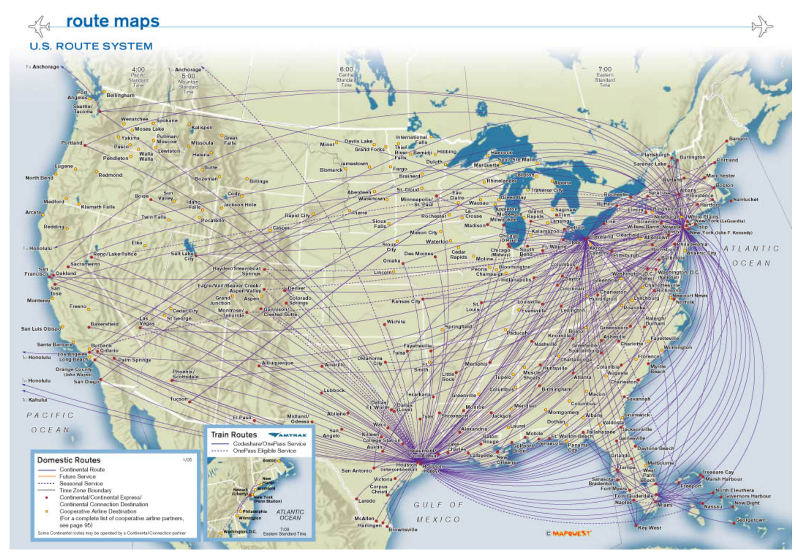
ECS 253 / MAE 253, Lecture 12

May 10, 2023



"Flows on spatial networks"

Announcements

- HW2b due ...

Topics

- Optimal allocation of facilities and transport networks:
  – Michael Gastner (SFI) and Mark Newman (U Mich)

- Network flows on road networks
  – Michael Zhang (UC Davis)

  (Details of demand, edge capacity, and feasible paths all extremely important)

  – I. Optimization and network flow

  – II. User vs System Optimal

  – III. Braess' Paradox

  – IV. Nash Equilibrium

  – V. Price of anarchy

Optimal design of spatial distribution systems:

(Download: Gastner.pdf)

Gastner and Newman, Summary

- I. Optimal allocation of facilities: Number of facilities within radius $n(r)$ scales sublinear of density: $n(r) \sim \rho(r)^{2/3}$.
  - Seems to hold true for distribution of public goods (hospitals, police stations, county seats, ...)
- II. Optimal connection of facilities into a network:
  - Linear tradeoffs between geometric and network metrics
  - From road networks to air transport

More flows and statistical physics

- David Aldous, “Spatial Transportation Networks with Transfer Costs: Asymptotic Optimality of Hub and Spoke Models”
- Marc Barthélemy, “Spatial networks” *Physics Reports* 499 (1), 2011.
- Flows of material goods, self-organization: Helbing et al.
- Jamming and flow (phase transitions):
  Nishinari, Liu, Chayes, Zechina.

Flows with edge constraints

- Network flows on road networks – Michael Zhang (UC Davis)
  (Details of demand, edge capacity, and feasible paths all extremely important)
  - I. Optimization and network flow
  - II. User vs System Optimal
  - III. Braess' Paradox
  - IV. Nash Equilibrium
  - V. Price of anarchy

User optimal versus system optimal (In the traffic context)

Act on self interests (User Equilibrium):

