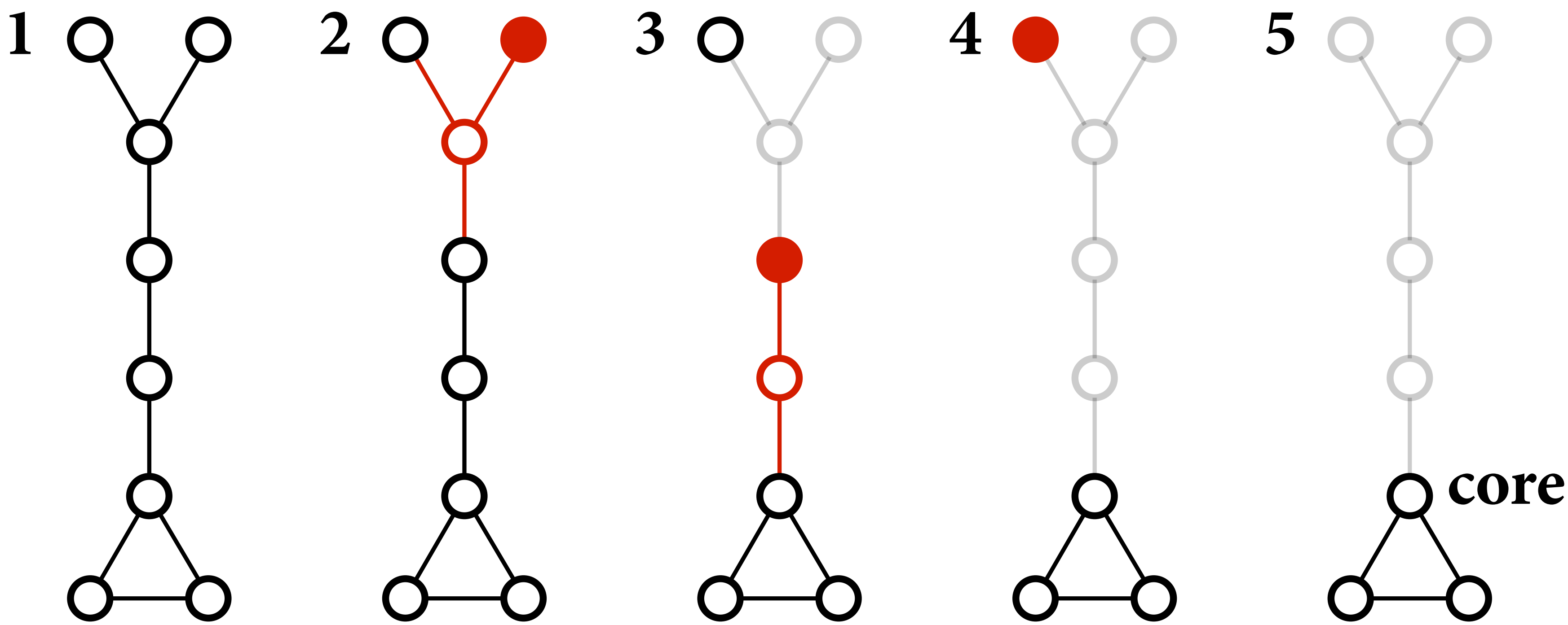
Greedy Leaf Removal



Greedy Leaf Removal

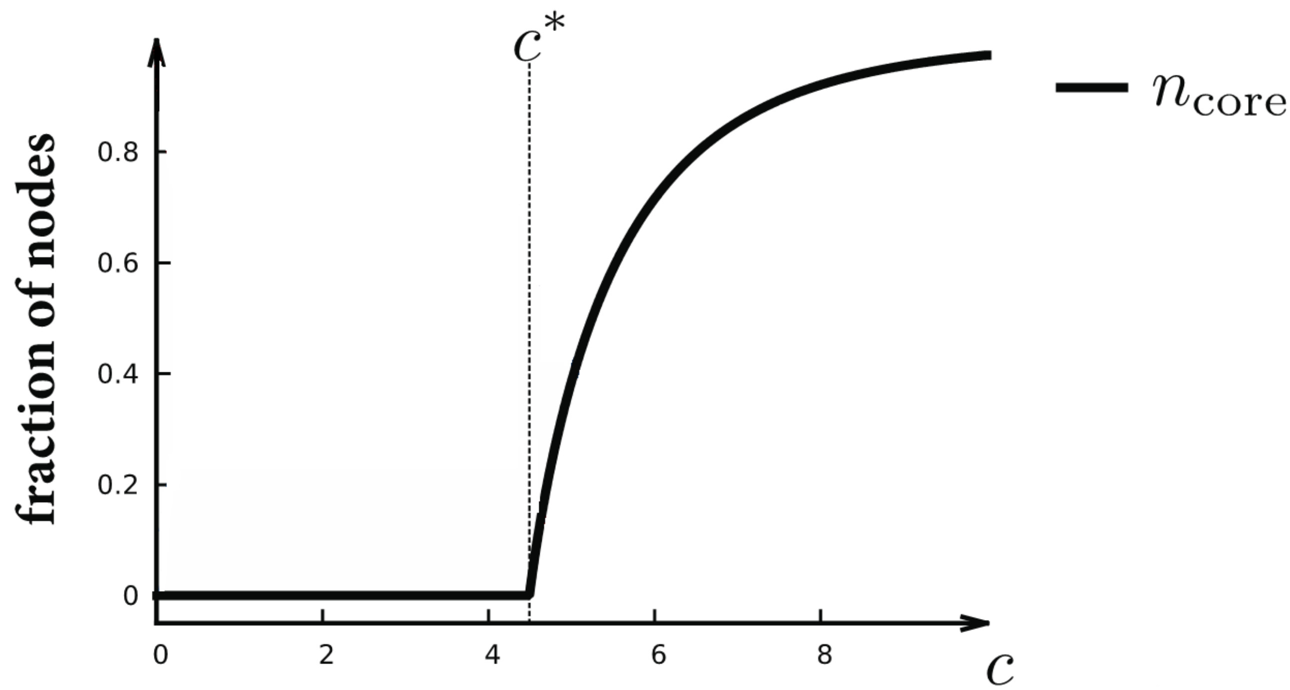


Greedy Leaf Removal



Core does not depend on the order of removal.

Core of Undirected Networks

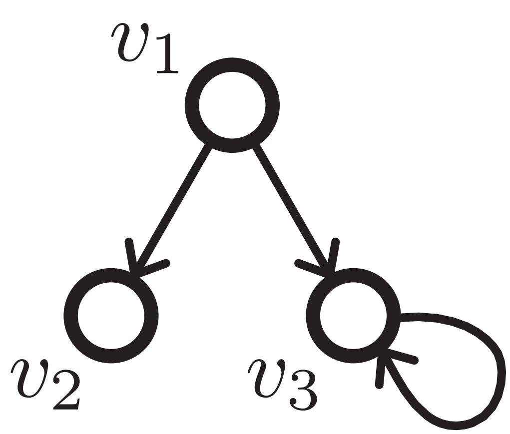


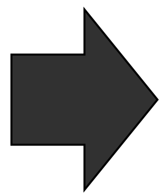
Directed SF network

Continuous phase transition.

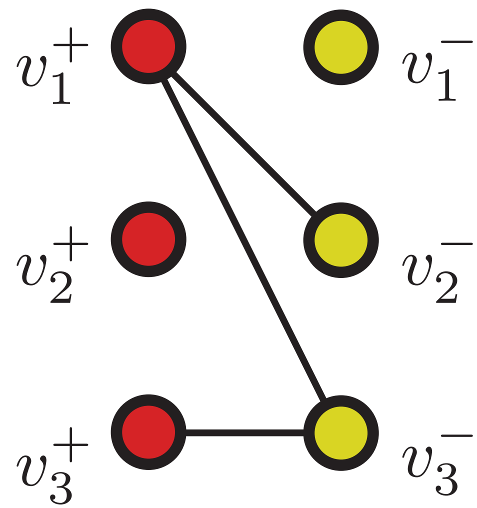
Directed Networks

Directed network





Bipartite representation



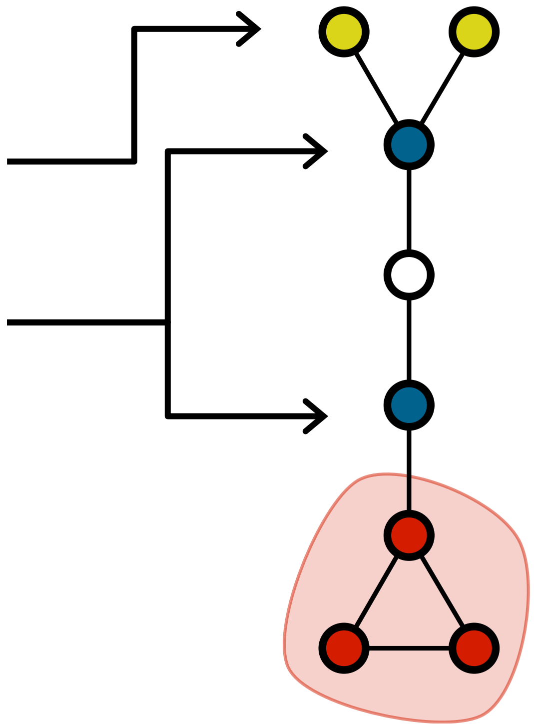
$(v_1 \to v_2)$

(v_1^+, v_2^-)

Greedy Leaf Removal is defined on the bipartite representation.

