Temporal Networks

aka time-varying networks, time-stamped graphs, dynamical networks...

Network Theory and Applications

ECS 253 / MAE 253

Spring 2016

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Sources

Reviews:

Physics Reports 519 (2012) 97–125

Contents lists available at SciVerse ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Temporal networks

Petter Holme [a,b,c,*], Jari Saramäki [d]

Eur. Phys. J. B (2015) 88: 234
DOI: 10.1140/epjb/e2015-60657-4

THE EUROPEAN PHYSICAL JOURNAL B

Colloquium

Modern temporal network theory: a colloquium*

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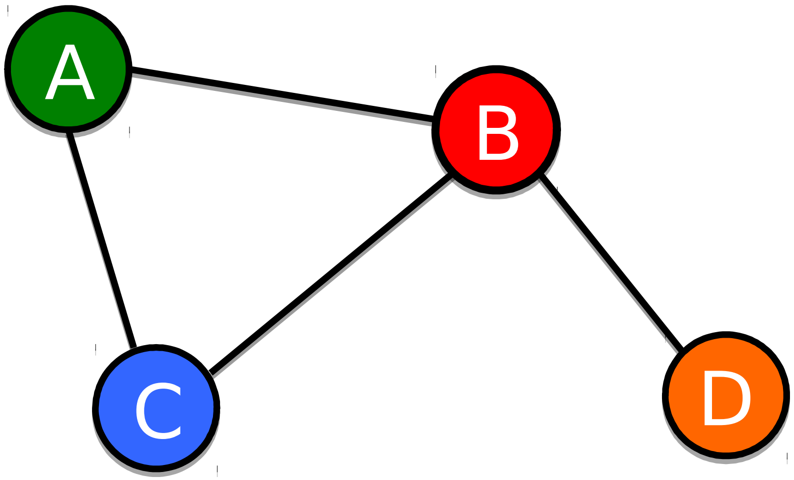
Courses:

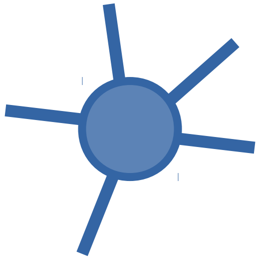
- ENS Lyon: Márton Karsai
- Northeastern University: Nicola Perra, Sean Cornelius, Roberta Sinatra

Temporal Networks

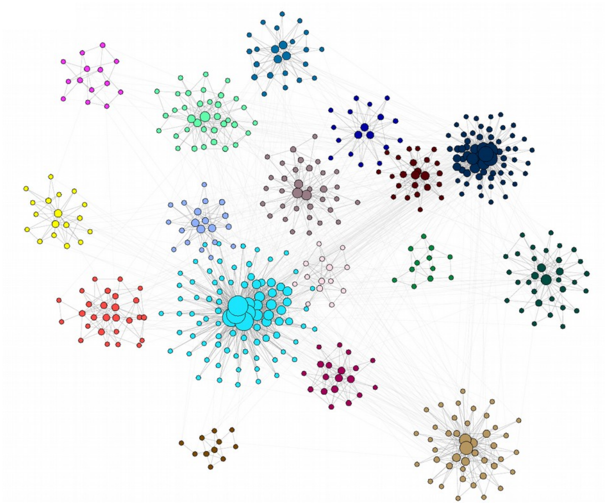
- So far: static network



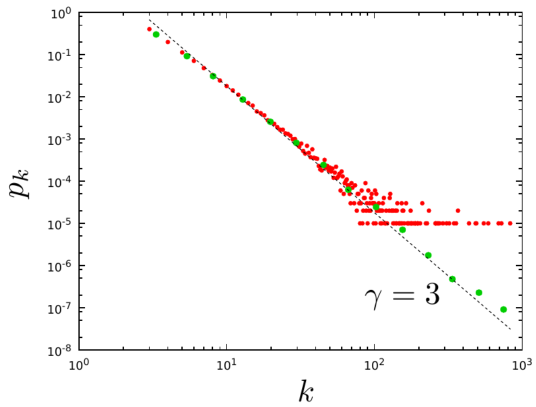
- Description:



Microscopic:
Node, link properties
(degree, centralities)



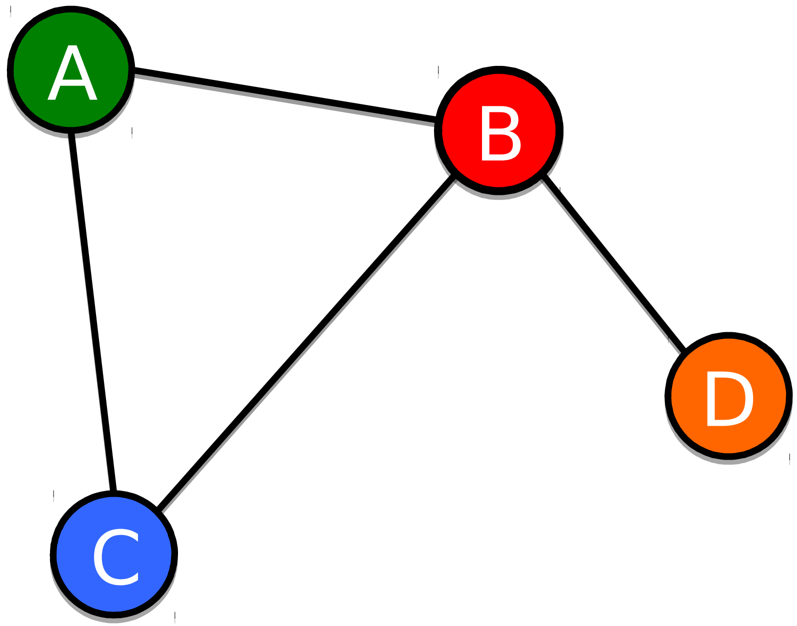
Mezoscopic:
Motives, communities



Macroscopic:
statistics

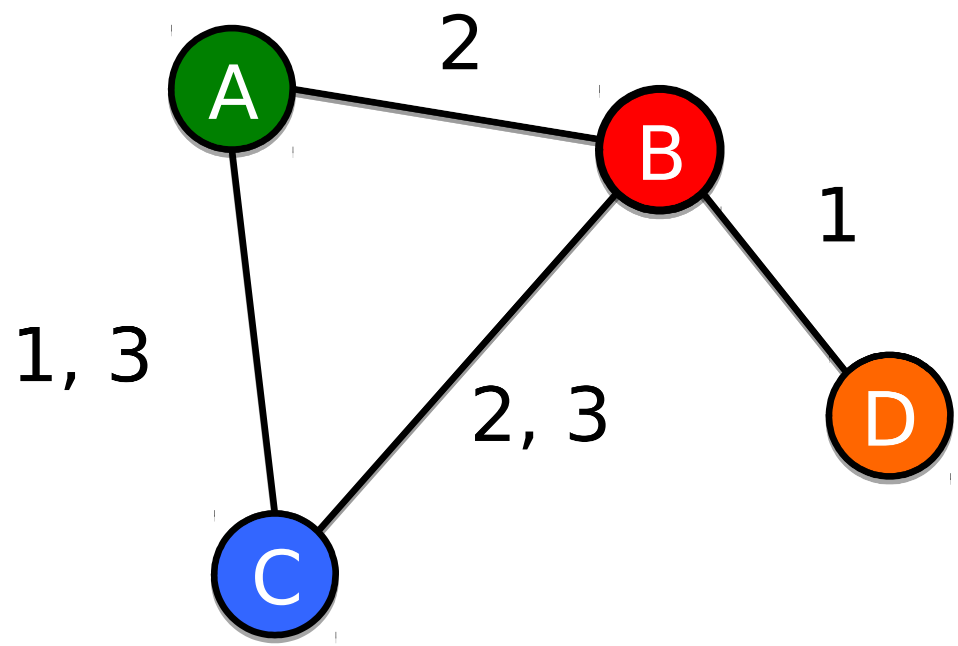
Temporal Networks

- Static network: Spreading process can reach all nodes starting from A.