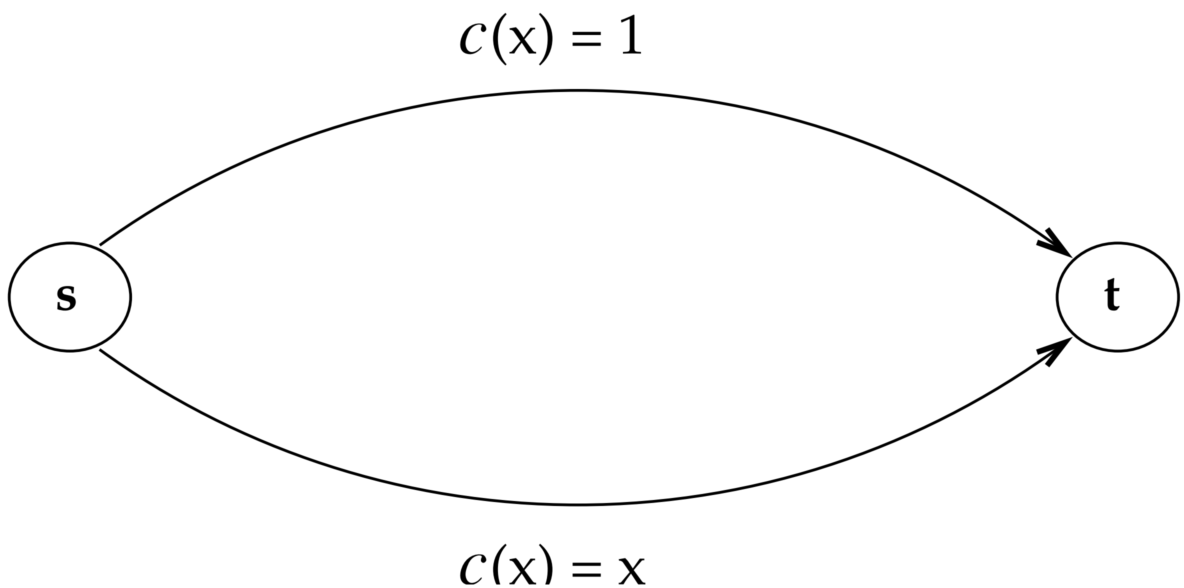
- Travelers have full knowledge of the network and its traffic conditions
- Each traveler minimizes his/her own travel cost (time)

Act on public interests (System Optimal):

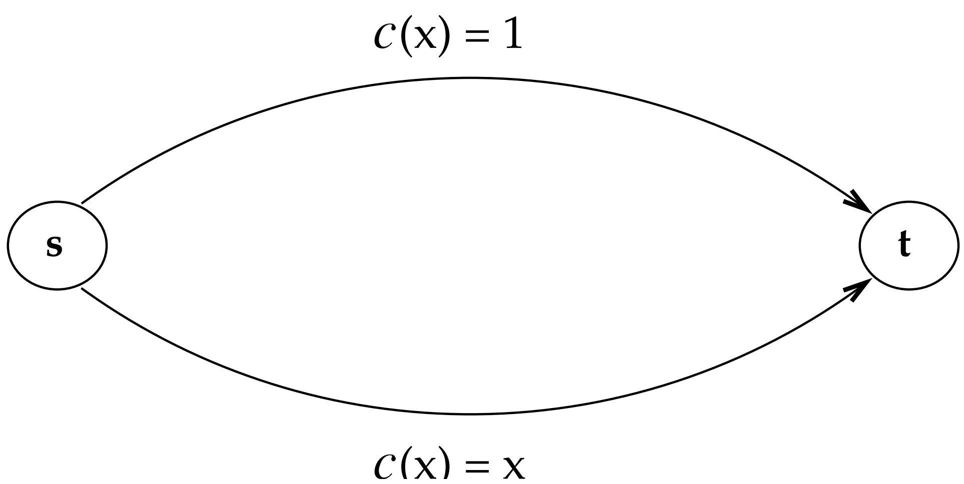
- Travelers choose routes to make the total travel time of all travelers minimal (which can be achieved through choosing the routes with minimal marginal travel cost)

