

Problem 1: Pigou's congestion example

Recall Pigou's example discussed in class, where there are two roads that connect a source, s , and destination, t . The roads have different travel costs. Fraction x_1 of the traffic flow on route 1, and the remainder x_2 on route 2. Here consider the following scenario.

- The first road has “infinite” capacity but is slow and requires 1 hour travel time, $T_1 = 1$.
- The second road always requires at least 15 mins, which then increases as a function of traffic density, $T_2 = 0.25 + 0.75x_2$.

If drivers act in a “selfish” manner – the user optimal scenario – all the traffic will flow on the second path, as one is never worse off. Worst case scenario for path 2, both paths take one hour. So no one is incentivized to change their behavior.

1. Assume user optimal behavior, and calculate τ the expected travel time per car.
2. If instead we could control the flows, we could minimize the expected travel time. Using the expression in part (a), calculate the optimal allocation of flows \bar{x}_1 and \bar{x}_2 that minimize the expected travel time per car.
3. What is τ_m , the expected travel time when the flow is optimized?