Temporal Networks

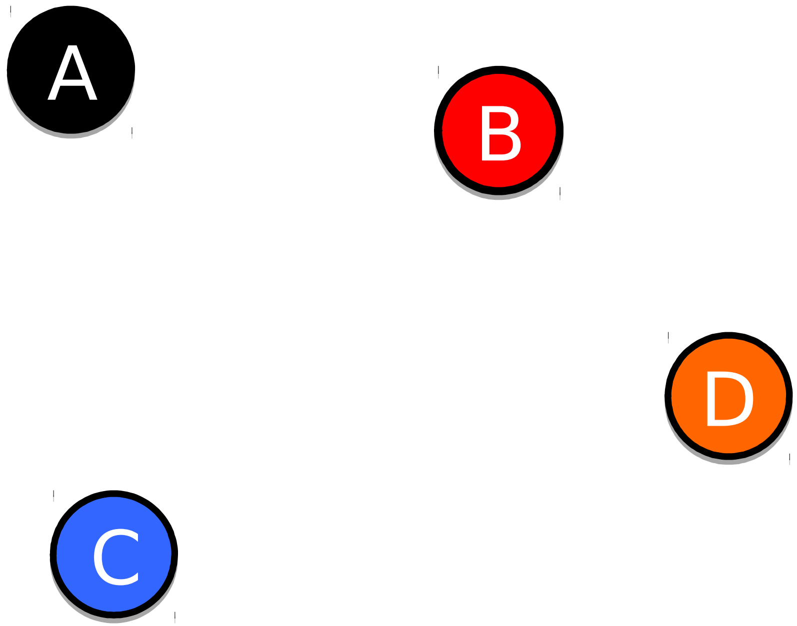
- Now: time of interaction



Temporal Networks

- Now: time of interaction

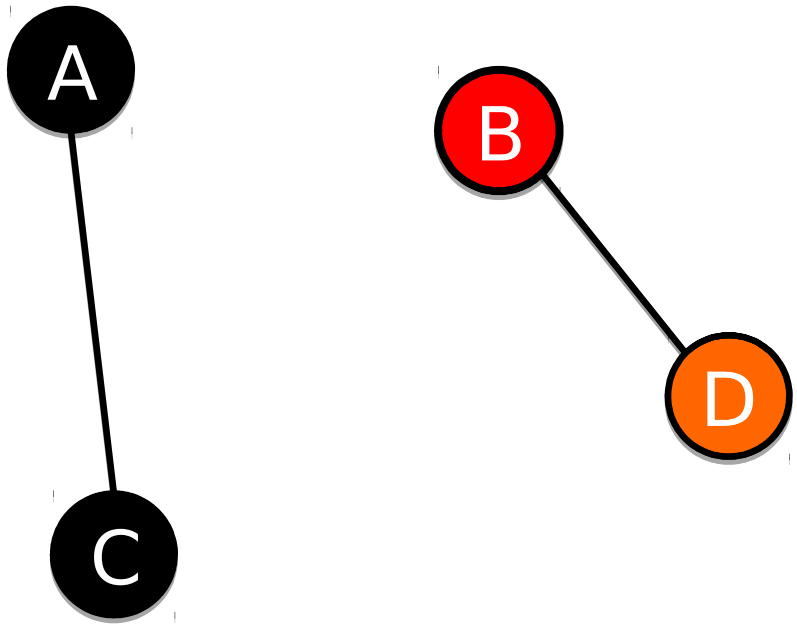
t=0



Temporal Networks

- Now: time of interaction

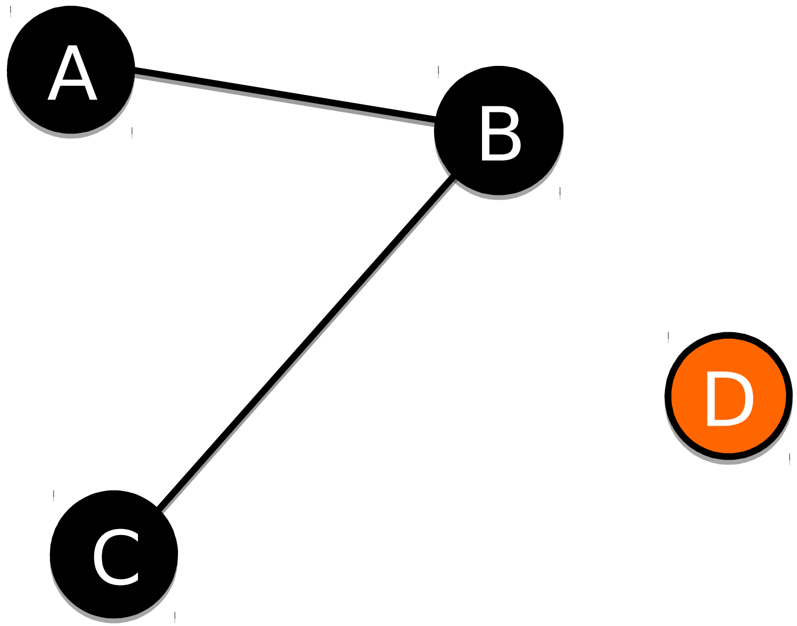
t=1



Temporal Networks

- Now: time of interaction

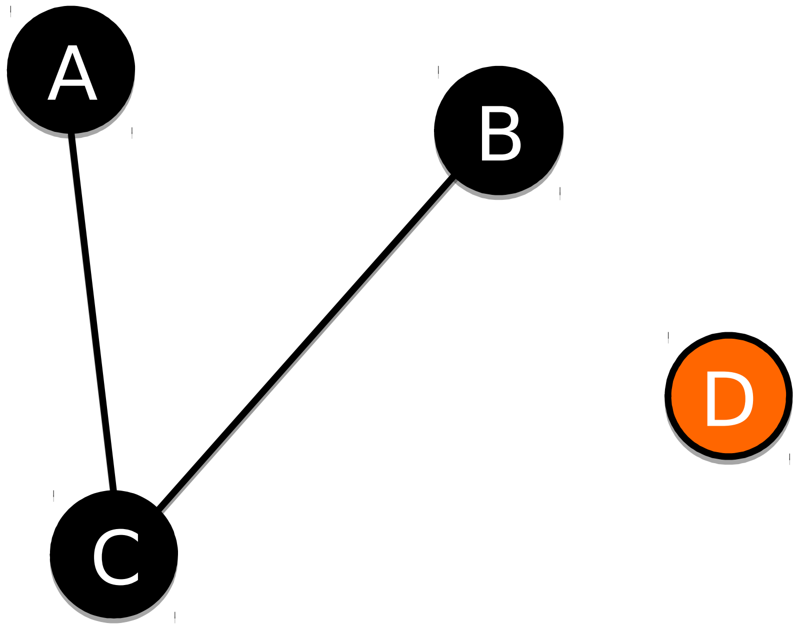
t=2



Temporal Networks

- Now: time of interaction

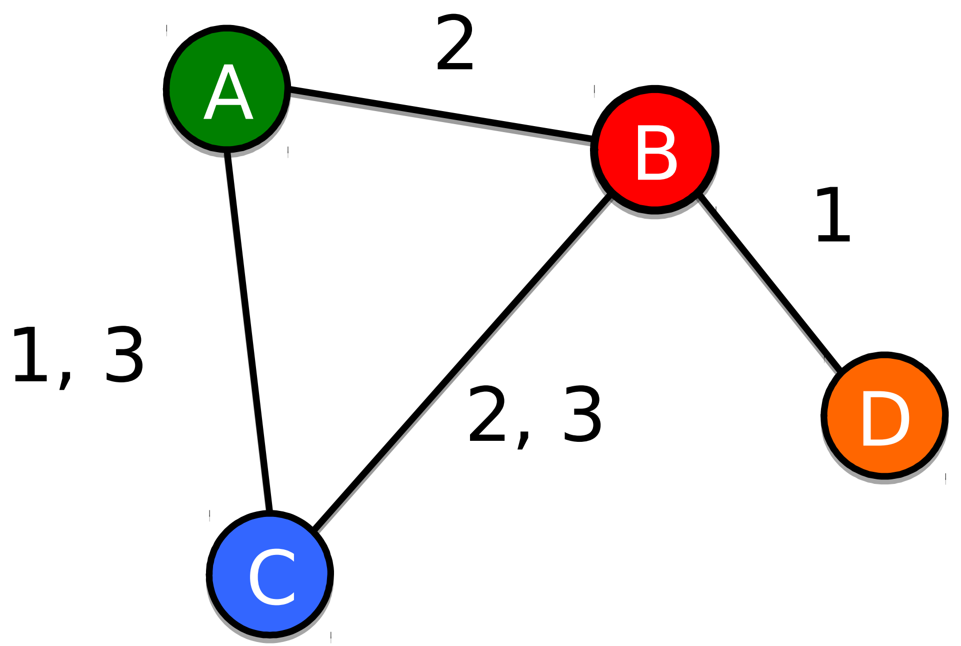
t=3



Temporal Networks

- Now: time of interaction

t=0



?

?

?

Microscopic:
Node, link properties
(degree, centralities)

Mezoscopic:
Motives, communities

Macroscopic:
statistics

When are temporal nets useful?

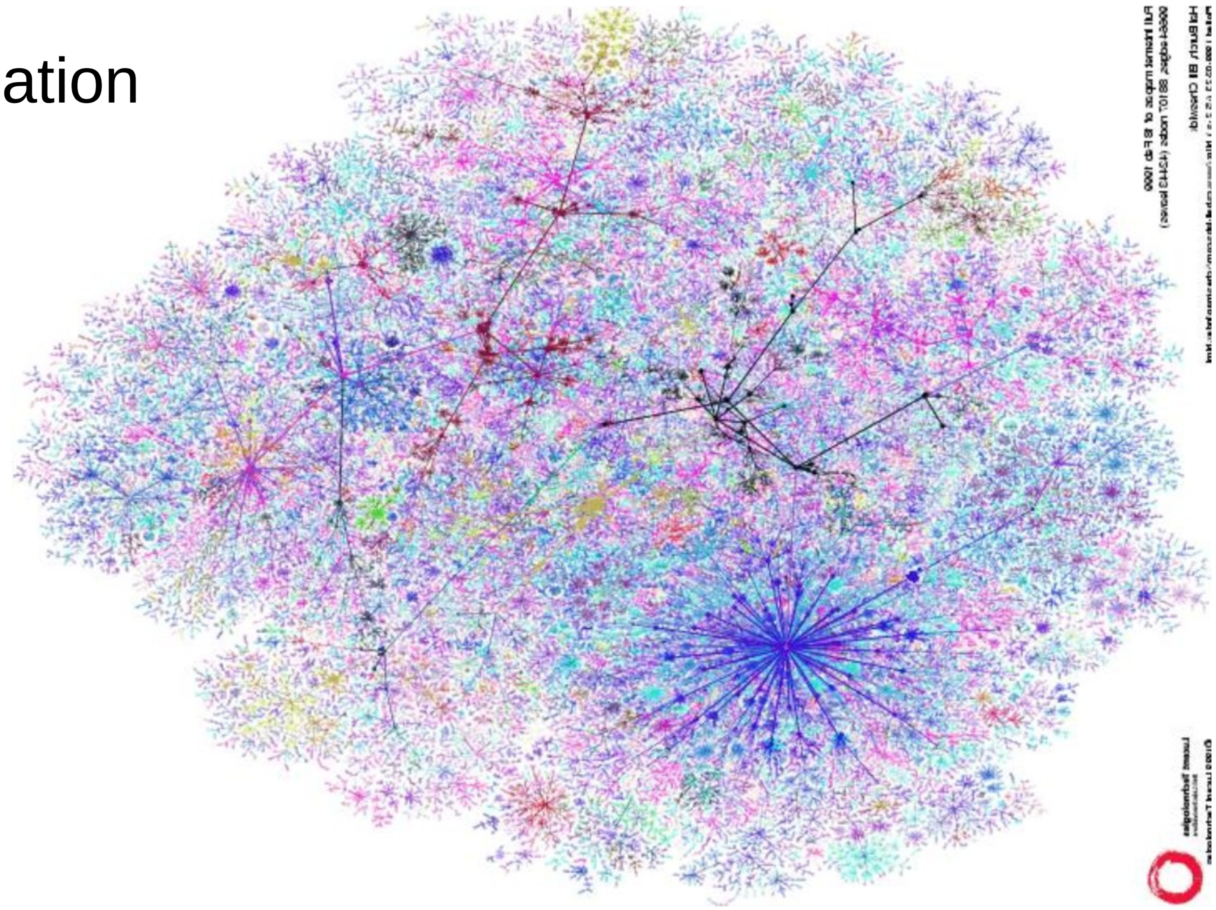
- Timescales:

τ_{D} : timescale of dynamics

τ_{N} : timescale of changes in network

- $\frac{\tau_{\mathrm{D}}}{\tau_{\mathrm{N}}} \gg 1$: static approximation

Example: Internet



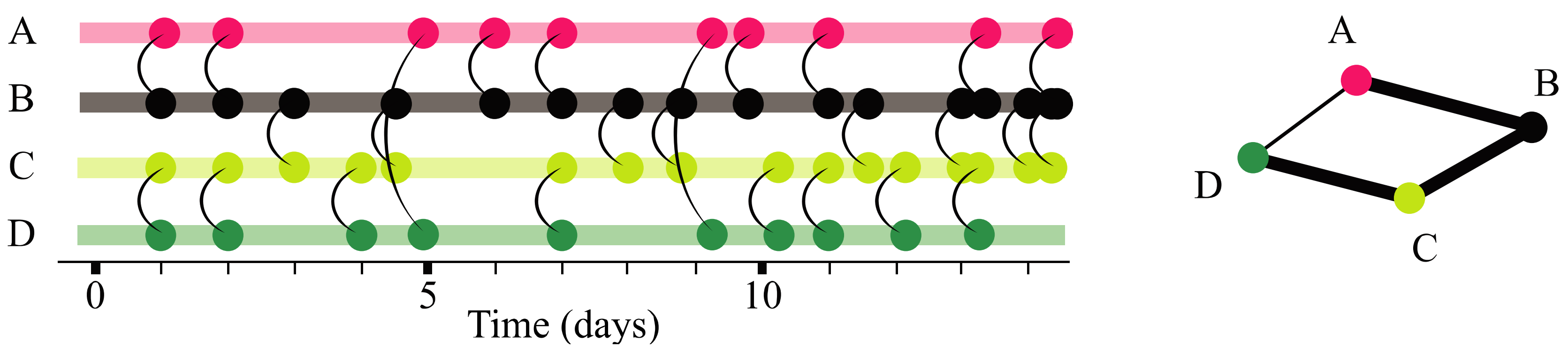
Branigan et al, *Internet Computing, IEEE* (2001)

When are temporal nets useful?

- Timescales:

τ_{D} : timescale of dynamics
τ_{N} : timescale of changes in network

- $\frac{\tau_{\mathrm{D}}}{\tau_{\mathrm{N}}} \ll 1$: annealed approximation



Example: PC or Mac

- Process slow enough that A meets all contacts
- Weight: how frequently they meet

P. Holme and J. Saramäki. "Temporal networks." Physics reports 519.3 (2012): 97-125.

When are temporal nets useful?

- Timescales:

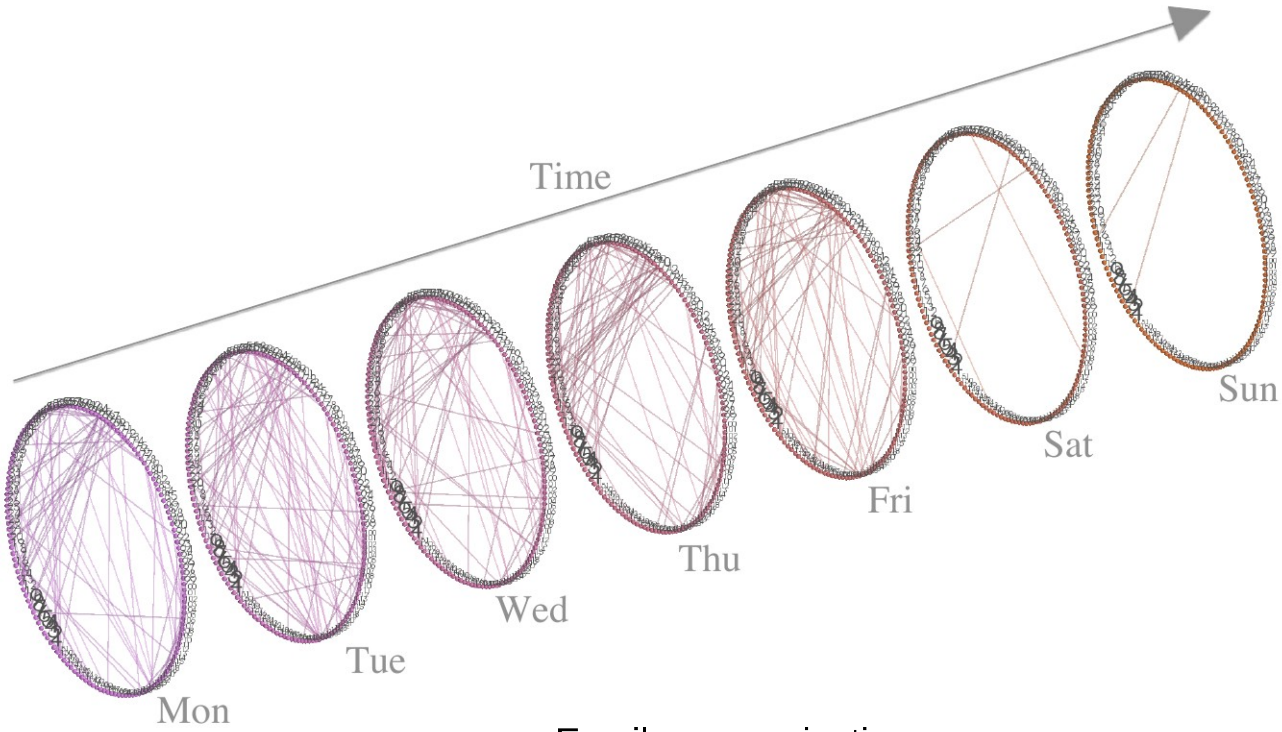
$\tau_{\rm D}$: timescale of dynamics
$\tau_{\rm N}$: timescale of changes in network

- $\dfrac{\tau_{\rm D}}{\tau_{\rm N}} \sim 1$:

TEMPORAL NETWORKS

Examples

- Communication: Email, phone call, face-to-face
- Proximity: same hospital room, meet at conference, animals hanging out
- Transportation: train, flights…
- Cell biology: protein-protein, gene regulation
- ...