$$\min \sum_{a \in A} t_a(v_a) v_a$$

Pigou's example: User versus system optimal

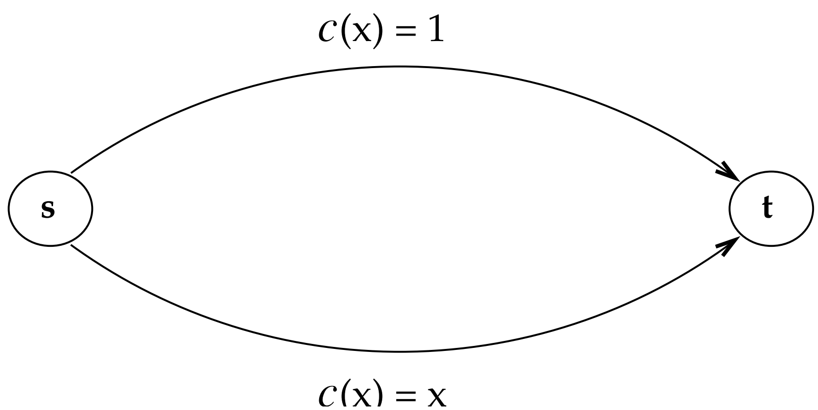


- Two roads connecting source, s, and destination, t
- Route 1, "infinite" capacity but circuitous; 1 hour travel time
- Route 2, direct but easily congested; travel time is 1 hour times the **fraction** of traffic on the route, x_2.
  - Route 1, $c_1 = 1$ hour
  - Route 2, $c_2 = x_2 \cdot 1$ hour.



- Everyone takes the bottom road!
  - It is never worse than the top road, and sometimes better.
  - In general, an equilibrium exists when the travel times on all routes are equal. (See HW and later in lecture.)

Average travel time



- Average travel time: $\tau = x_1 \cdot c_1 + x_2 \cdot c_2$.

- Average travel time in equilibrium = 1 hour = 60 mins

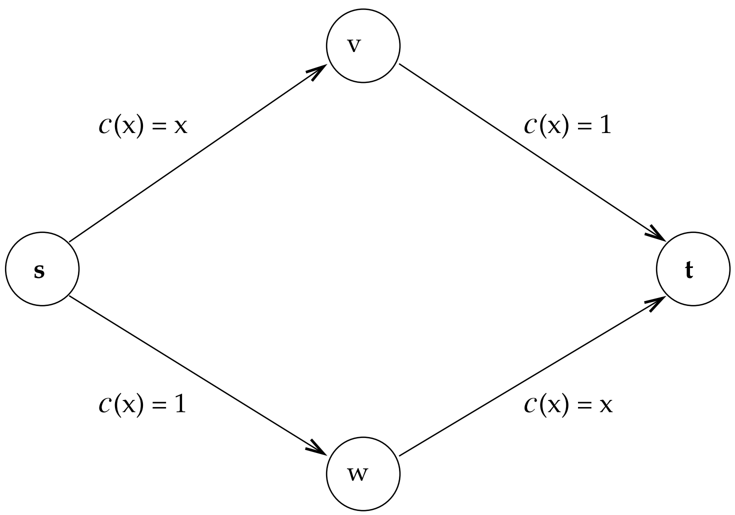
- If could incentivize half the people to take the upper road, then the cost of the lower road is one-half hour.
  - Average travel time: 0.5*1 + 0.5*0.5 = 0.75 hour = 45 mins!

See Michael Zhang's slides (zhang.pdf)

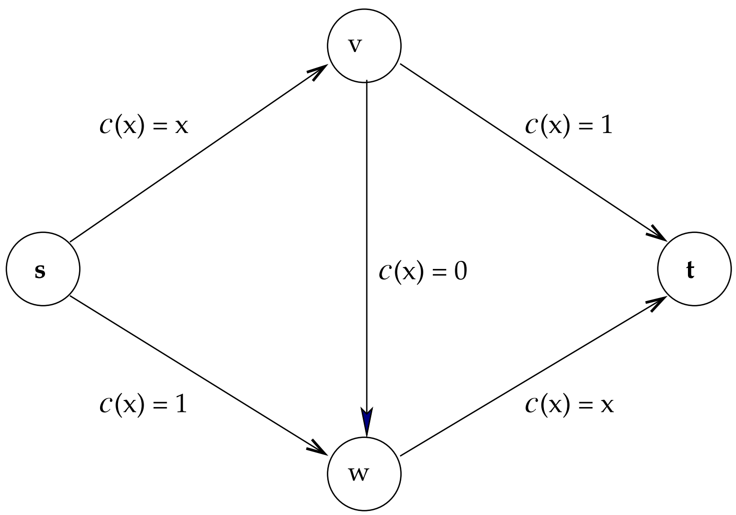
Braess Paradox

- Dietrich Braess, 1968
  (Braess currently Prof of Math at Ruhr University Bochum, Germany)

- In a user-optimized network, when a new link is added, the change in equilibrium flows might result in a higher cost, implying that users were better off without that link.