Analytical solution

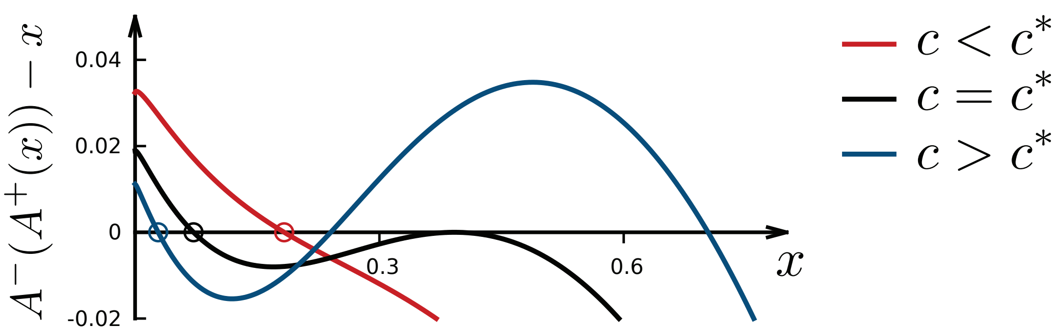
- **α-removable:** Nodes that can become isolated.
- **β-removable:** Nodes that can become neighbors of a leaf.



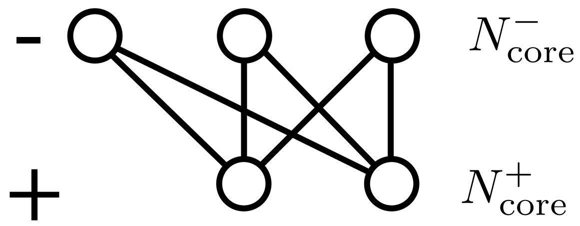
$$\alpha^{\pm} = A^{\pm}(1-\beta^{\mp})$$
$$1-\beta^{\pm} = A^{\pm}(\alpha^{\mp})$$

$$A^{\pm}(x) = \sum_{k=1}^{\infty} Q^{\pm}(k)(1-x)^k$$

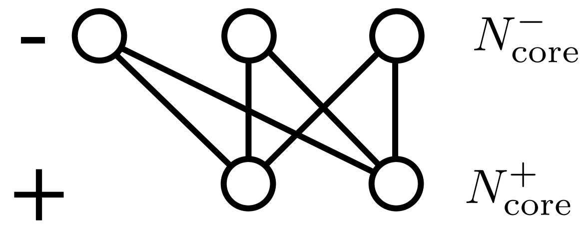
α^{-} :



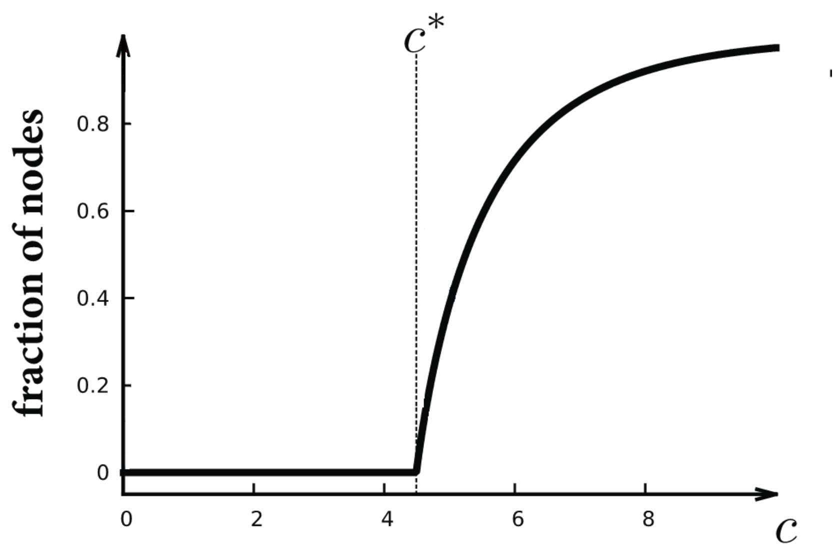
Core of Directed Networks



Core of Directed Networks

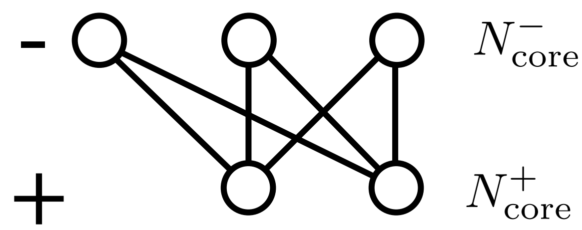


$$P^+(k) \equiv P^-(k)$$

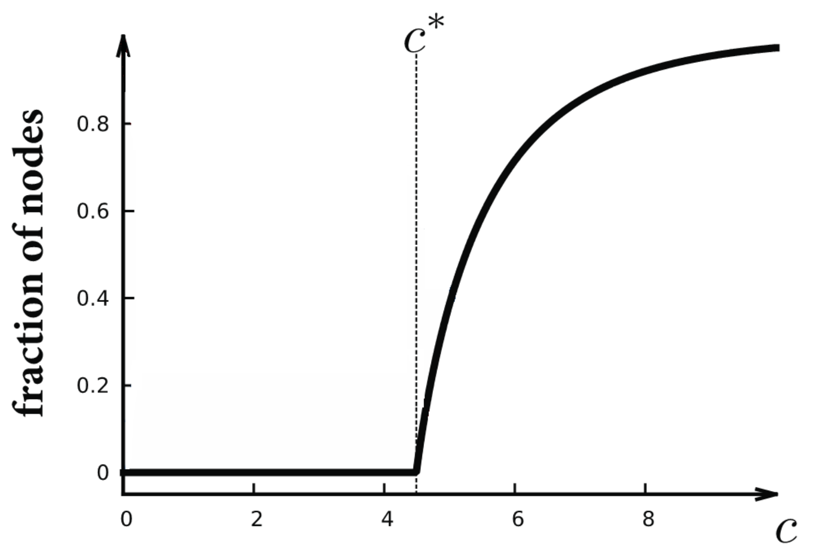


Continuous.

Core of Directed Networks



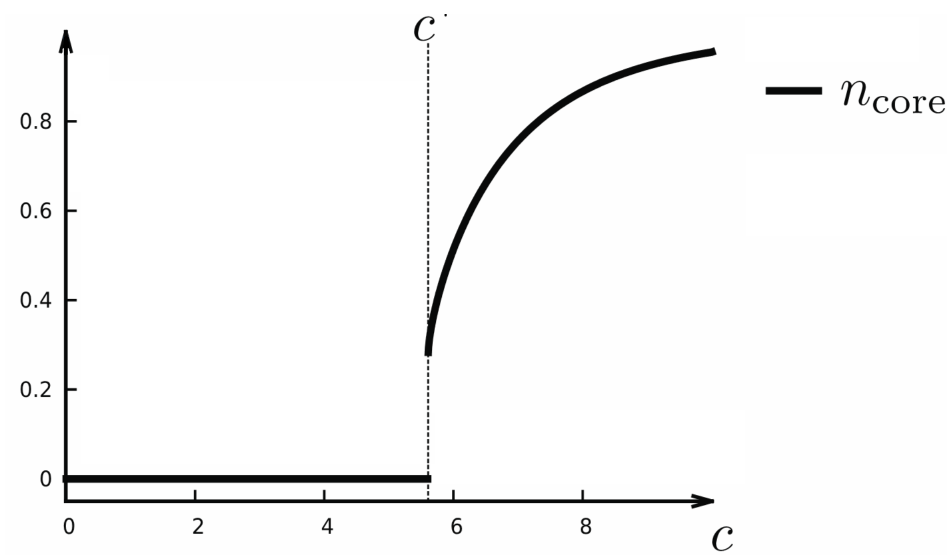
$P^+(k) \equiv P^-(k)$



Continuous.

$$E[N^+_{\text{core}}] - E[N^-_{\text{core}}] = 0$$

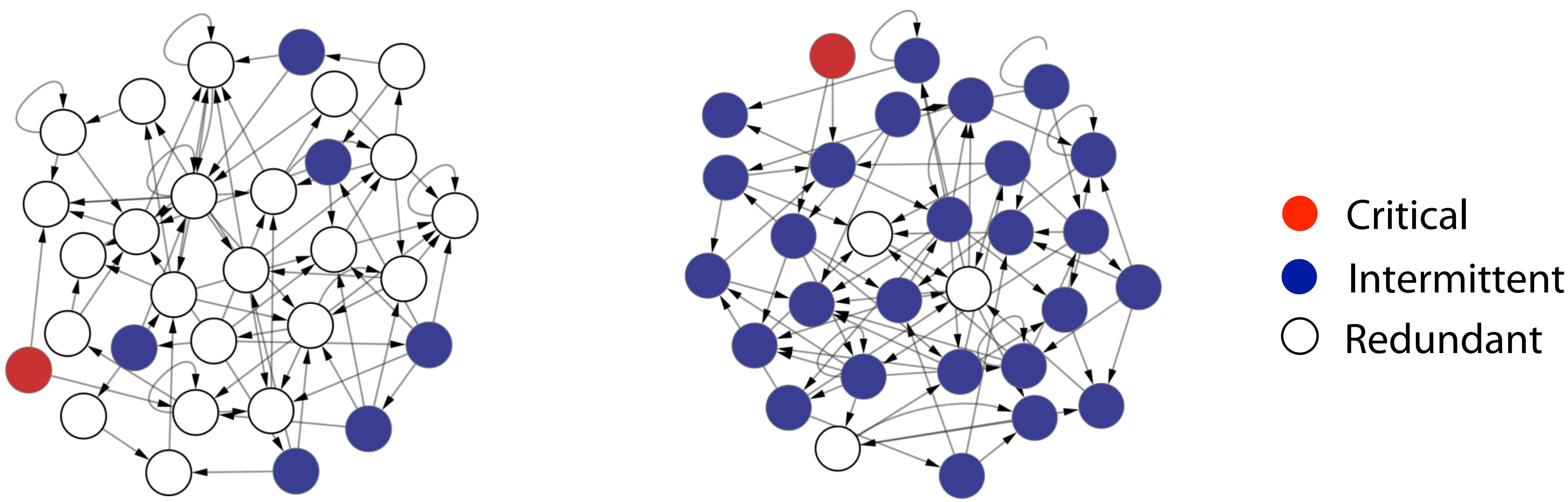
$P^+(k) \not\equiv P^-(k)$



Discontinuous.