Email communication

J. Tang, John, Social Network Systems. ACM, 2010.

Examples

- Communication: Email, phone call, face-to-face
- Proximity: same hospital room, meet at conference, animals hanging out
- Transportation: train, flights…
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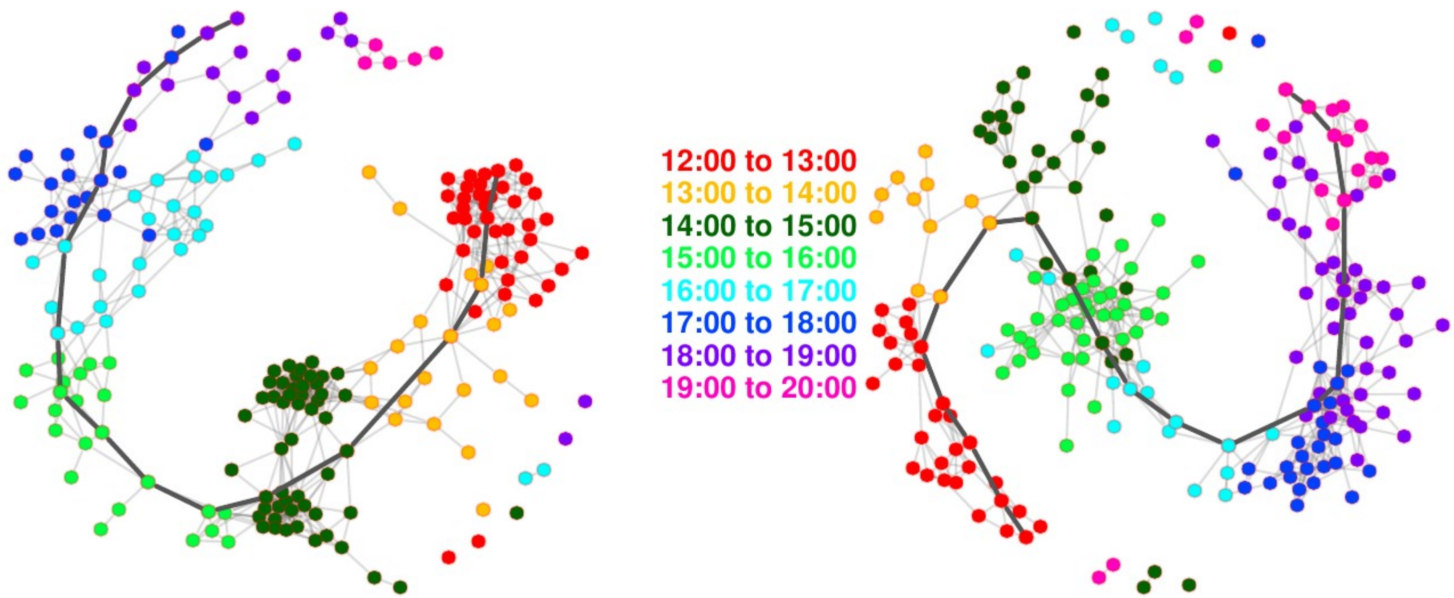
- ...



Visitors at exhibit.

Isella, Lorenzo, et al. Journal of theoretical biology 271.1 (2011): 166-180.

Examples

- Communication: Email, phone call, face-to-face
- Proximity: same hospital room, meet at conference, animals hanging out
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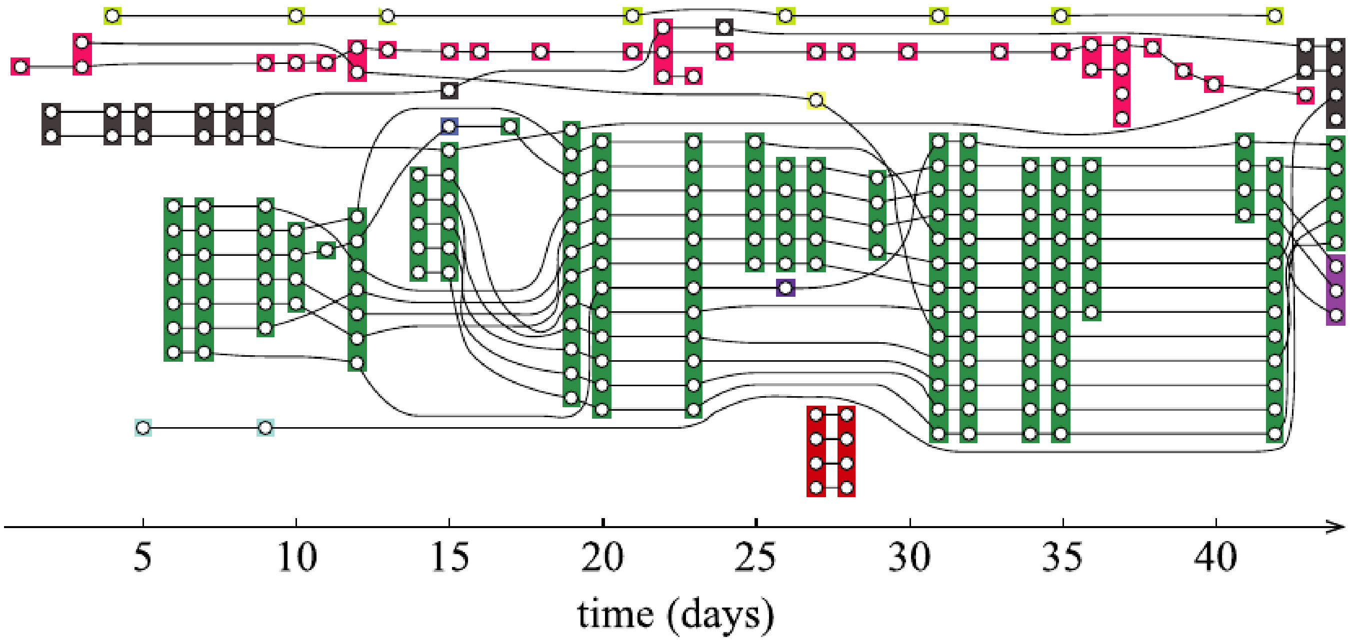


Temporal network of zebras
C. Tantipathananandh, et.al. (2007)

Plan

1) Mathematical representation
2) Path- based measures of temporal
3) Temporal heterogeneity
4) Processes and null models
5) Motifs

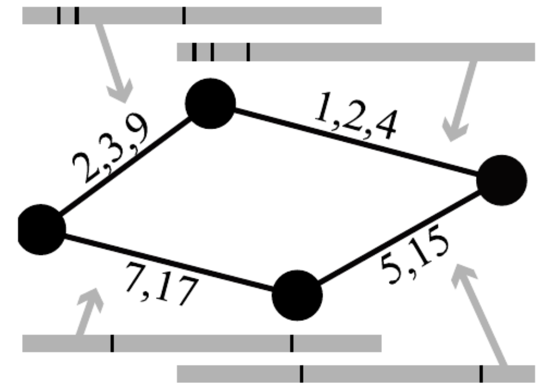
Mathematical Description

Mathematical representation

- **Temporal graph:** $G_t = (V, E, T_e)$

Set of vertices. Set of edges. $T_e = \{t_1, t_2, ..., t_n\}$

Set of times when edge *e* is active.



1. Contact sequence

$$E \subset T \times V \times V$$

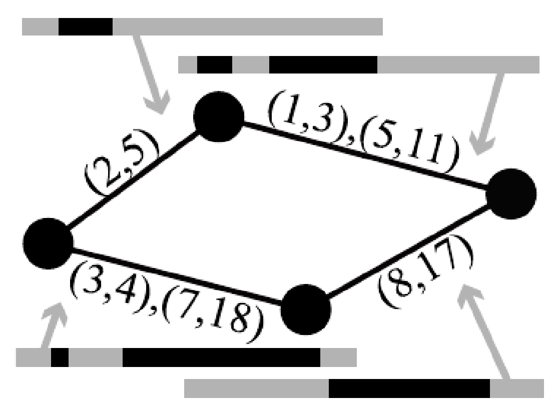
2. Adjacency matrix sequence

$$A_{ij}(t)$$

- **Interval graph:** $G_t^d = (V, E, T_e^d)$

$$T_e^d = \{(t_1, t_1'), ..., (t_n, t_n')\}$$

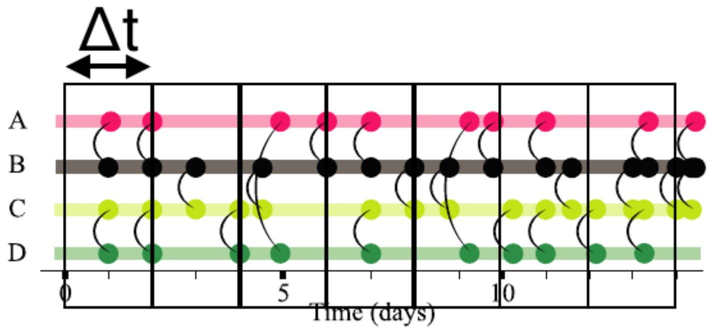
Set of intervals when edge *e* is active.



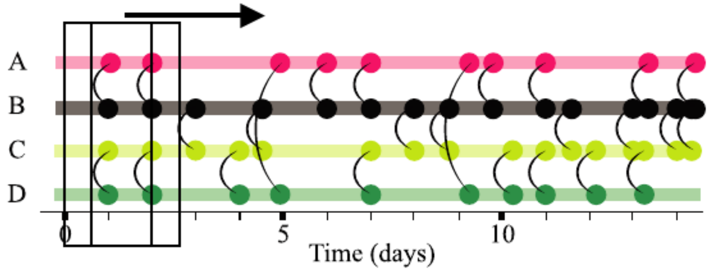
P. Holme and J. Saramäki. "Temporal networks." Physics reports 519.3 (2012): 97-125.

Aggregating in time windows

- Sequence of snapshots



Consecutive windows



Sliding windows

- Lossy method
- Sometimes data is not available
- Convenient: Static measures on snapshots → Time series of measures

- Problem: snapshots depend on window size?
- How to choose?

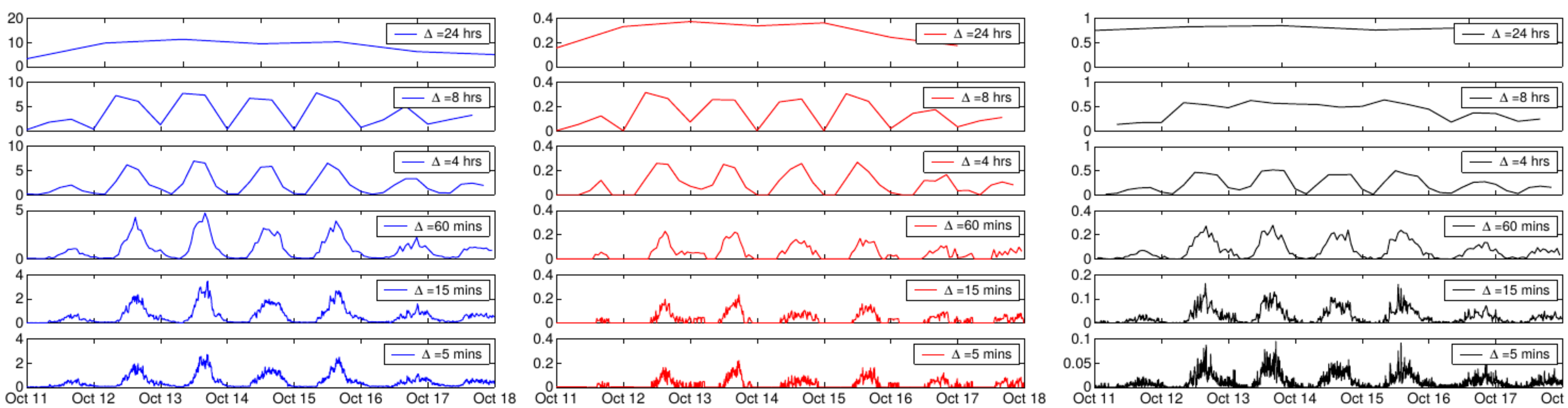
Window size?