(a) Initial network



(b) Augmented network

Recall Zhang notation

Flows in a Highway Network

$N:$ set of nodes

$A:$ set of links

$I:$ set of origins

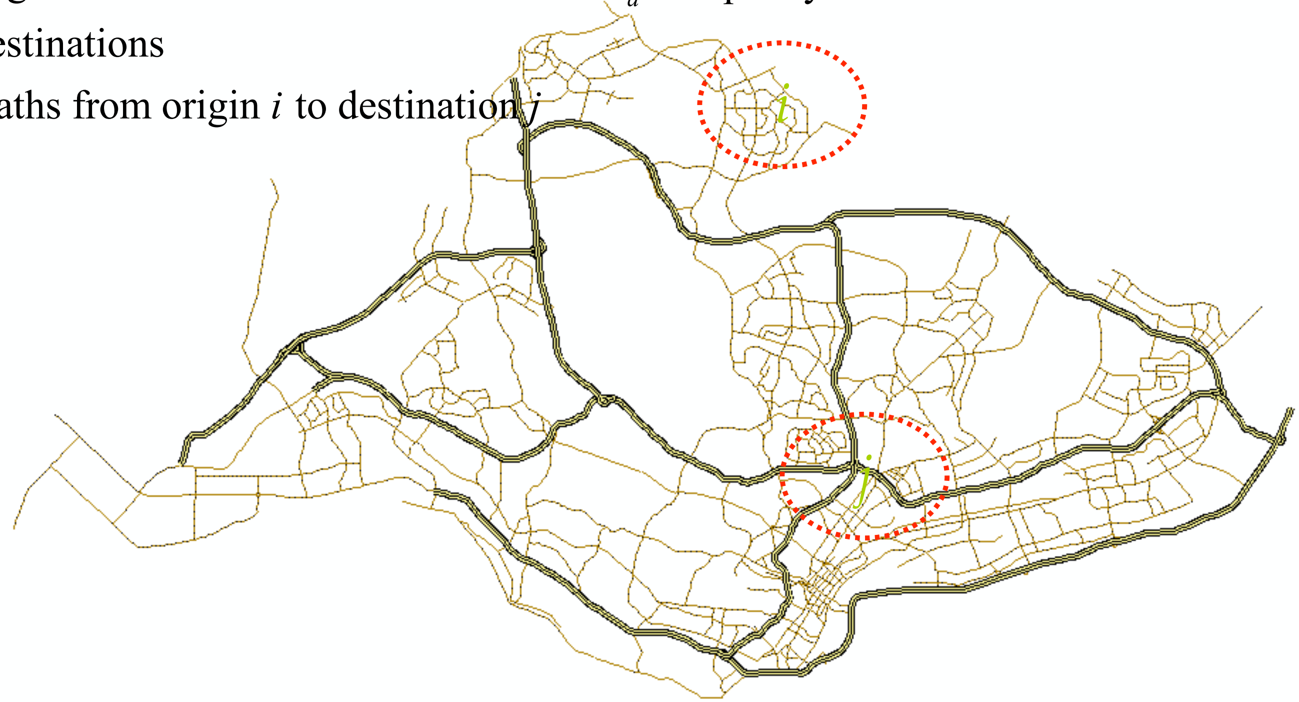
$J:$ set of destinations

$R_{ij}:$ set of paths from origin i to destination j

$t_a(v_a, C_a):$ link travel cost function

$q_{ij}:$ Traffic demand from origin i to destination j

$C_a:$ Capacity on link a



- Recall Zhang notation
  - q_{ij} is overall traffic demand from node i to j.
  - $t_a(\nu_a, C_a)$ is travel cost along link a,
  - which is a function of total flow that link ν_a and capacity C_a.
- Equilibrium is when the cost on all feasible paths is equal