$$E[N^+_{\text{core}}] - E[N^-_{\text{core}}] \sim N$$

Control modes + core percolation

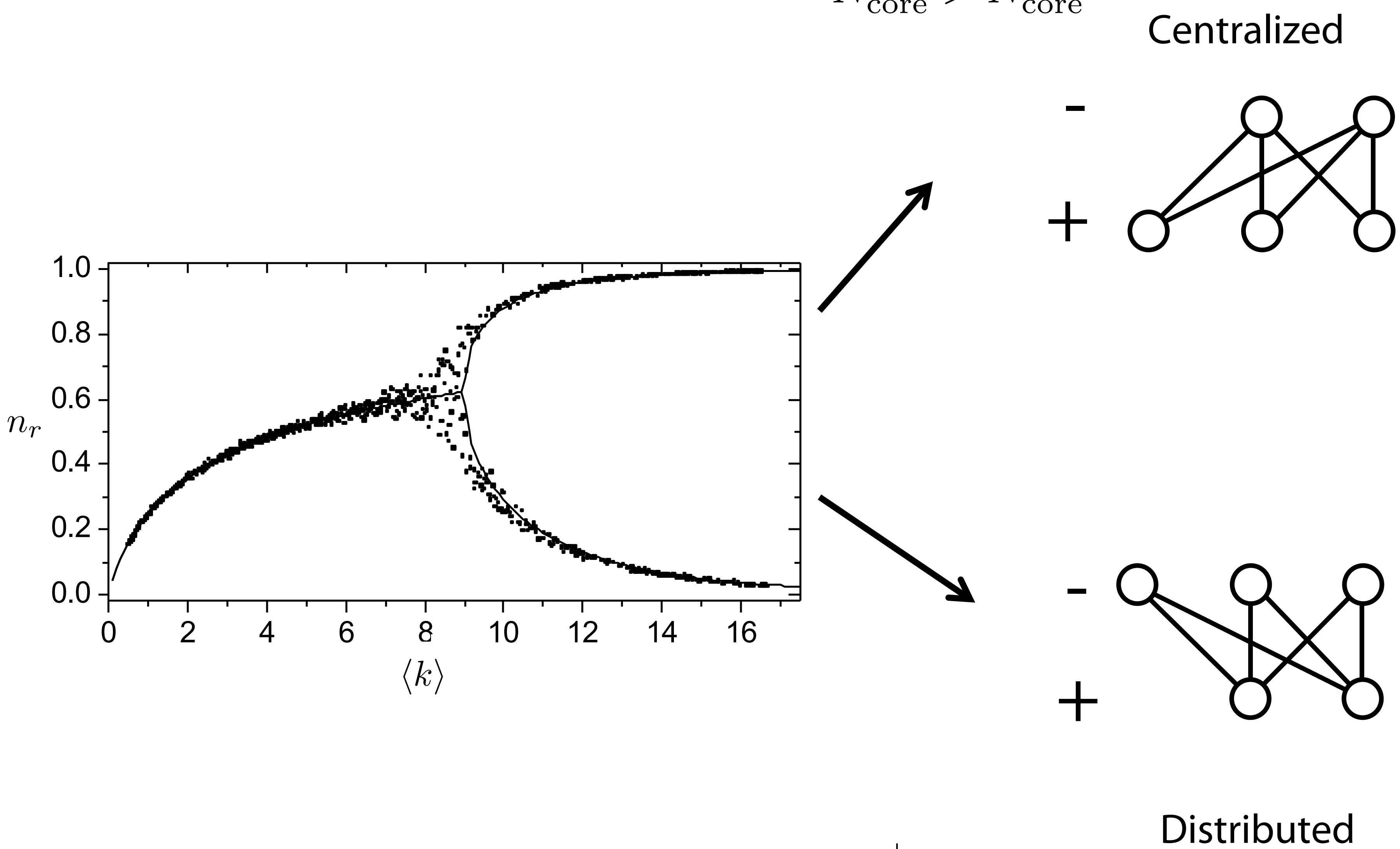


Centralized control. Distributed control.

$$N^{+}_{\mathrm{core}} > N^{-}_{\mathrm{core}} \qquad N^{+}_{\mathrm{core}} < N^{-}_{\mathrm{core}}$$

$P^+(k) \equiv P^-(k)$

$N^+_{\mathrm{core}} > N^-_{\mathrm{core}}$



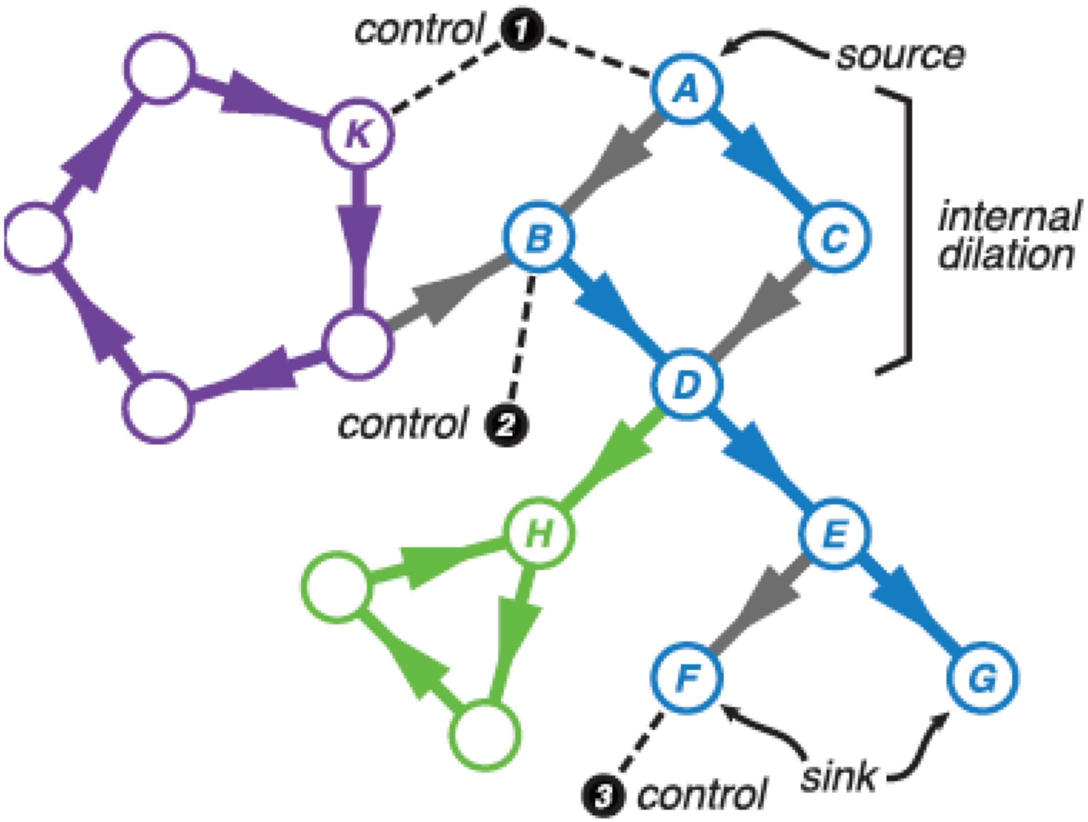
$N^+_{\mathrm{core}} < N^-_{\mathrm{core}}$

Back to degree distribution discussion

(Still considering LTI system)

- How does degree-distribution impact controllability?
- Hubs are almost always matched! (They have so many edges.)
- Low degree nodes matter
  - As we saw, if in-degree = 0, must drive that node directly.
- Some deeper answers (beyond degree distribution)
  - G. Bianconi (low degree nodes)
  - Ruths ”control profiles”

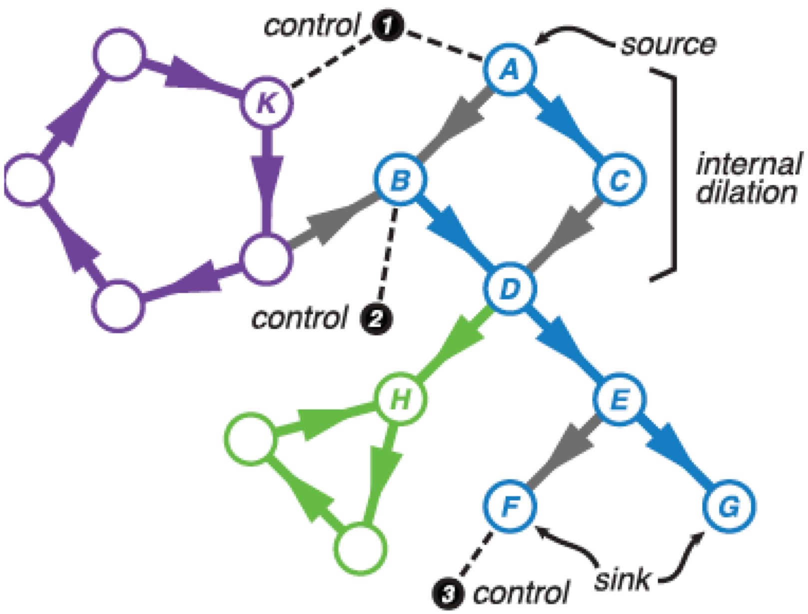
Control profiles (for LTI complex networks)