- MIT reality mining project: high resolution proximity data
- Snapshots: $A^{(1)}, A^{(2)}, \ldots, A^{(T)}$ Time window: Δ
- Adjacency correlation:

$$\gamma_j = \frac{\sum_{i \in N(j)} A_{i,j}^{(x)} \; A_{i,j}^{(y)}}{\sqrt{\left(\sum_{i \in N(j)} A_{i,j}^{(x)}\right) \left(\sum_{i \in N(j)} A_{i,j}^{(y)}\right)}}$$

$N(j)$: set of nodes that are connected to *j* at *x* of *y*

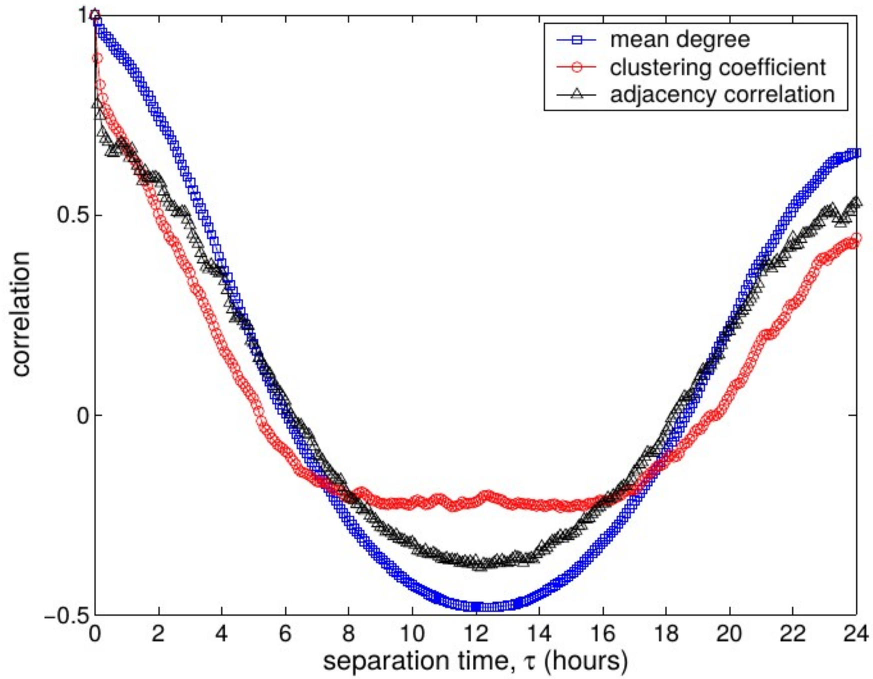
- 0 uncorrelated, 1 if the same



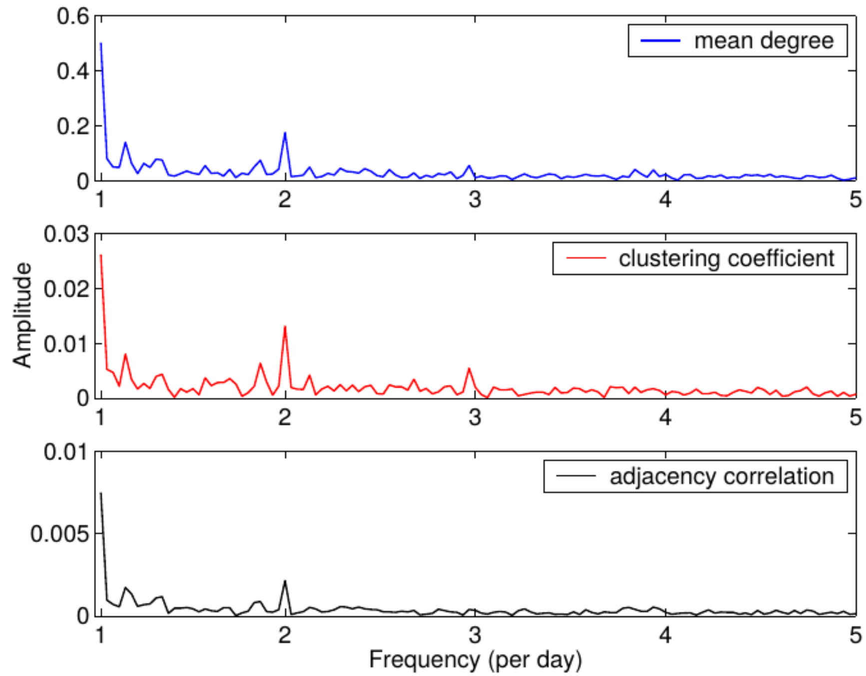
Average degree Clustering coef. 1-Adjacency corr.

Clauset and Eagle (2012)

Window size?



Time series autocorrelation
for Δ=5 min



Fourrier spectrum

Driven by periodic patterns → Sampling rate should be twice the highest frequency

$\Delta = 4$ hours

Clauset and Eagle (2012)

Path-based measures

Path-based measures

Time-respecting paths:

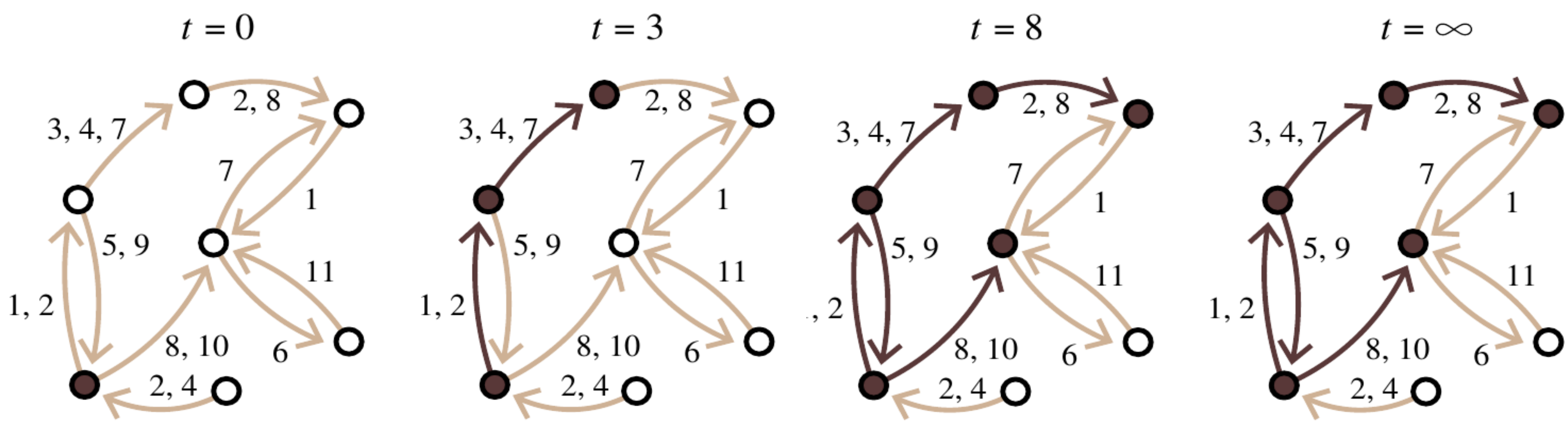
- Takes into account the temporal order and timing of contacts.

$$\{(i,k,t_1),(k,l,t_2),\ldots,(p,j,t_n)\} \qquad t_1<t_2<\ldots<t_n$$

- Optional: maximum wait time

Properties:

- **No reciprocity:** path $i \to j$ does not imply path $j \to i$.
- **No transitivity:** path $i \to j$ and path $j \to k$ does not imply path $i \to j \to k$.
- **Time dependence:** path $i \to j$ that starts at t does not imply paths at $t' > t$.



P. Holme, Physical Review E 71.4 (2005)

Path-based measures

Observation window $[t_0, t_1]$

Influence set of node *i*

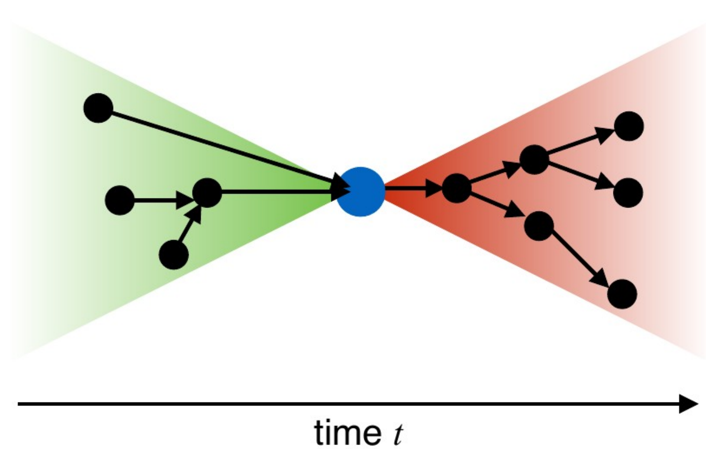
- Nodes that can be reached from node *i* within the observation window.
- Reachability ratio *f*: fraction of nodes that can be reached

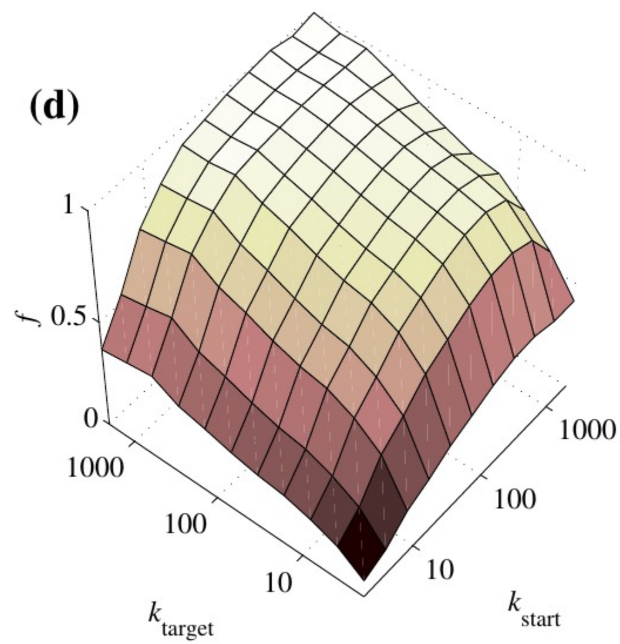
Source set of node *i*

- Nodes from node *i* is reached within the observation window.

Reachability ratio

- Fraction of node pairs *(i,j)* such that path $i \rightarrow j$ exists.





P. Holme, Physical Review E 71.4 (2005)

Path-based measures

Temporal path length – Duration

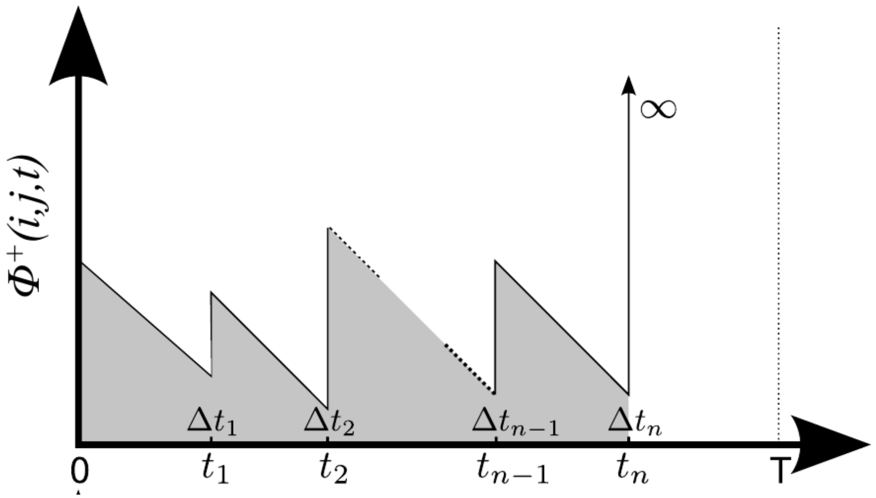
- Duration = $t_n - t_0$

Temporal distance – Latency

- $\Phi^{+}{}_{i,t}(j)$ the shortest (fastest) path duration from *i* to *j* starting at *t.*

Information latency

- $\lambda_{i,t}(j)$ the age of the information from *j* to *i* at *t*



- End of the observation window: paths become rare.
- Solution: periodic boundary, throw away end

R. K. Pan, and J. Saramäki.Physical Review E 84.1 (2011)

Path-based measures

Strongly connected component

- All node pairs are connected in both directions within *T*.