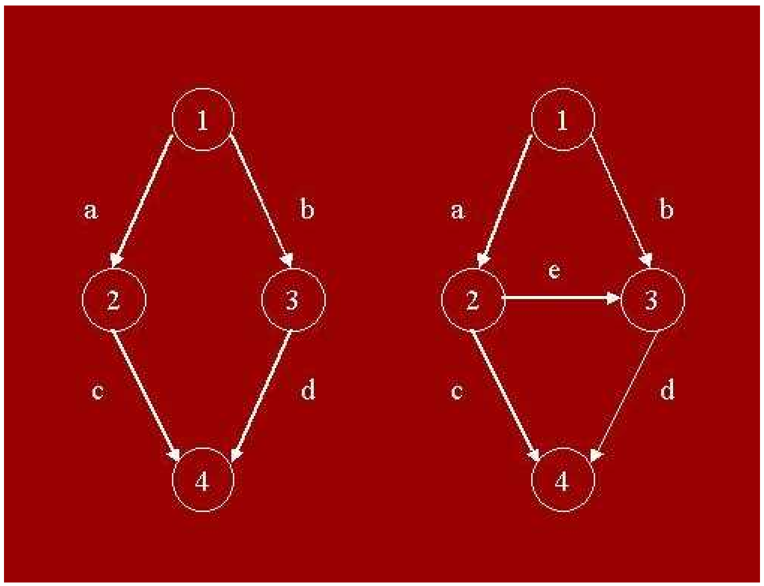
Getting from 1 to 4

Assume traffic demand $q_{14} = 6$. Originally 2 paths (a-c) and (b-d).

- $t_a(\nu_a) = 10\nu_a$
- $t_b(\nu_b) = \nu_b + 50$
- $t_c(\nu_c) = \nu_c + 50$
- $t_d(\nu_d) = 10\nu_d$

$\Longrightarrow$ Eqm: $\nu = 3$ on each link

$\mathbf{C_1 = C_2 = 83}$



Add new link with $t_e(\nu_e) = \nu_e + 10$

Now three paths:

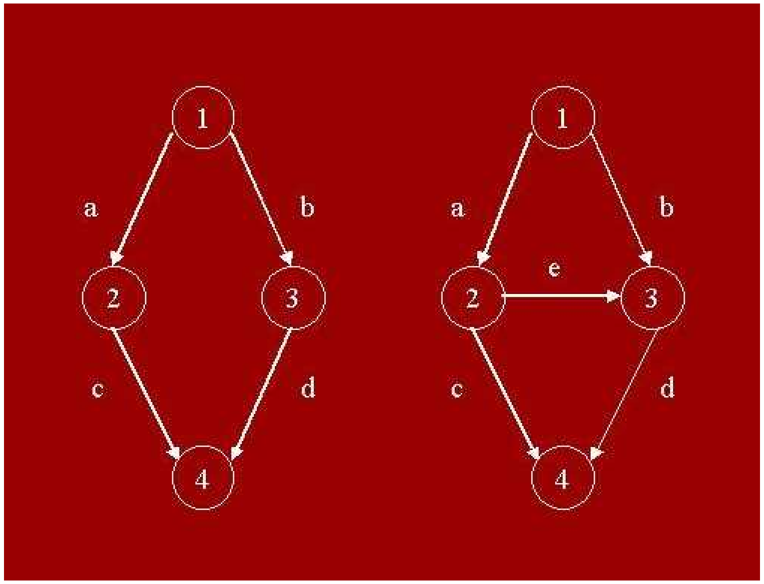
Path 3 (a - e - d), with $\nu_e = 0$ initially, so $C_3 = 0 + 10 + 0 = 10$

$C_3 < C_2$ and C_1 so a new equilibrium is needed.

- By inspection, shift one unit of flow form path 1 and from 2 respectively to path 3.