- Source node
- Sink node
- Internal dilation
- External dilation

D. Ruth, J. Ruths, "Control Profiles of Complex Networks", *Science*, 343, 2014.

Control profiles --- categories of nodes

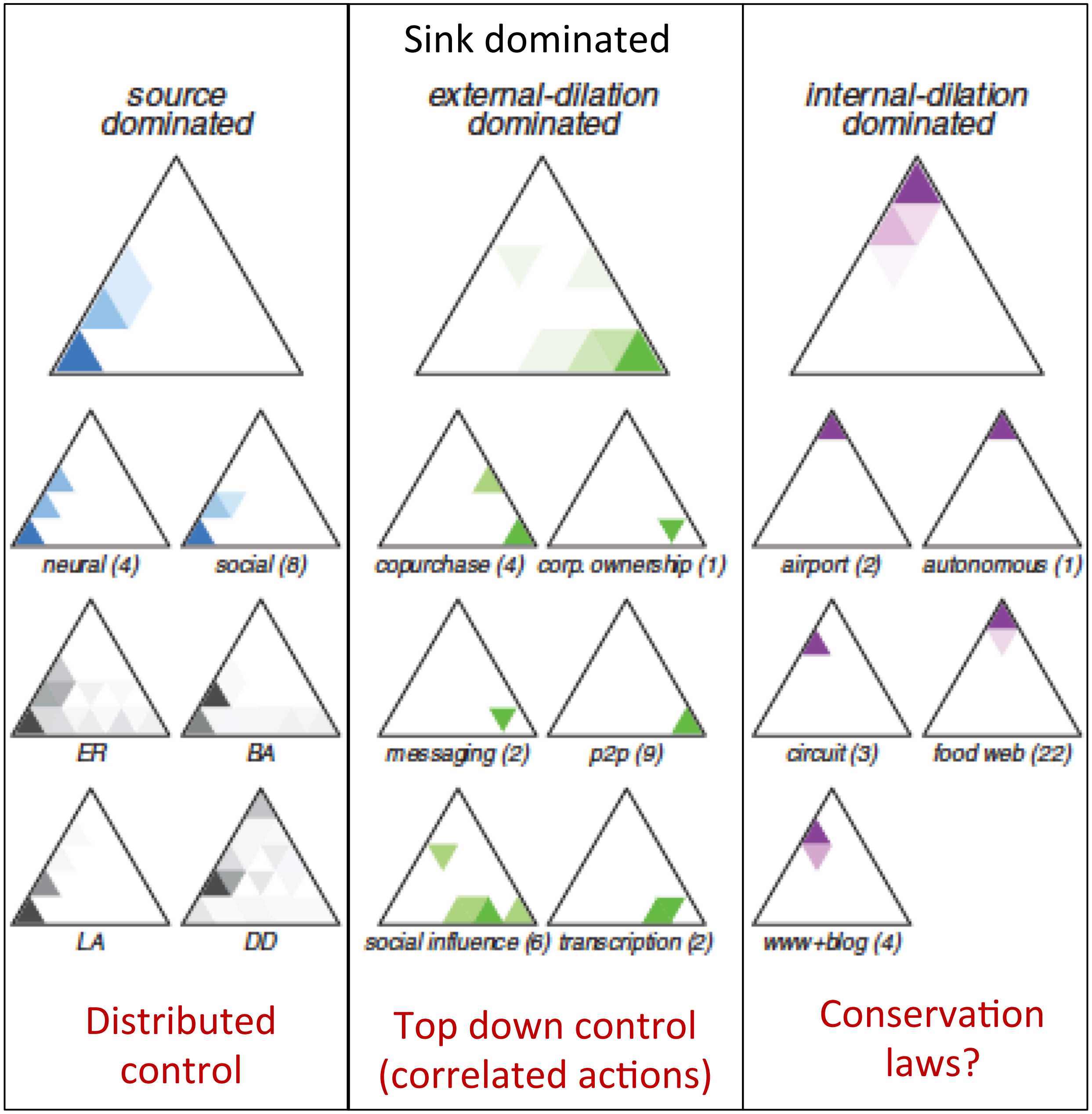


They conclude what matters:

- #sources, n_s
- #internal dilations, n_i
- #external dilations, n_e

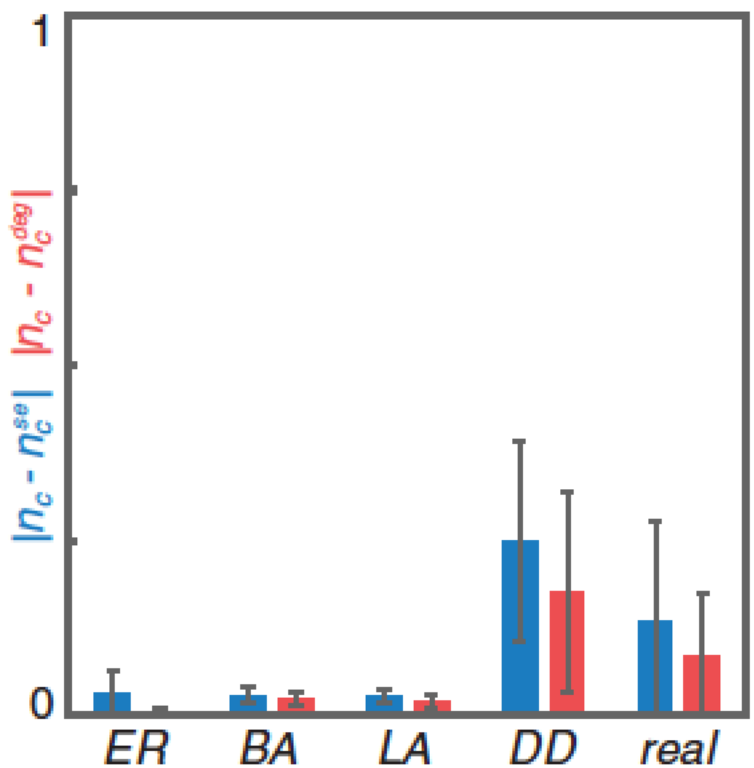
- Source node – no in-links (must drive this node directly)
- Sink node – no out-links (cannot control another node)
- Internal dilation – a branch point (cannot control both out-edges independently)
- External dilation – excess sink nodes (#sources – #sinks)

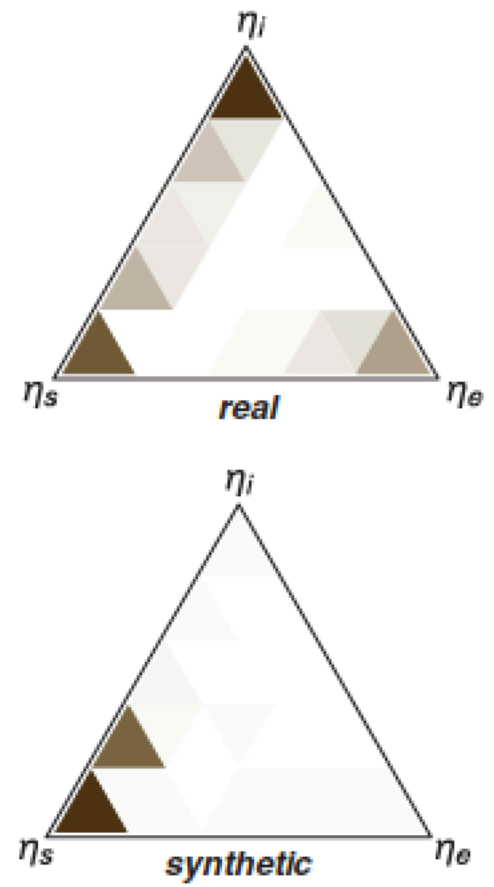
Real-world networks fall into different classes/ control profiles



”Control profiles” of real versus synthetic network models

(They don’t match, e.g., PA adds only source nodes)



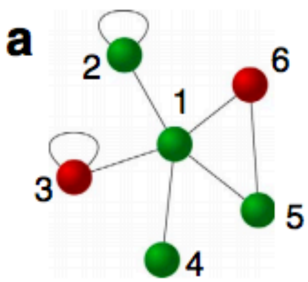


C. Campbell, K. Shea, R. Albert, Comment on “Control profiles of complex networks”, *Science*, 346, 2014. (Add a parameter for added node has in- or out-edges.)

"Exact controllability" of LTI networks

Note on Structural control: weak versus strong notions but the non-zero edge weights are unspecified; directed edges.

Z. Yuan, C. Zhao, Z. Di, W-X Wang, and Y-C Lai
"Exact controllability of complex networks". Nature Communications, 4, 2013



$A\text{-}\lambda^M I_N$ —Elementary column Transformation→ Column canonical form; Eigenvalues

$$\begin{bmatrix} -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \end{bmatrix} \quad \lambda = \begin{bmatrix} -1.9032 \\ -1.0000 \\ 0.1939 \\ 1.0000 \\ 1.0000 \\ 2.7093 \end{bmatrix}$$

$\lambda^M = 1.00$

$\mu(\lambda^M) = 2$

More recently applied to multiplex networks, *New Journal of Physics,* 2014.

- Undirected edges allowed
- Edge weights specified
- Consider the eigenvalues of A
- N_D = maximum geometric multiplicity of eigenvalues

Energy/cost of control

G.Yan, J. Ren, Y-C Lai, C-H Lai, and B. Li. "Controlling complex networks: How much energy is needed?" *Physical Review Letters*, 108(21), 2012.

- LTI system
- Know the adjacency matrix (including edge weights)
- Drive the system from state x_0 to x_{Tf} in time T_f
- Restrict to structures that only need one driver node
- T_f -> 0 energy diverges
- T_f -> ∞ energy required can go to zero!