Weakly connected component

- All node pairs are connected in at least one direction within *T*.

Temporal betweenness centrality

- Static:

$$b(i) = \frac{\sum_{i \neq j \neq k} v_{jk}(i)}{\sum_{j \neq k} v_{jk}}$$

Temporal:

$$b(i,T) = \frac{\sum_{i \neq j \neq k} v_{jk}(i,T)}{\sum_{j \neq k} v_{jk}(T)}$$

Etc.

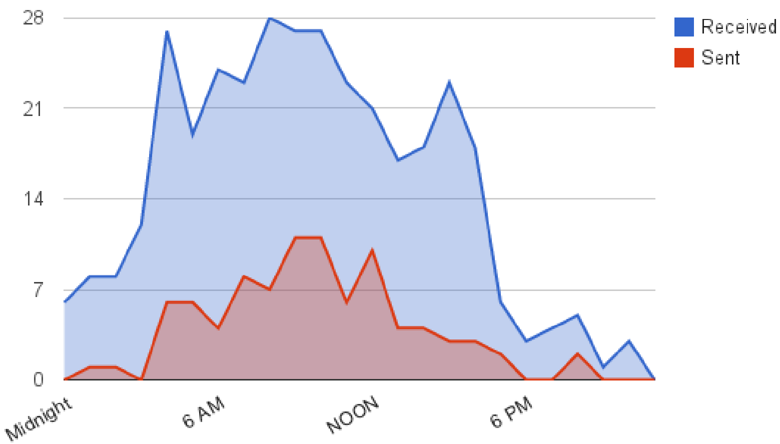
R. K. Pan, and J. Saramäki.Physical Review E 84.1 (2011)

Temporal heterogeneity

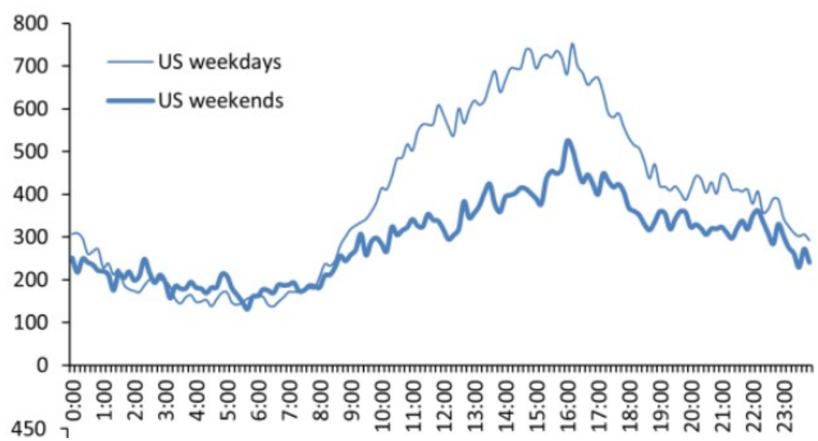
Source 1: Periodic patterns

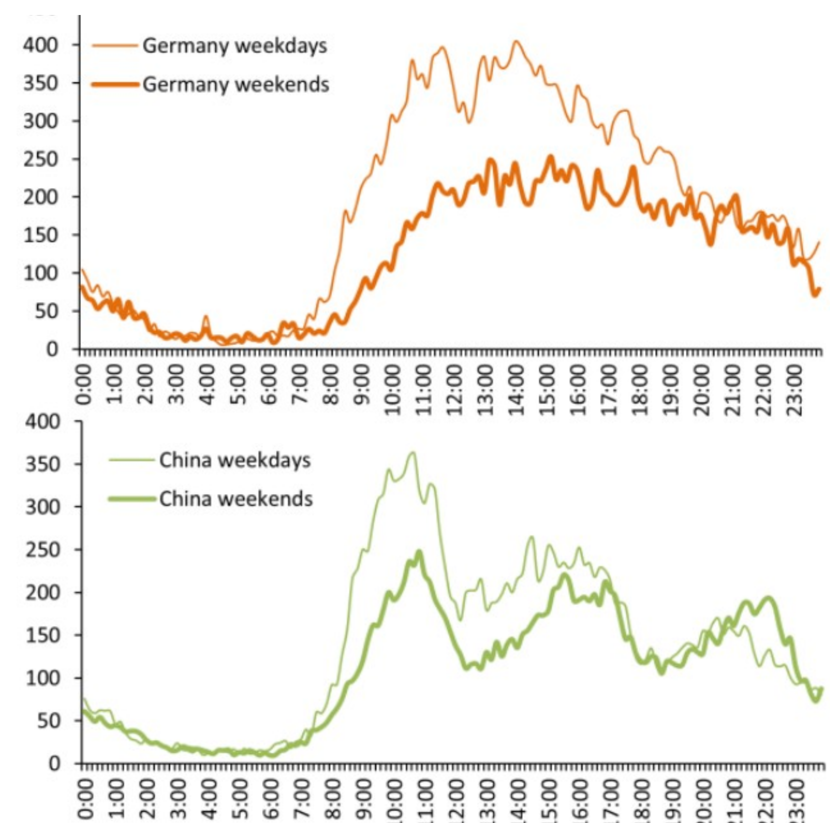
For example, circadian rythm

- My email usage



- Scientists work schedule

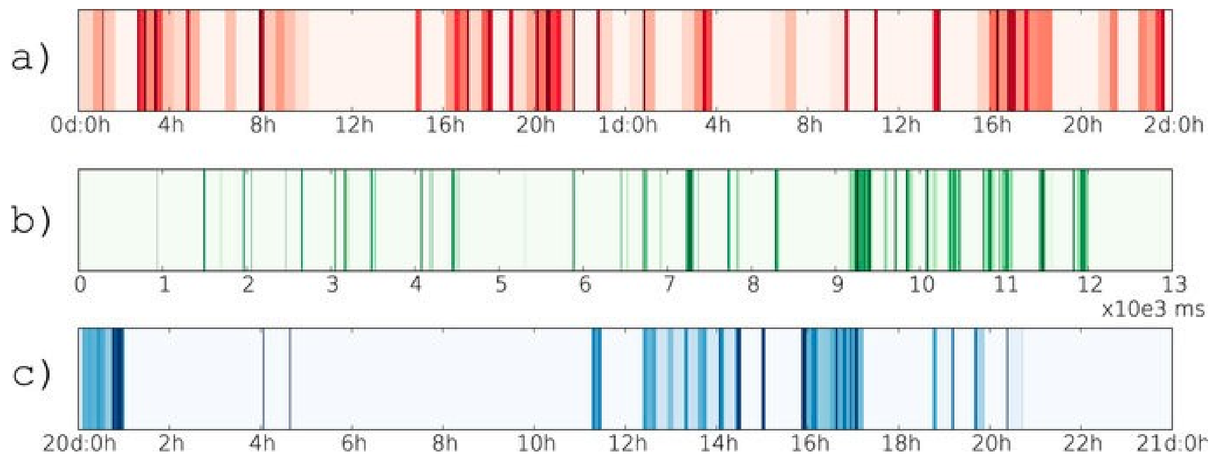




Wang et al, *Journal of infometrics* (2012)

Source 2: Burstiness

- Humans and many natural phenomena show heterogeneous temporal behavior on the individual level.
- Switching between periods of low activity and high activity bursts.
- Sign of correlated temporal behavior



Earthquakes in Japan

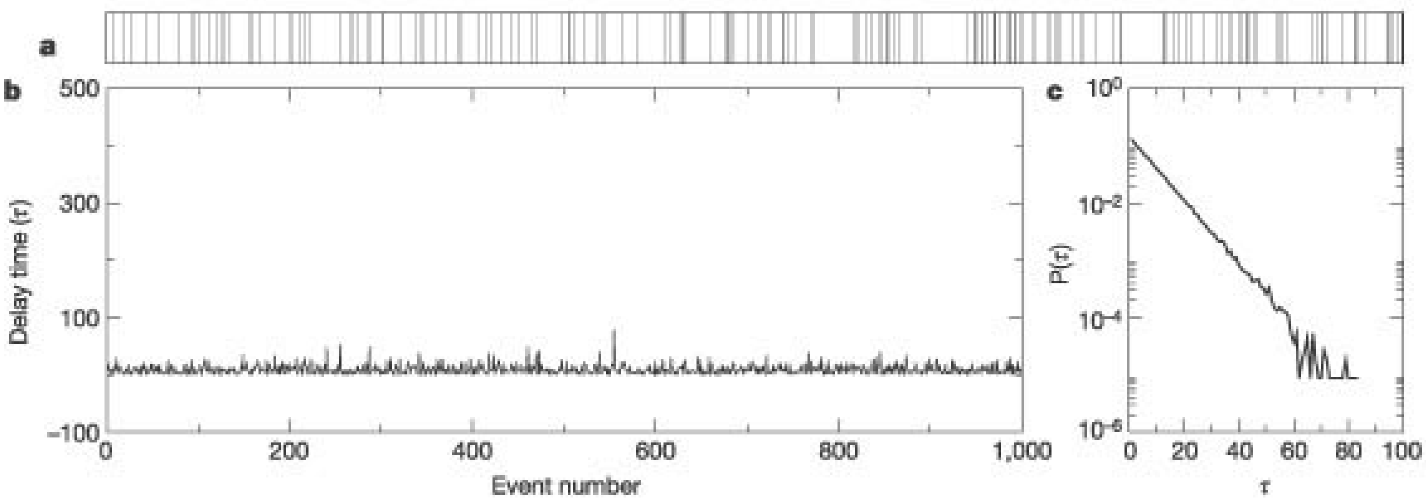
Neuron firing

Phone calls

Karsai et al, *Scientific Reports* (2011)

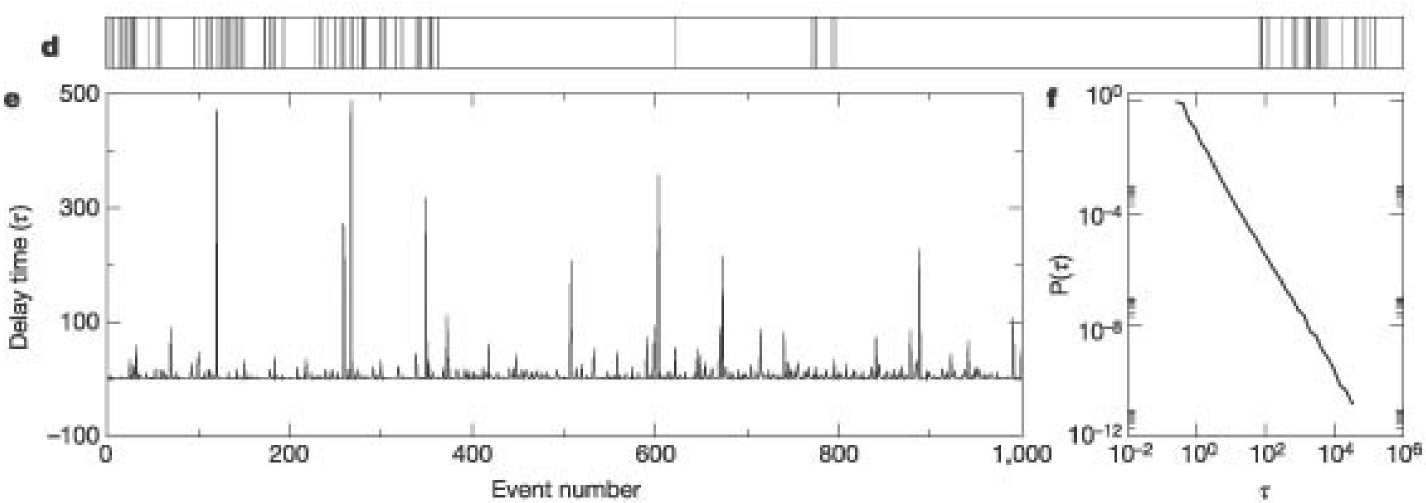
Source 2: Burstiness

- Sign of correlated temporal behavior
- Reference: Poisson process, events uncorrelated



$$P(\tau) \sim e^{-q\tau}$$

Poisson process



$$P(\tau) \sim \tau^{-\gamma}$$

Bursty signal

Barabási, *Nature* (2005)