- Now all paths have flow $f_1 = f_2 = f_3 = 2$.

- Link flow $\nu_a = 4$, $\nu_b = 2$, $\nu_c = 2$, $\nu_d = 4$, $\nu_e = 2$.



$t_a = 40$, $t_b = 52$, $t_c = 52$, $t_d = 40$, $t_e = 12$.

$C_1 = t_a + t_c = \mathbf{92}$; $C_2 = t_b + t_d = \mathbf{92}$; $C_3 = t_a + t_e + t_d = \mathbf{92}$.

- $92 > 83$ **so just increased the travel cost!**

Braess paradox – Real-world examples

- 42nd street closed in New York City. Instead of the predicted traffic gridlock, traffic flow actually improved.

- A new road was constructed in Stuttgart, Germany, traffic flow worsened and only improved after the road was torn up.

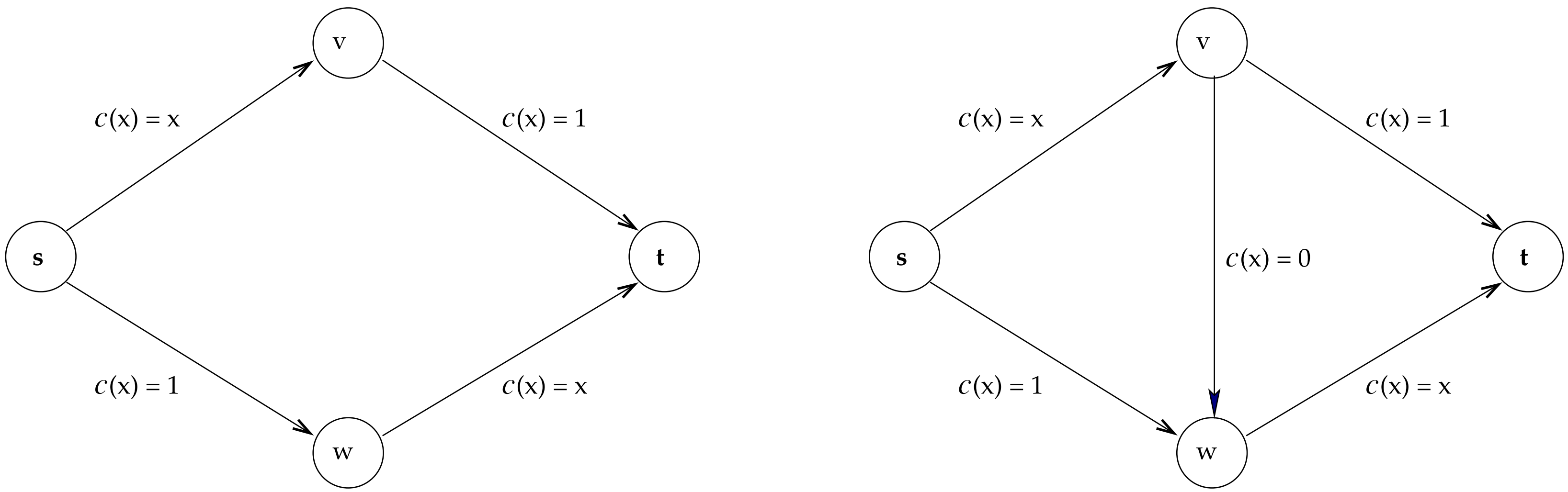
Braess paradox depends on parameter choices

- "Classic" 4-node Braess construction relies on details of q_{14} and the link travel cost functions, t_i.

- The example works because for small overall demand (q_{14}), links a and d are cheap. The new link e allows a path connecting them.

- If instead demand large, e.g. $q_{14} = 60$, now links a and d are costly! ($t_a = t_d = 600$ while $t_b = t_c = 110$). The new path a-e-d will always be more expensive so $\nu_e = 0$. No traffic will flow on that link. So Braess paradox does not arise for this choice of parameters.

Another example of Braess



(a) Initial network (b) Augmented network

How to avoid Braess?