The control energy:

$$\mathcal{E}(T_f) \equiv \int_0^{T_f} \|\mathbf{u}_t\|^2 dt$$

Target control:

When full control is unnecessary or infeasible

- **Motivation:**
  - Internet (Unnecessary full control)
  - Social networks (Unnecessary full control)
  - Financial networks (Infeasible full control)
  - Flights network (Infeasible full control)
  - Biology networks (Infeasible full control)
  - …

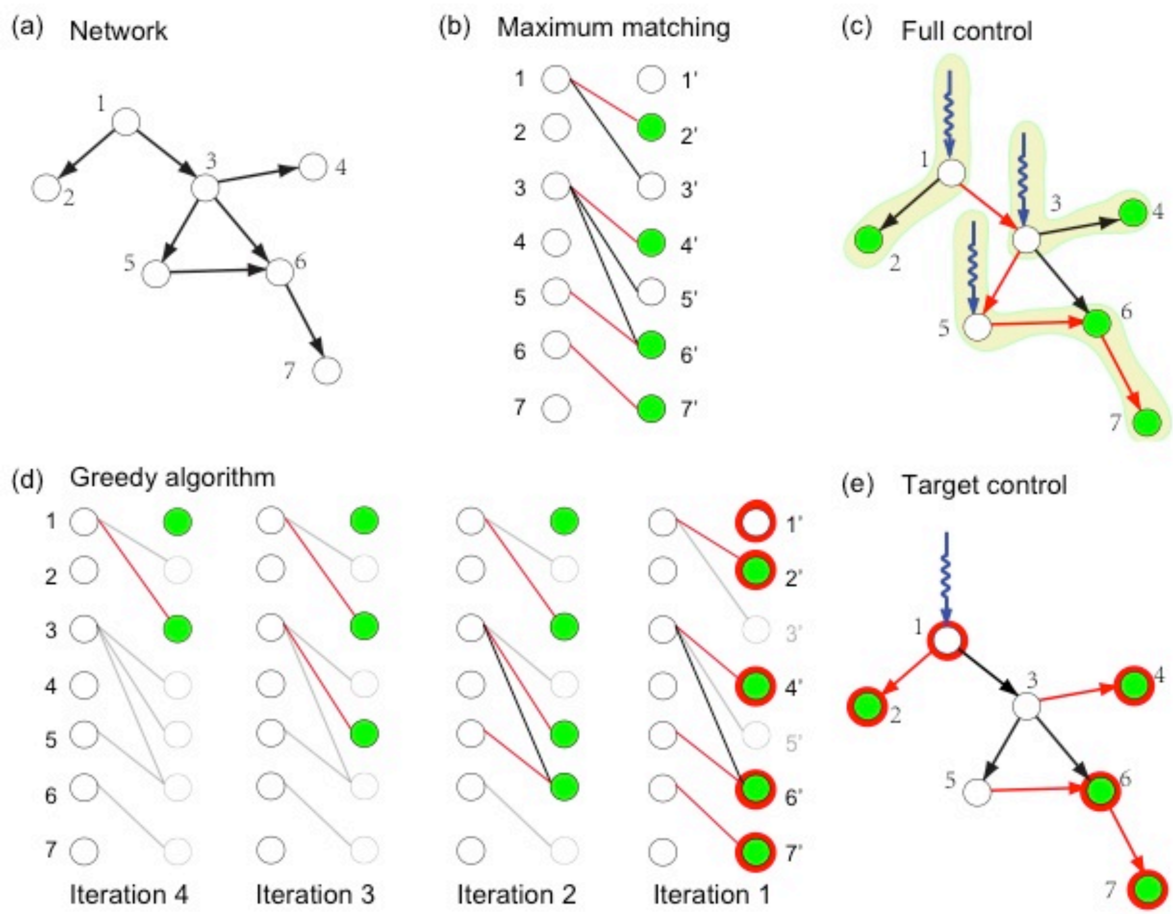
Target Control

the ability to efficiently control a preselected subset of nodes.

Structural control can miss important partial (target) control

k-walks Theory: The driver node can control a set of nodes where each node has a distinct path length to the driver.

Alternative solution:



Structural control predicts N_D=3

k-walk theory correctly predicts N_D=1

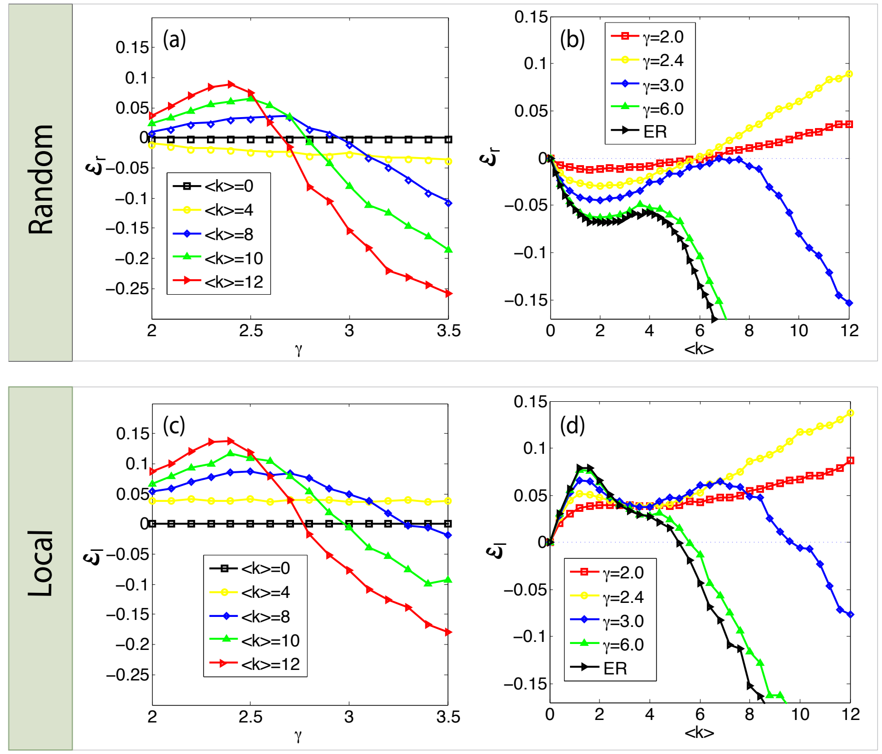
Target control efficiency

N_D Number of driver nodes to control the entire network

$P_D(f)$ Number of driver nodes to control *f* fraction of nodes

$$\alpha_D(f) = \frac{P_D(f)}{N_D}$$ Target control controllability

$$\varepsilon = 0.5 - \int_0^1 \alpha_D(f)\,\mathrm{d}f$$ Target control efficiency

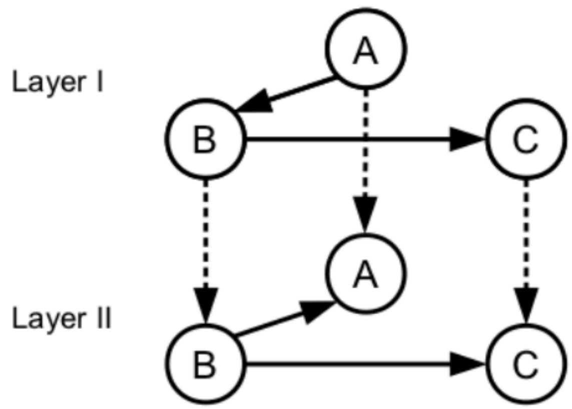


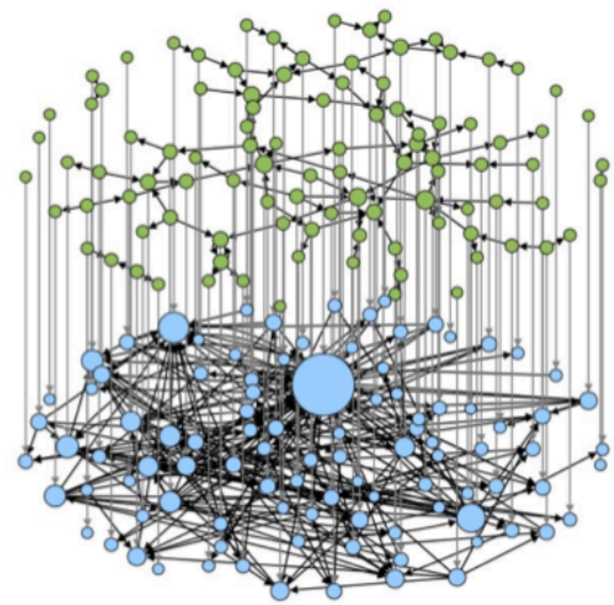
Results:

1. **In general local target control is more efficient than random target control.**
2. **More surprisingly, we find that degree heterogeneous networks like scale-free networks have higher specific and overall target control efficiency than degree homogeneous networks, for both random and local schemes.**

J. Gao, Y.-Y. Liu, R.D., A-L Barabasi, “Target control of complex networks”, *Nat. Comm., 2014*

Control of LTI multiplex, multi-timescale networks





$$\mathbf{x}_{\mathrm{I}}(t) = \mathbf{A}_{\mathrm{I}}\mathbf{x}_{\mathrm{I}}(t-\tau_{\mathrm{I}}) + \mathbf{B}\mathbf{u}(t-\tau_{\mathrm{I}}) \qquad \text{if } (t \bmod \tau_{\mathrm{I}}) = 0,$$

$$\mathbf{x}_{\mathrm{II}}(t) = \mathbf{A}_{\mathrm{II}}\mathbf{x}_{\mathrm{II}}(t-\tau_{\mathrm{II}}) + \Delta_{\tau_{\mathrm{I}}}(t)\mathbf{D}\mathbf{x}_{\mathrm{I}}(t-\tau_{\mathrm{I}}) \qquad \text{if } (t \bmod \tau_{\mathrm{II}}) = 0,$$

M Pósfai, J Gao, SP Cornelius, AL Barabási, R.D., *Physical Review E* 94 (3), 032316, 2016.

Rather than node control what about edge control?