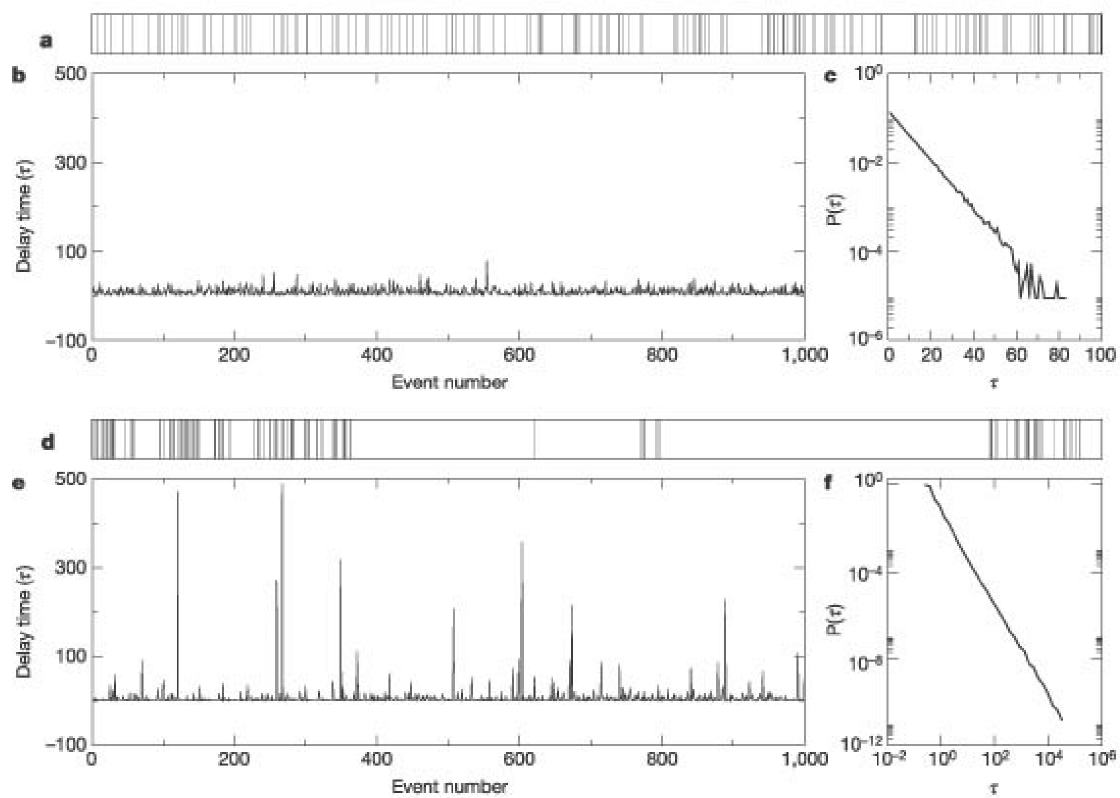
Source 2: Burstiness

- Measure of burstiness: $$B \equiv \frac{(\sigma_\tau/m_\tau - 1)}{(\sigma\tau/m_\tau + 1)} = \frac{(\sigma_\tau - m_\tau)}{(\sigma_\tau + m_\tau)}$$

m_τ - average inter-event time

σ_τ - STD of inter-event time

B=-1	B=0	B=1
Max. regular	Poisson	Max bursty



Barabási, *Nature* (2005)

Possible explanation for burstiness

- Executing tasks based on prioirity
- *L* types of tasks, one each (e.g. work, family, movie watching,...)
- Each task *i* has a priority x_i draw uniformly from [0,1]
- Each timestep one task is executed, probability of choosing *i*:

$$\Pi(i) = \frac{x_i^\gamma}{\sum_{j=1}^{L} x_j^\gamma}$$

- And a new task is added of that type

$\gamma = 0$

Random

$$P(\tau) \sim e^{-\tau}$$

$\gamma = \infty$

Deterministic

$$P(\tau) \sim \tau^{-1}$$

Barabási, *Nature* (2005)

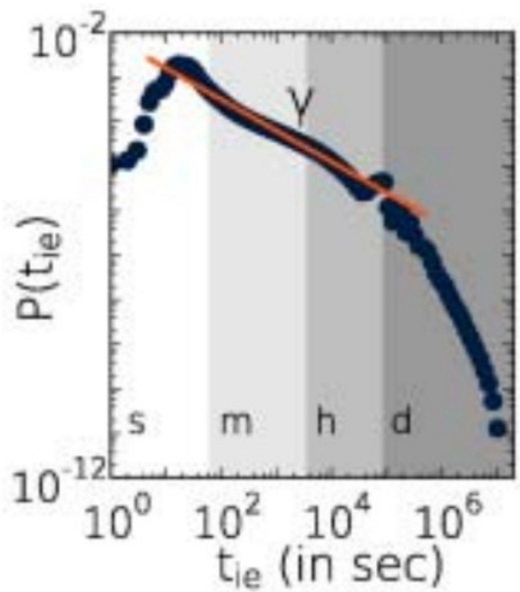
Is inter-event time power-law?

$$P(\tau) \sim e^{-q\tau}$$

Poisson process

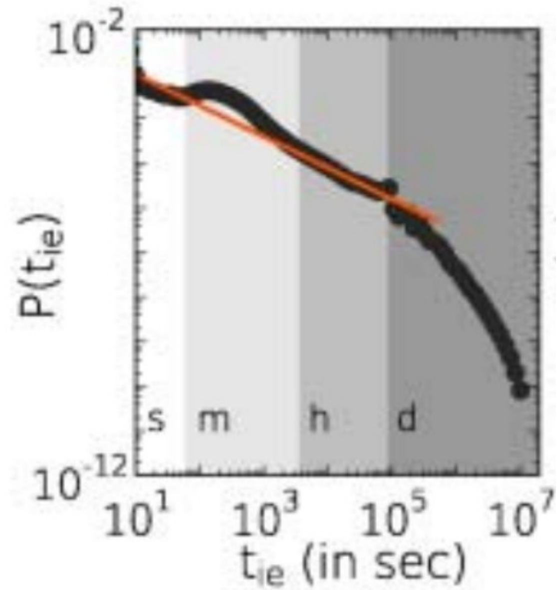
$$P(\tau) \sim \tau^{-\gamma}$$

Bursty signal



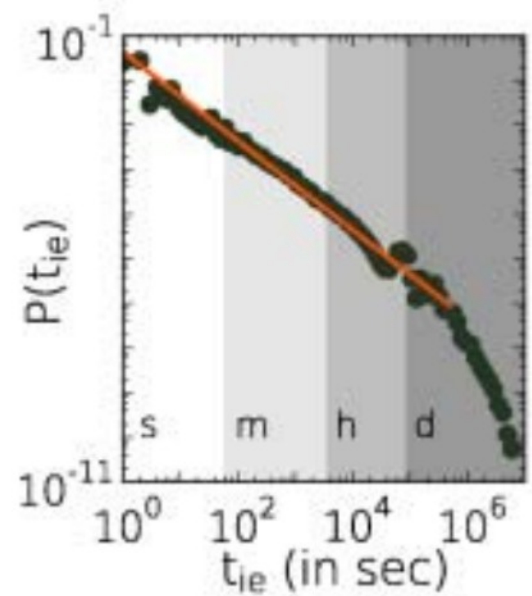
Phone calls

$\gamma \approx 0.7$



Text message

$\gamma \approx 0.7$



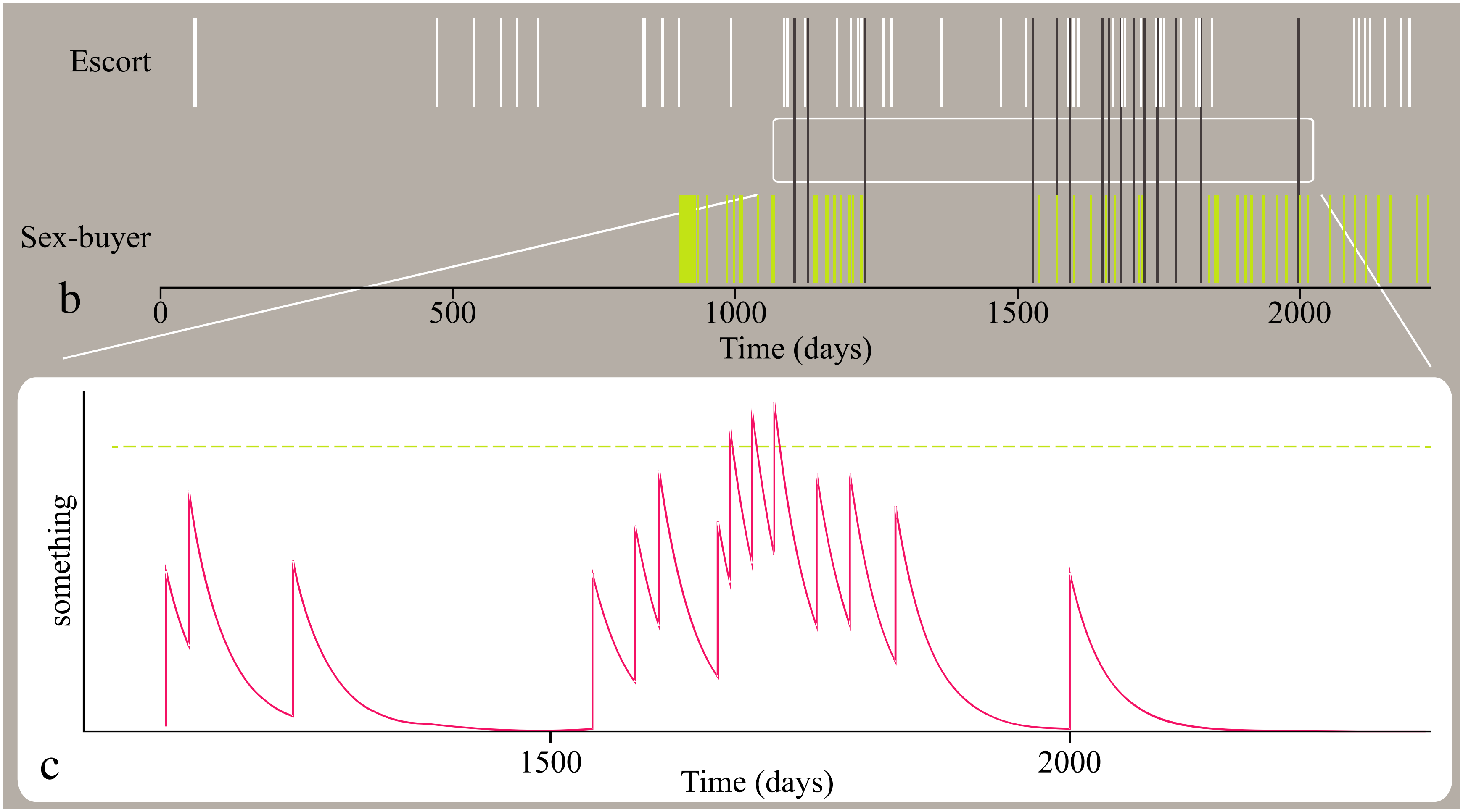
Email

$\gamma \approx 1.0$

- Is this a power-law? Definitely not exponential.

Karsai et al, *Scientific Reports* (2012)

Does burstiness matter?



L. EC Rocha, F. Liljeros, and P. Holme. PNAS 107.13 (2010)

Processes and null models

Structure and dynamics

- So far: various measures to characterize network
- How does structure affect processes on the network?
- Possibility:

  1) Generate model networks with given parameters.

  2) Run dynamics model → measure outcome.

  3) Scan parameters.
- Models from scratch: many parameters to set → few models, no dominant

- Instead:

  1) Take empirical network.

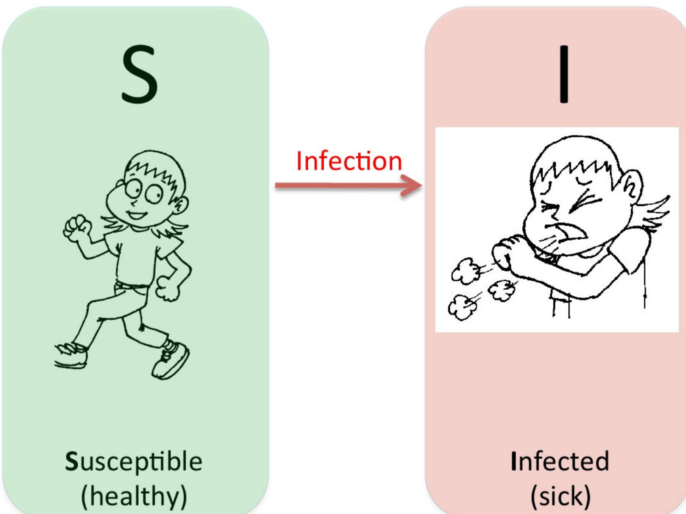
  2) Remove correlations by randomization.

  3) Run dynamics model → measure outcome.