- Back to Zhang presentation .... typically solve for optimal flows numerically using computers. Can test for a range of choices of traffic demand and link costs.

Power grid cascades similarly depend on details

"Small vulnerable sets determine large network cascades in power grids", Yang Yang, Takashi Nishikawa, Adilson E. Motter, *Science* 358, 886 (2017). *See web link*

- The failure of a small set of edges is implicated in large-scale failure. But membership in the set varies with specific details of the operating conditions.

- Intriguing connections to k-core which suggest theoretical understanding of flow-rerouting and vulnerability is possible.

- R. D. Accompanying perspectives piece, "Curtailing cascading failures" *Science 358, 860 (2017)*

More flows and equilibirum

- David Aldous, "Spatial Transportation Networks with Transfer Costs: Asymptotic Optimality of Hub and Spoke Models"
- Marc Barthélemy, "Spatial networks" *Physics Reports* 499 (1), 2011.
- Flows of material goods, self-organization: Helbing et al.
- Jamming and flow (phase transitions): Nishinari, Liu, Chayes, Zechina.
- Algorithmic game theory: Multiplayer games for users connected in a network / interacting via a network.
  - Designing algorithms with desirable Nash equilibrium.
  - Computing equilibrium when agents connected in a network.

User-centric behavior

- Utility functions
- Game theory
  - Normal form games & Nash equilibrium:

    - Prisoner's dilemma
    - Stag hunt

Prisoner's Dilemma

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

Blue has two strategies:

- Cooperates/Red Cooperates — Blue gets payout "3"
- Cooperates/Red Defects – Blue gets "0"
- Defects/Red Defects – Blue gets "1"
- Defects/Red Cooperates – Blue gets "5"

Ave payout: Cooperate = 1.5, Defect = 3

Nash equilibrium

No player has anything to gain by changing only his or her own strategy.

– **Blue always chooses to Defect!** Likewise Red always chooses Defect.

– Both defect and get “1” (Nash), even though each would get a higher payout of “3” if they cooperated (Pareto efficient).

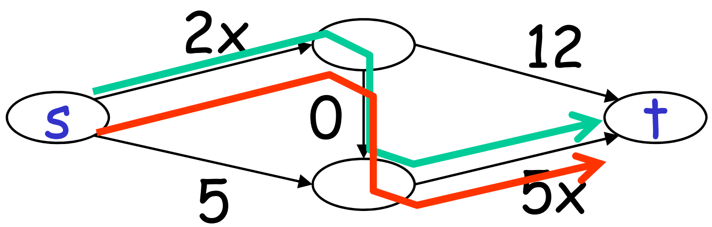
“The price of anarchy”

E. Koutsoupias, C. H. Papadimitriou
“Worst-case equilibria,” STACS 99.

Cost of worst case Nash equilibrium / cost of system optimal solution.

The Price of Anarchy

Nash Equilibrium:

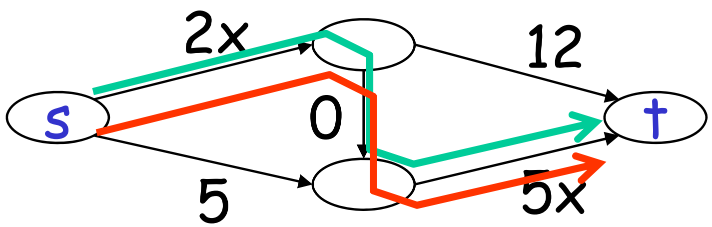


cost = 14+14 = 28

(From Roughgarden Barbados Talk, http://theory.stanford.edu/ tim/)

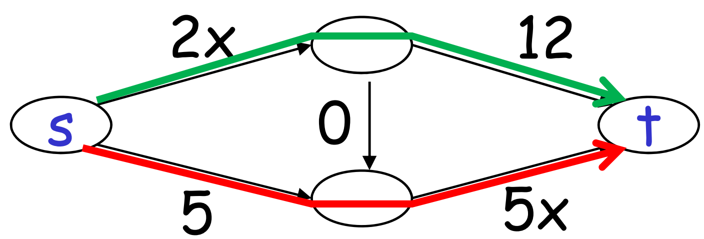
The Price of Anarchy

Nash Equilibrium:



cost = 14+14 = 28

To Minimize Cost:



cost = 14+10 = 24

Price of anarchy = 28/24 = 7/6.