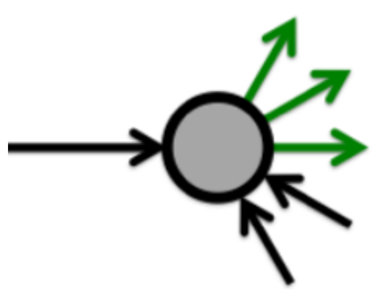
(i.e., dynamics on edges, not on the nodes)

T. Nepusz and T. Vicsek , "Controlling edge dynamics in complex networks", *Nature Physics*, **8**, 2012.

Commentary: "Complex networks: The missing link", Slotine & Liu

Why edge dynamics?

- Airline networks
- Social communication patterns
  - Uni-cast (unique message on each outgoing edge) versus
  - Broadcast (same message on all outgoing edges)



- Hubs have so many out edges, they facilitate control
- Heterogeneous degree distribution easier to control

Traditional control theory -- beyond LTI

- Linearization:
  - Around fixed points
  - Around a trajectory
  - Open loop control (feedback!)

- Lie algebra and Lie brackets for non-linear formulations of Kalman

Part II.

Non-linear dynamics and “control of chaos”

II. Nonlinear dynamics

- Controlling chaos:

  - Ott, Gregobi, Yorkes, “Controlling chaos”. Physical Review Letters, 64(11):1196, 1990.

- Pinning control

  - Used extensively for synchronization
  - RO Grigoriev, MC Cross, and HG Schuster. Pinning control of spatiotemporal chaos. Physical Review Letters, 79(15):2795, 1997.
  - F. Sorrentino, M. di Bernardo, F. Garofalo, and G. Chen. Controllability of complex networks via pinning. Physical Review E, 75(4):046103, 2007.

II. Nonlinear dynamics

- Attractor switching networks & “compensatory perturbations”

  - S P Cornelius, WL Kath, and AE Motter. Realistic control of network dynamics. Nature Communications, 4, 2013.
  - Wells, Daniel K., William L. Kath, and Adilson E. Motter. "Control of stochastic and induced switching in biophysical networks." *Physical Review X* 5.3 (2015): 031036.
  - Ying-Cheng Lai. Controlling complex, non-linear dynamical networks. National Science Review, 1(3):339–341, 2014.

- Control of phase transitions in complex networks

  - Explosive percolation in random networks, D Achlioptas, RM D’Souza, J Spencer, Science 323 (5920), 1453-1455, 2009.

II. Nonlinear dynamics

- Control of SOC and Dragon Kings

  - P-A. Noel, C. D. Brummitt, and R. D'Souza. Controlling self-organizing dynamics on networks using models that self-organize. Phys. Rev. Lett., 111:078701, 2013.
  - P-A. Noël, C. D. Brummitt, and R. D'Souza. "Bottom-up model of self-organized criticality on networks." *Physical Review E* 89.1 (2014): 012807.
  - The Self-Organization of Dragon Kings, Y Lin, K Burghardt, M Rohden, PA Noël, RM D'Souza arXiv:1705.10831

In the beginning

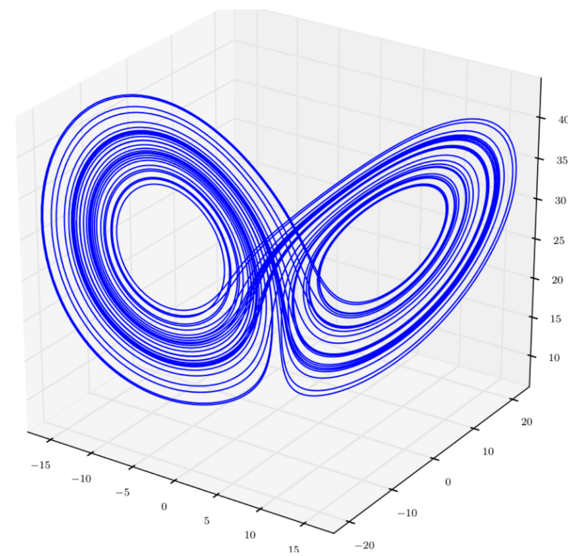
VOLUME 64, NUMBER 11 PHYSICAL REVIEW LETTERS 12 MARCH 1990

Controlling Chaos

Edward Ott,[(a),(b)] Celso Grebogi,[(a)] and James A. Yorke[(c)]
University of Maryland, College Park, Maryland 20742
(Received 22 December 1989)

- Convert a chaotic attractor to an one of a many possible time-periodic motions by making only small time-dependent perturbations of a system parameter

- Uses delay coordinate embedding (can measure an empirical system and determine the control action)



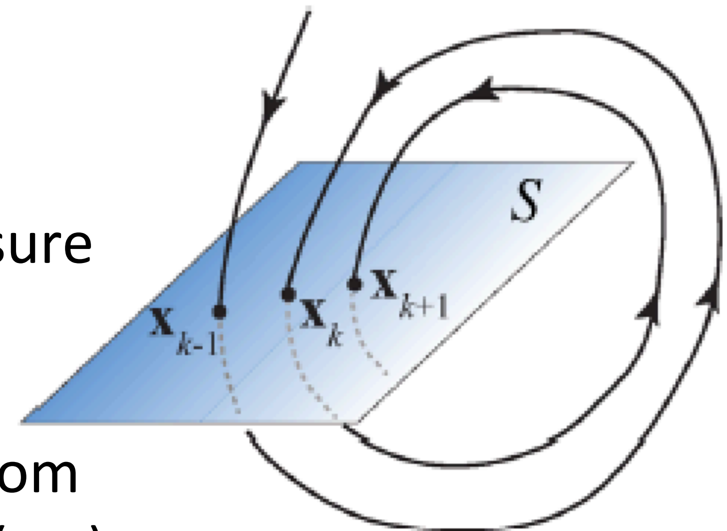
Chaotic attractor control

- An infinite number of unstable periodic orbits typically embedded in a chaotic attractor.
- Determine some of the low-period unstable periodic orbits
- Find a desired one (that improves "system performance")
- Tailor time-dependent parameter perturbations to stabilize that orbit
- Note: Many different orbits can be stabilized showing that chaos can be a big advantage for control or system adaptation. (Simple period orbit can't be significantly altered, but a chaotic one can.)

Controlling chaos (OGY theory, cont)

- Consider the trajectory of a three-dimensional continuous time dynamical system:
  - d**x**/dt = F(**x**,p)
  - **x** is the three-dimensional vector
  - p is the parameter that can vary between some range p* > p > -p*

- Consider a surface of section and measure every time the dynamical trajectory pierces the surface.
  - Accumulate a time series of data from the trajectories and approximate F(**x**,p)
  - Linearize around a fixed point
  - Linear state feedback law

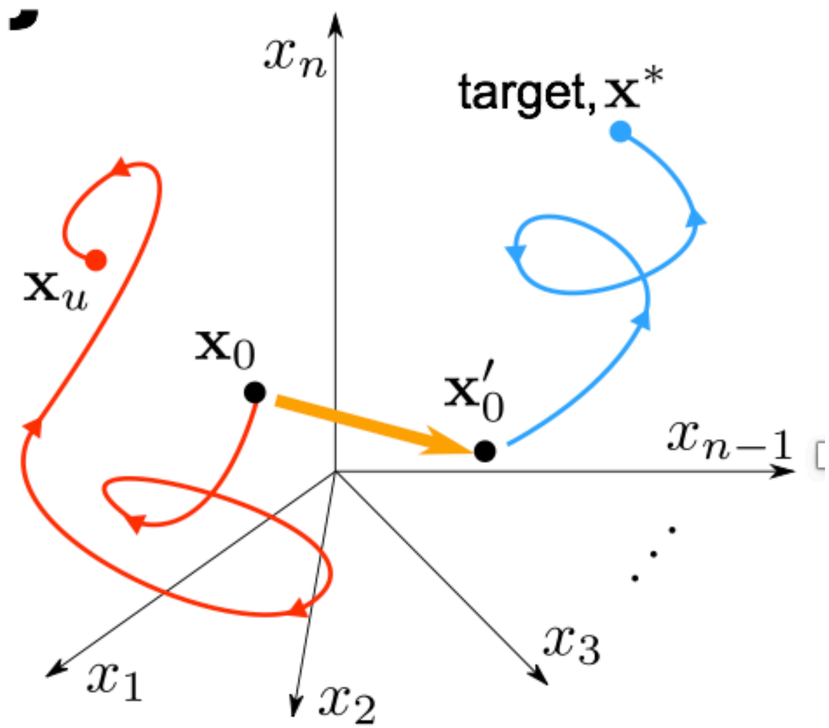


(Image Y.-Y. Liu)

OGY is for low-dimensional systems, classic example **x** is 3-dimensional.