L. EC Rocha, F. Liljeros, and P. Holme. PNAS 107.13 (2010)

Dynamics

- Model: SI model



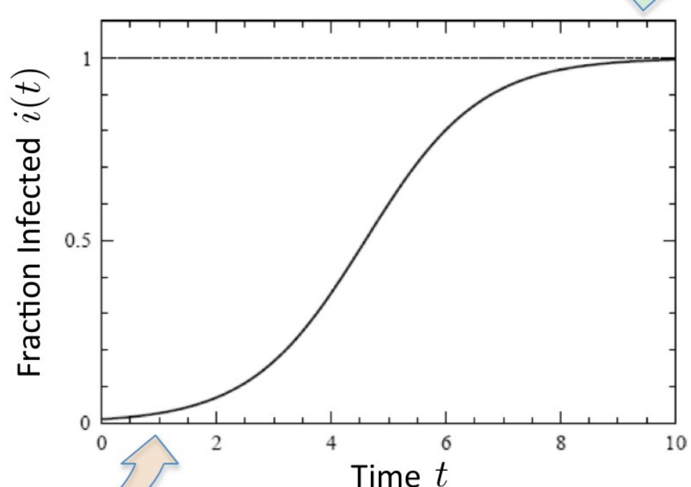
- Infection rate: β

If $i(t)$ is small,

$$\frac{di(t)}{dt} \approx \beta\langle k\rangle$$

$$i(t) \approx i_0 e^{\beta\langle k\rangle t}$$

exponential outbreak



As $i(t) \rightarrow 1$

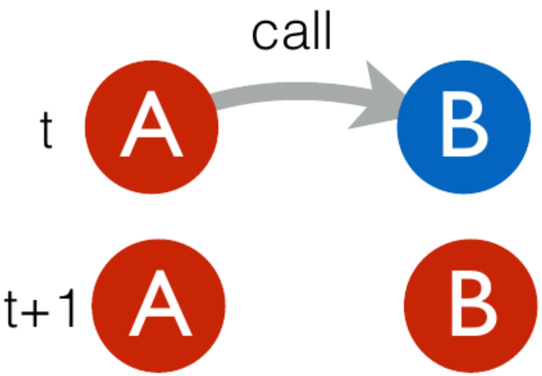
$$\frac{di(t)}{dt} \rightarrow 0$$

saturation

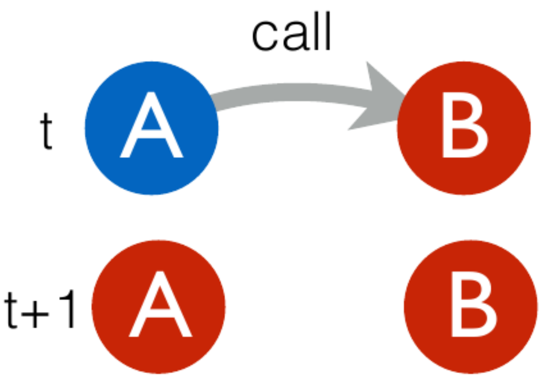
If connected

Dynamics

- Model: SI model on a temporal network
- Simplest model of information spreading
- Infection only spreads along active contacts
- Infection can spread both ways
- Infection rate $\beta = 1$
- Single seed at *t=0*



- Mobile call data as underlying network

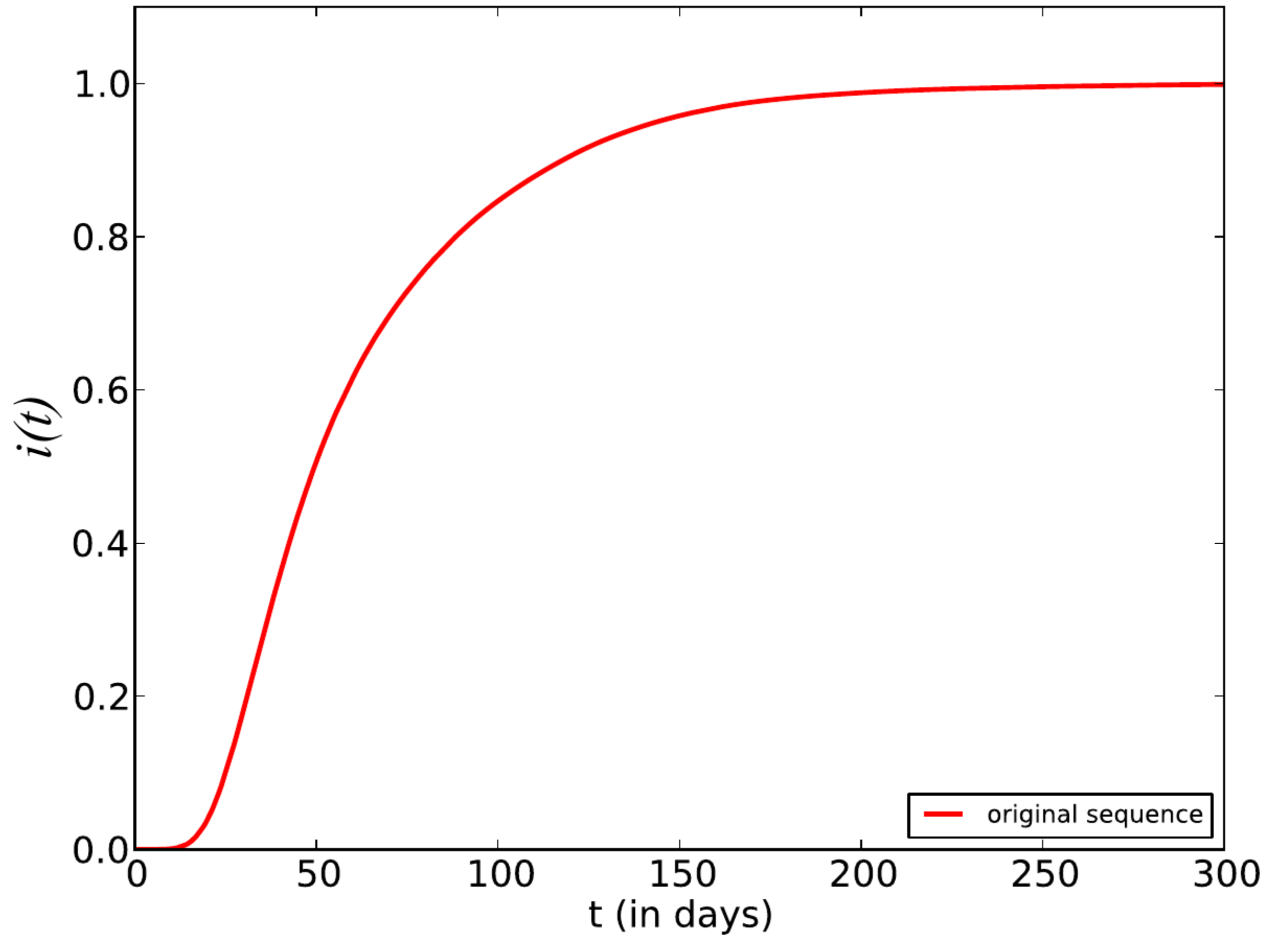


Slide adopted from Márton Karsai

Original temporal net

- We run SI model and measure $i(t) = \frac{I(t)}{N}$

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7



- Now what? Is this because burstiness, community structure?

Slide adopted from Márton Karsai

Properties of the original

- **Bursty dynamics (BD)**
  Heterogeneous inter-event time distribution

- **Community structure (CS)**
  Densely connected subgroups
  (Any other structure beyond degree distribution)

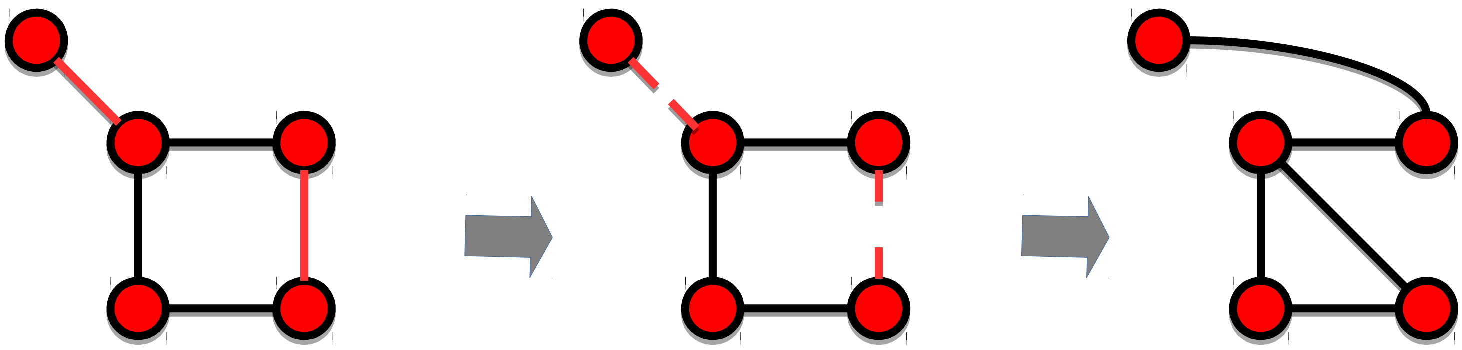
	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7

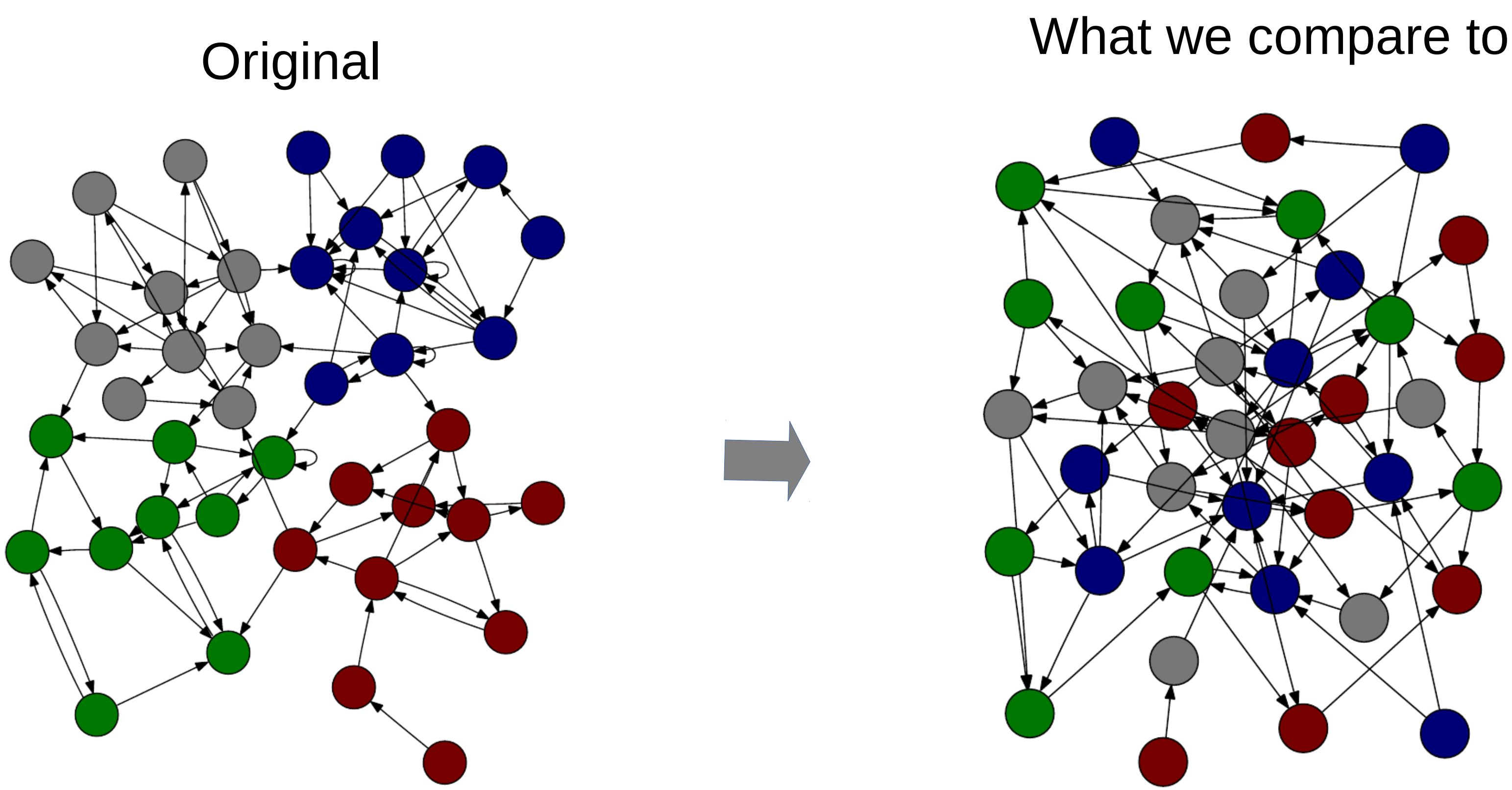
- **Link-link correlations (LL)**
  Causality between consecutive calls

- **Weight-topology correlation (WT)**
  Strong ties within local communities, weak ties connect different communities
  Weight = total call time
  s = strength of a node = some of adjacent link weights
  Onnela, J-P., et al. "Structure and tie strengths in mobile communication networks." PNAS 104.18 (2007).