- if multiple equilibria exist, look at the *worst* one

(From Roughgarden Barbados Talk, http://theory.stanford.edu/ tim/)

Selfish routing and the POA on the Internet

T. Roughgarden and E. Tardos, How Bad is Selfish Routing?, FOCS ’00/JACM ’02

- Routing in the Internet is *decentralized*: Each router makes a decision, so path dynamically decided as packet passed on.

- Cost of an edge $c(e)$, may be constant (infinite capacity) or depend on the load.

- *“Shortest path”* routing (really lowest $\sum c(e)$ routing) typically implemented.

- This is equivalent to “selfish routing” (each router chooses best option available to it).

- **Resulting POA = 2!**

Braess and the POA for Internet traffic

Greg Valiant, Tim Roughgarden, Eva Tardos
"Braess's paradox in large random graphs", Proceedings of the 7th ACM conference on Electronic commerce, 2006.

- Removing edges from a network with "selfish routing" can decrease the latency incurred by traffic in an equilibrium flow.

- With high probability, (as the number of vertices goes to infinity), there is a traffic rate and a set of edges whose removal improves the latency of traffic in an equilibrium flow by a constant factor.

- Braess paradox found in random networks often (not just "classic" 4-node construction).

Algorithmic game theory

- Since we know users act according to Nash, can we design algorithms (mechanisms) that bring Nash and System Optimal as close together as possible?

- Typically we think of players who interact via a network, or who's connectivity is described by a network of interactions.

  – Multiplayer games for users connected in a network or interacting via a network.

  – Designing algorithms with desirable Nash equilibrium.

  – Computing equilibrium when agents connected in a network.

mechanism design
(or *inverse* game theory)

- agents have utilities – but these utilities are known *only to them*
- game designer prefers certain outcomes *depending on players' utilities*
- designed game (mechanism) has designer's goals as dominating strategies

(Papadimitriou, "Algorithms, Games, and the Internet" presented at STOC/ICALP 2001.)

Some traditional games:

e.g.

matching pennies

1,-1	-1,1
-1,1	1,-1

prisoner's dilemma

3,3	0,4
4,0	1,1

chicken

0,0	0,1
1,0	-1,-1

auction

	1 … n
1	$0,\ v-y$
.	
.	$u-x,\ 0$
n	

(Papadimitriou, "Algorithms, Games, and the Internet" presented at STOC/ICALP 2001.)

Mechanism design example:

e.g., Vickrey auction

- sealed-highest-bid auction encourages gaming and speculation
- Vickrey auction: Highest bidder wins,

  pays second-highest bid

Theorem: Vickrey auction is a truthful mechanism.

Theorem: It maximizes social benefit *and* auctioneer expected revenue.

(Papadimitriou, “Algorithms, Games, and the Internet” presented at STOC/ICALP 2001.)

(Modified) Vickrey auctions in real life – Google AdWords, and Yahoo's ad sales

- Bidding on a "keyword" so that your advertisement is displayed when a search user enters in this keyword
- You can safely bid the maximum price you think is fair, and if you win, you may actually pay less!
- Mechanism design
  – Incentivizes users to bid what they think is fair (reveal their true utilities)
  – Keeps more people in the bidding
  – Does not necessarily maximize profits for seller

Summary of spatial flows and games

- Optimal location of facilities to maximize access for all.
- Designing “optimal” spatial networks (collection/distribution networks – subways, power lines, road networks, airline networks).
- Details of flows on actual networks make all the difference!
  – Users act according to Nash
  – Braess paradox (removing edges may improve a network’s performance!)
  – The “Price of Anarchy” (cost of worst Nash eqm / cost of system optimal)
- Mechanism design / algorithmic game theory