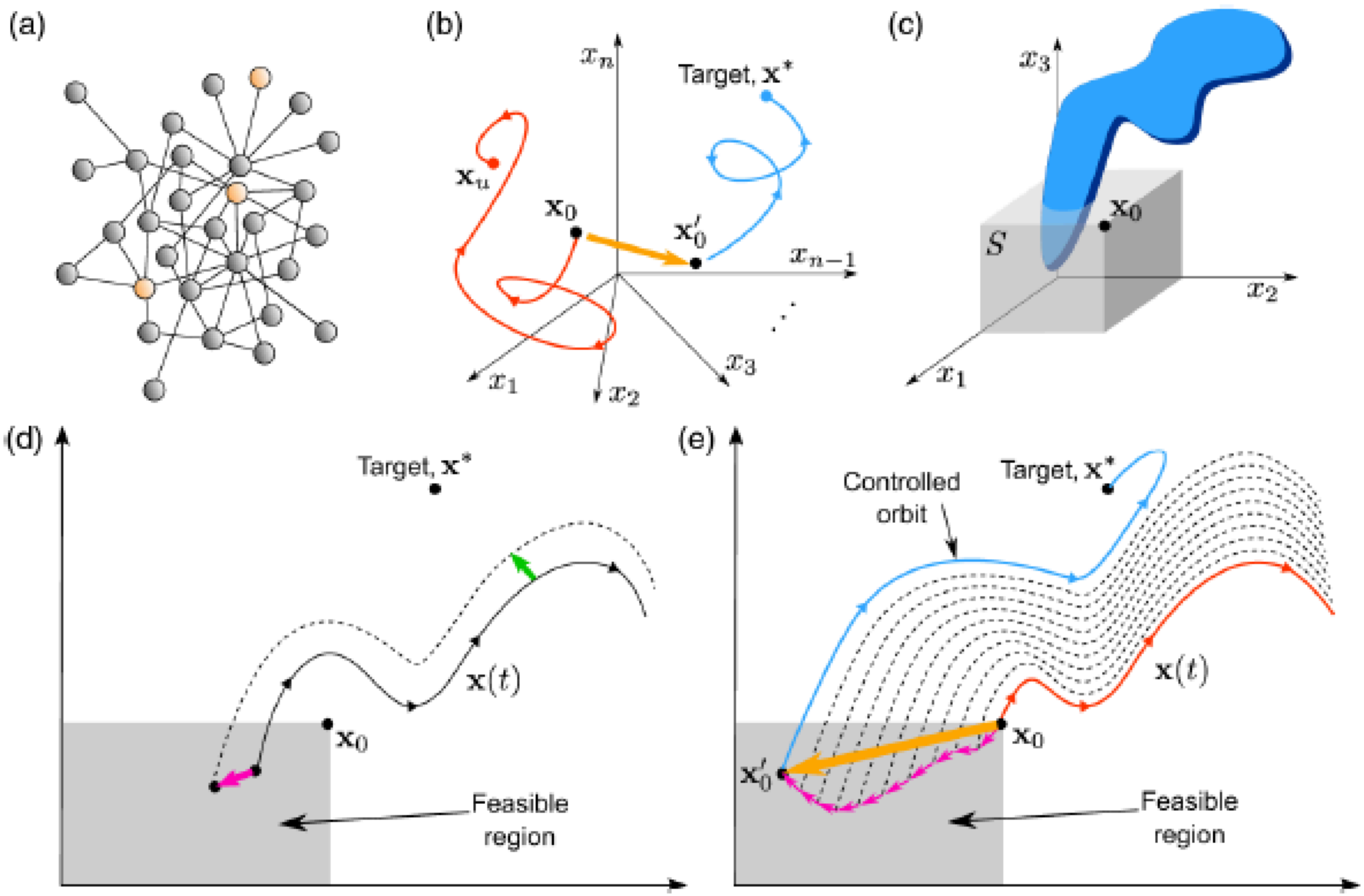
(Complex networks massive number of dimensions)

Kicking control and compensatory perturbations



Exploit basins of attraction and natural phase-space trajectories

S P Cornelius, WL Kath, and AE Motter. "Realistic control of network dynamics". *Nature Communications*, 4, 2013.

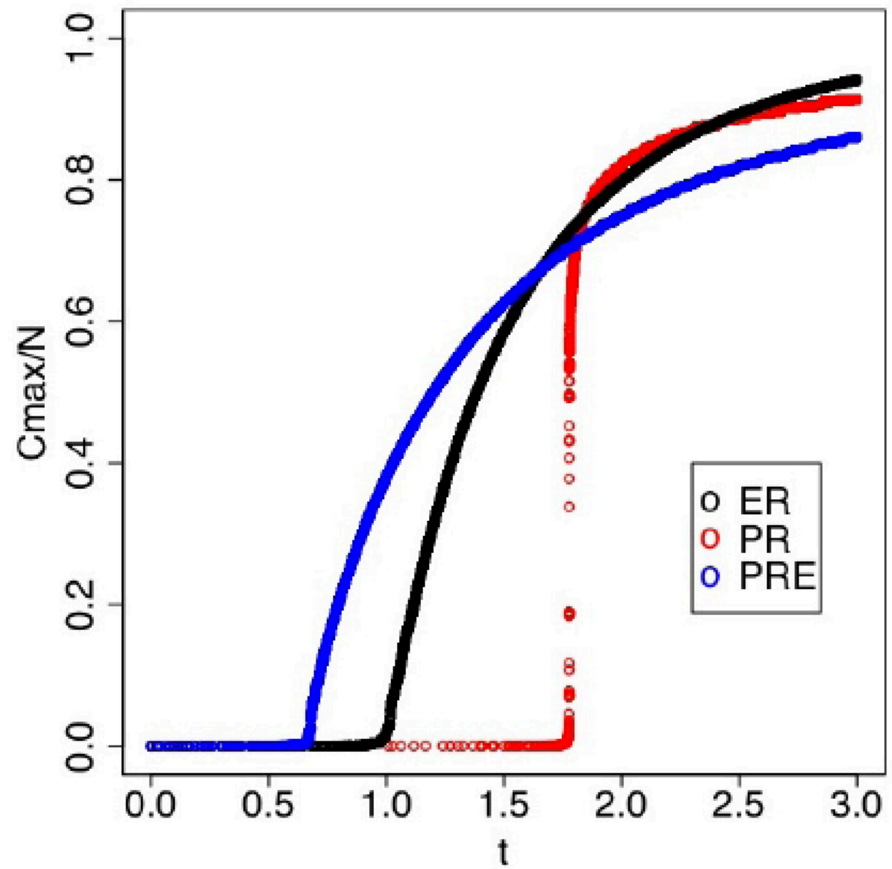


Control of phase transitions

Design small interventions that enhance or delay the onset of phase transitions in a complex network.

- **Enhance** – similar to **ER** but with earlier onset.

- **Delay** – Extremely abrupt



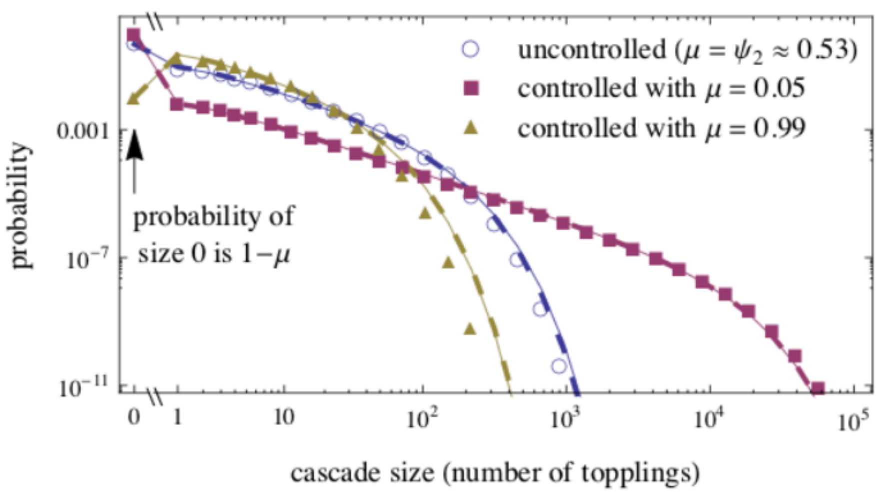
"Explosive percolation in random networks", D Achlioptas, RM D'Souza, J Spencer, *Science* 323 (5920), 1453-1455, 2009.

Control of self organized cascades

Controlling the BTW model away from the SOC state

Noël, Brummitt, R.D., Phys. Rev. Lett. **111** *0780701, 2013*

Control parameter μ:
probability grain lands on a node at threshold*



- Avoid cascades, $\mu = 0.05 \rightarrow$ larger cascades when they do occur.
- Ignite cascades, $\mu = 0.99 \rightarrow$ smaller cascades, but more frequent.

- Interventions can drive a sub-critical system to critical point
- Optimal levels of control exist to maximize profit

III. Social Systems

Modeling Influence & Opinion Dynamics

Mathematical models of social behavior

Analyze extent of epidemic spreading, product adoption, etc:

- Thresholds models
- Voter models
- Opinion dynamics
  (e.g. The Naming game)
- Percolation
- Game theory

INSIDE SCIENCE NEWS SERVICE

Zealots Help Sway Popular Opinions

Image credit: Gabriel Saldana via Flickr | http://bit.ly/1E9IjCQ
Rights information: http://bit.ly/1dWcOPS

Enthusiasts can greatly influence the adoption of new ideas.

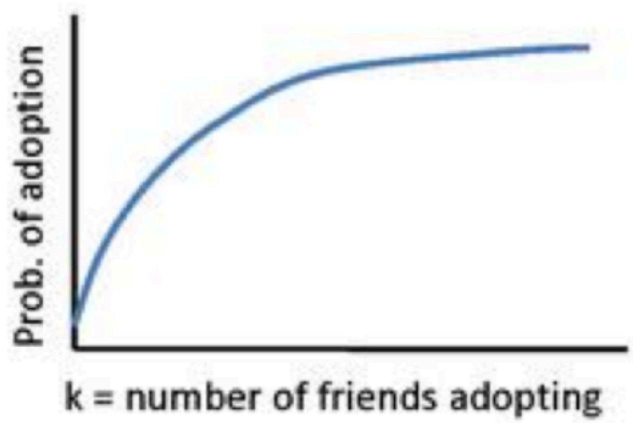
Originally published: Feb 19 2015 - 10:45am

By: Ker Than, Contributor

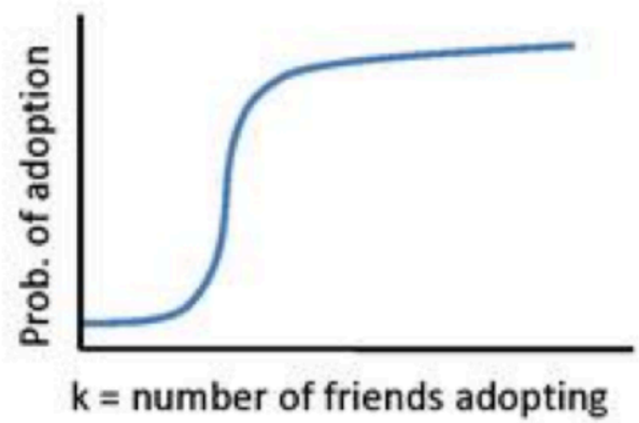
A. Waagen, G. Verma, K. Chan, A. Swami, R. D. *PRE*, 2015.

What mechanism makes an individual change their mind?

I. Diminishing returns versus thresholds



Diminishing returns?



Critical mass?

Kleinberg, Leskovec, Kempe
e.g., *KDD* 2003.

"Hill climbing" / best response
Algorithms for influential seed nodes

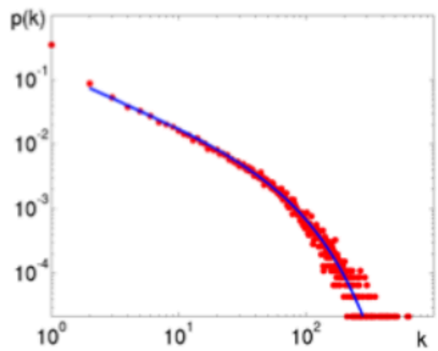
Watts, Dodds
e.g. *PNAS* 2002.

Percolation & generating functions
Susceptibles vs influentials/mavens
(Depends on active vs passive influence.)

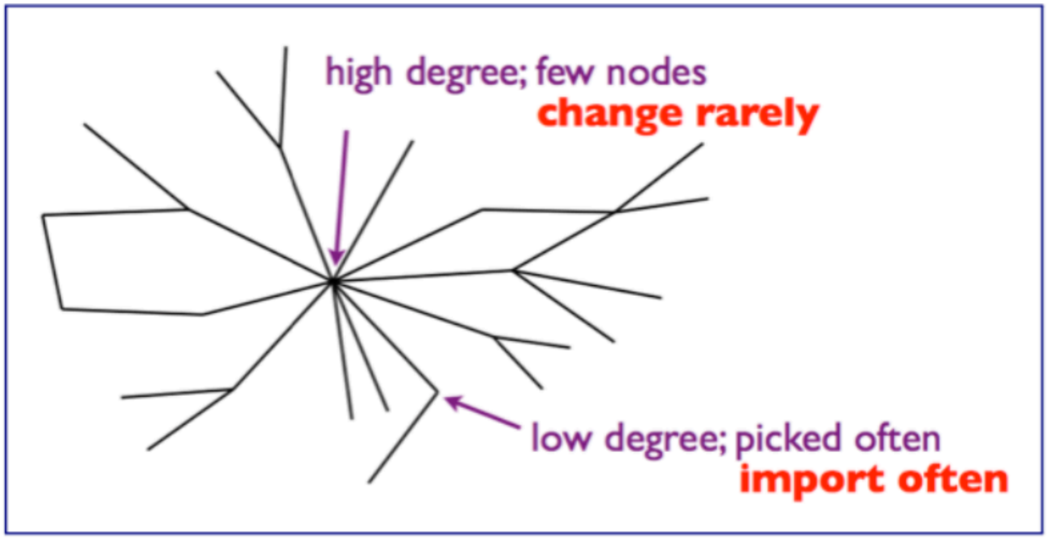
II: The Voter model, “Tell me what to think”

V. Sood, S. Redner, *Phys. Rev. Lett.* 94, 2005.

- At each time step in the process, pick a node at random.
- That node picks a random neighbor, and adopts the opinion of the neighbor.
- Ultimately, only one opinion prevails. The high degree nodes (hubs) win.



Degree distribution

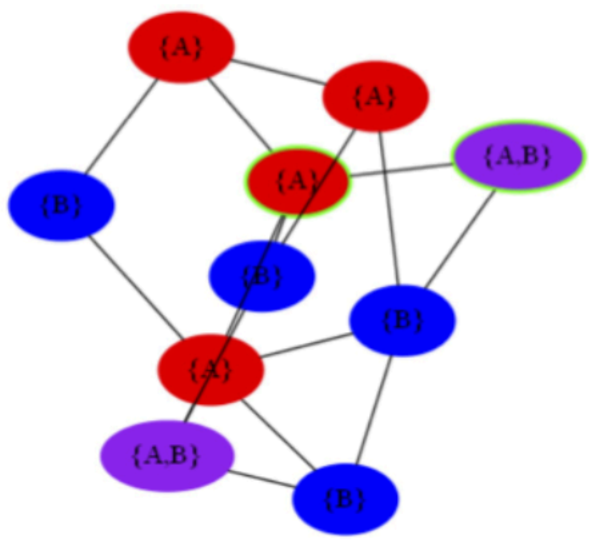


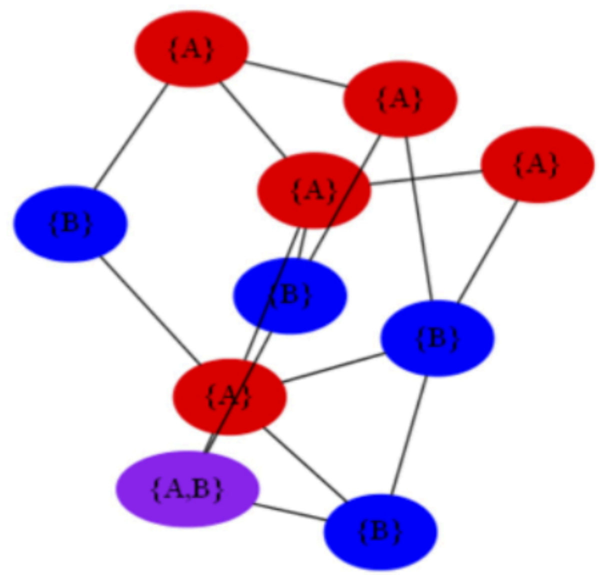
- *Invasion percolation* *(the “bully” model) yields the opposite: leaf nodes propagate opinions.*

III: "The Naming Game" / open minded individuals

Steels, *Art. Life* 1995; Barrat *et al.*, *Chaos* 2007; Baronchelli *et al.*, *Int. J. Mod. Phys.* 2008.

- Originally introduced for linguistic convergence. Two opinions, A and B.
- And each individual can hold A, B, or $\{A, B\}$.
- Exchange opinions with neighbors and update





The impact of Zealots

Committed individuals who will never change opinions

$$x' = z\left(x + z + \frac{p}{2}\right) - y(x - p),$$
$$y' = z\left(y + z + \frac{q}{2}\right) - x(y - q),$$
$$z = 1 - x - y.$$

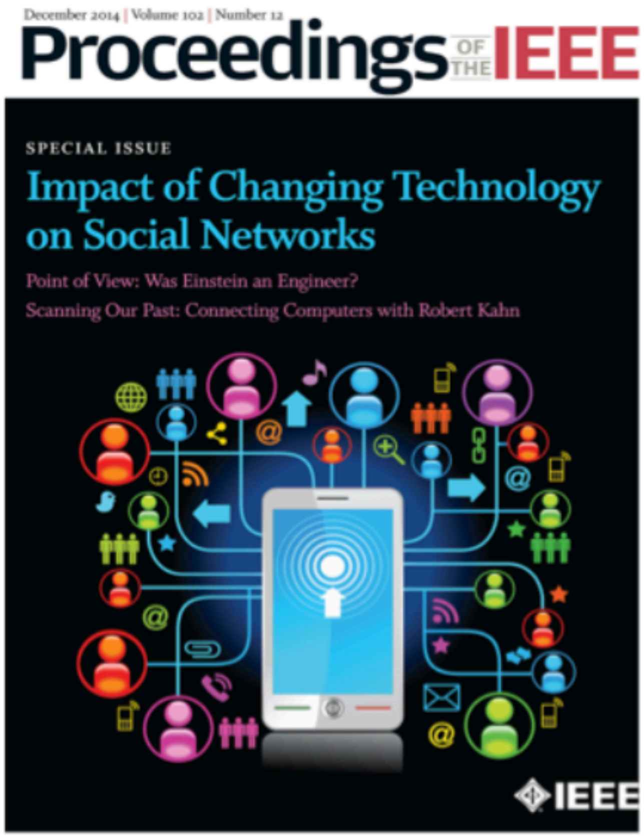
- p is fraction of A zealots; q is fraction of B zealots.
- **Voter model**: A finite ***number*** of zealots can sway the outcome.
- **Naming game**: A small ***fraction*** of zealots can sway the outcome.
- **Naming game with multiple choices, k:**

  Operating systems; cell phones; political parties; etc

  - Zealots of only one kind: Quickly obey the zealot.
  - Equal fractions of zealots of all kinds: Quickly reach stalemate.

Collective phenomena in social networks
How the online world is changing the game

J. Flack, R.D., editors, PIEEE (2014)



Past: Small, geographically localized social networks, concentrated power and influence

Present: Digital footprint, massive online experimentation, global information, rapid rate of change.

Social networks (feedback from the real social scientists)

- First need to build the influence network
- Then understand who to influence and how to influence (strategies of persuasion)
- Interventions are just at the infancy
- Existing interventions
  - Identify opinion leaders
  - Segmentation
  - Induction
  - Alteration
- But how do we make them lasting changes?
  - exploration, adoption, implementation, sustaining and monitoring.

Commonalities
Control of complex networks across genres

What to influence and how to influence and when to influence?

Commonalities

- **Do we want to control the global behavior or behavior of every single node?**

e.g. Large-scale connectivity in a network vs eliminating smoking in a particular individual

Commonalities

- **What is the minimal amount of knowledge of the system we need in order to control it?**