Recap from community detection

- Randomization to remove community structure of a static network



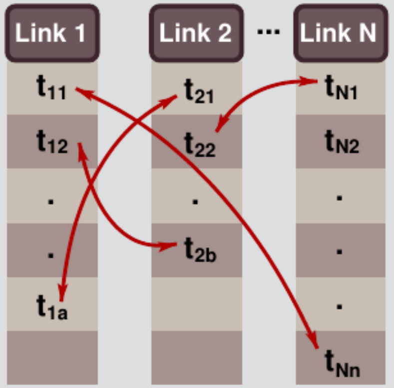


Randomization 1: temp. config. model

- Degree preserved randomization to remove **CS** and **WT**
- Shuffle event times to remove **BD** and **LL**

Shuffling

- Shuffle the event times of calls and destroy temporal heterogeneities
- keep P(w), P(k), P(s), w-top correlations
- destroy $P(t_{ie})$, link-link correlations



Time shuffling

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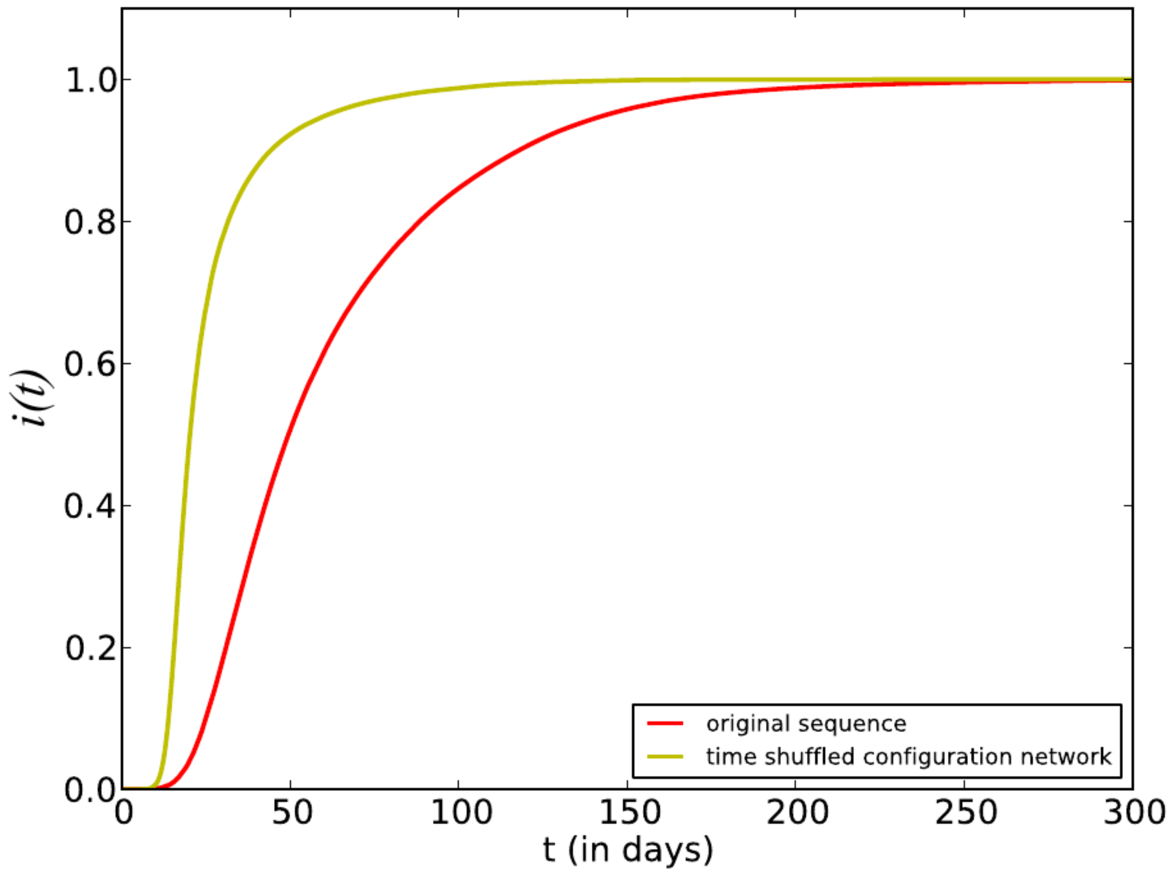
Randomization 1: temp. config. model

- Degree preserved randomization to remove **CS** and **WT**
- Shuffle event times to remove **BD** and **LL**
- **No correlation left.**

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	✕	✕	✕	✕	16,4

→ **Correlations slow the spread of information.**



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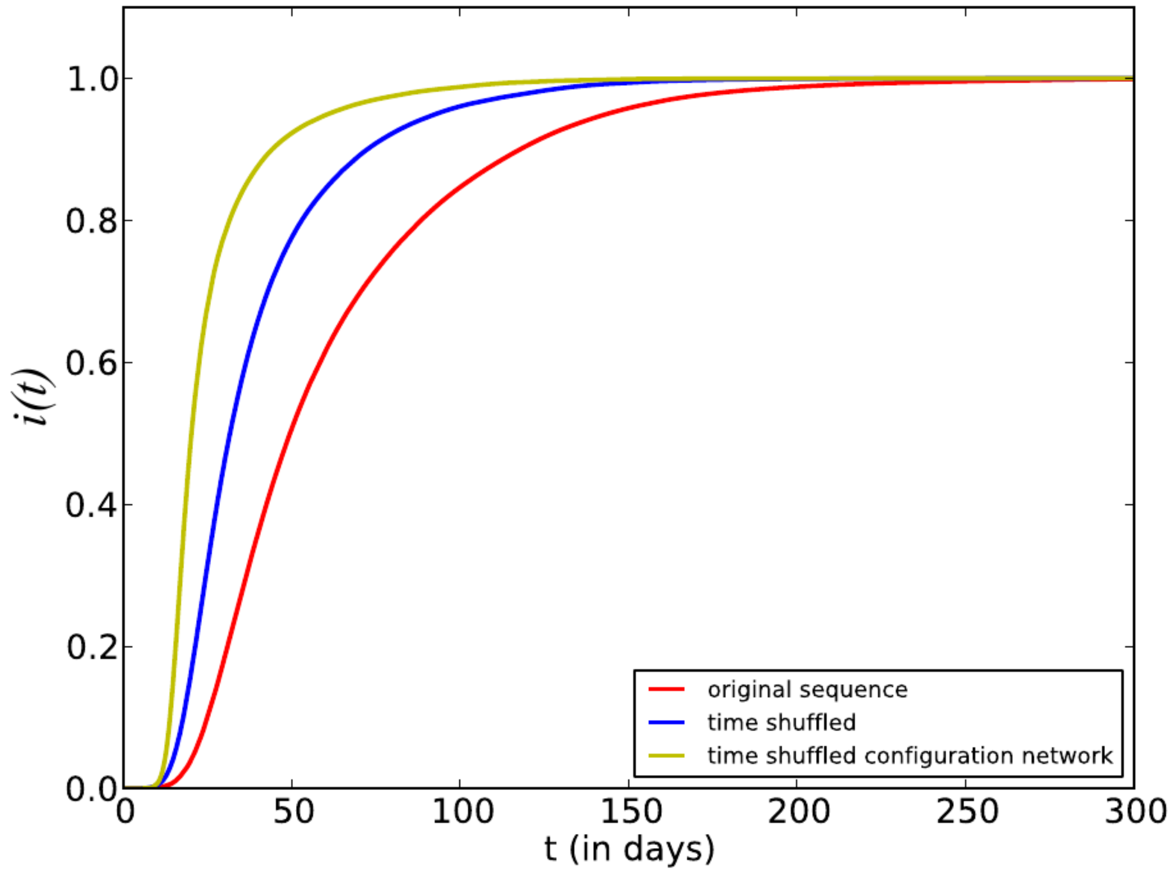
Randomization 2: time shuffled network

- Shuffle event times to remove **BD** and **LL**
- **CS** and **WT** remain.

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	×	×	×	×	16,4
Time	✓	×	×	✓	22,9



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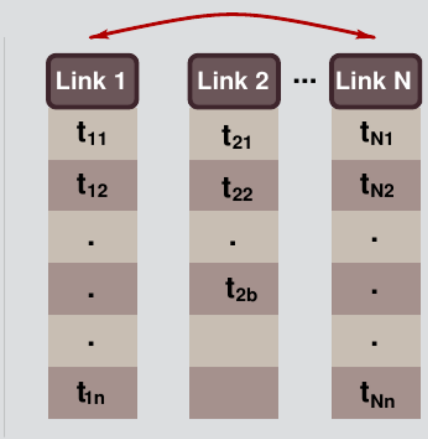
Randomization 2: time shuffled network

- Shuffle event sequences to remove **WT** and **LL**

Shuffling

- Change complete call sequences of individuals regardless of their edge weight
- keep P(w), P(k), $P(t_{ie})$
- destroy P(s), link-link correlations, w-top correlations



Eq.-w. link-seq. shuffling

- **CS** and **BD** remain.

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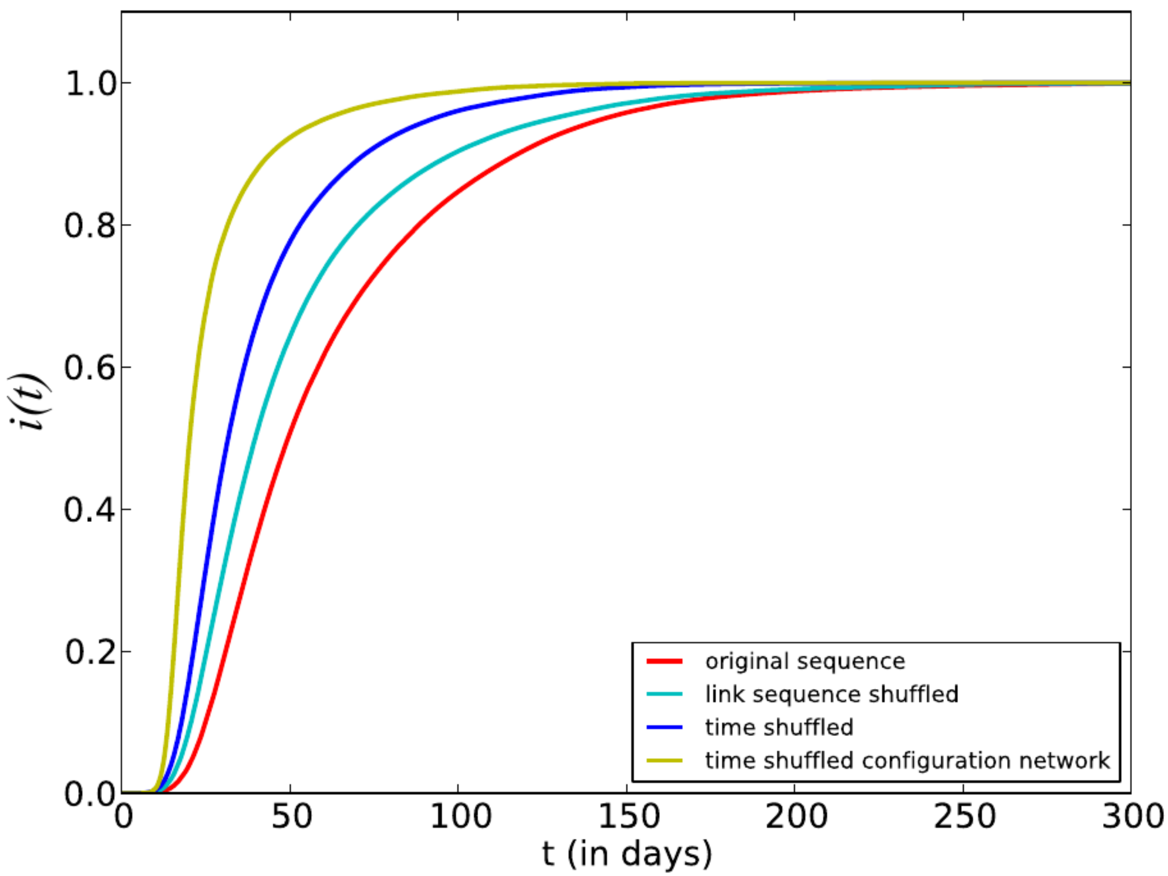
Randomization 3: time seq. shuffled

- Shuffle event sequences to remove **WT** and **LL**
- **CS** and **BD** remain.

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	×	×	×	×	16,4
Time	✓	×	×	✓	22,9
Link	×	✓	×	✓	27,5



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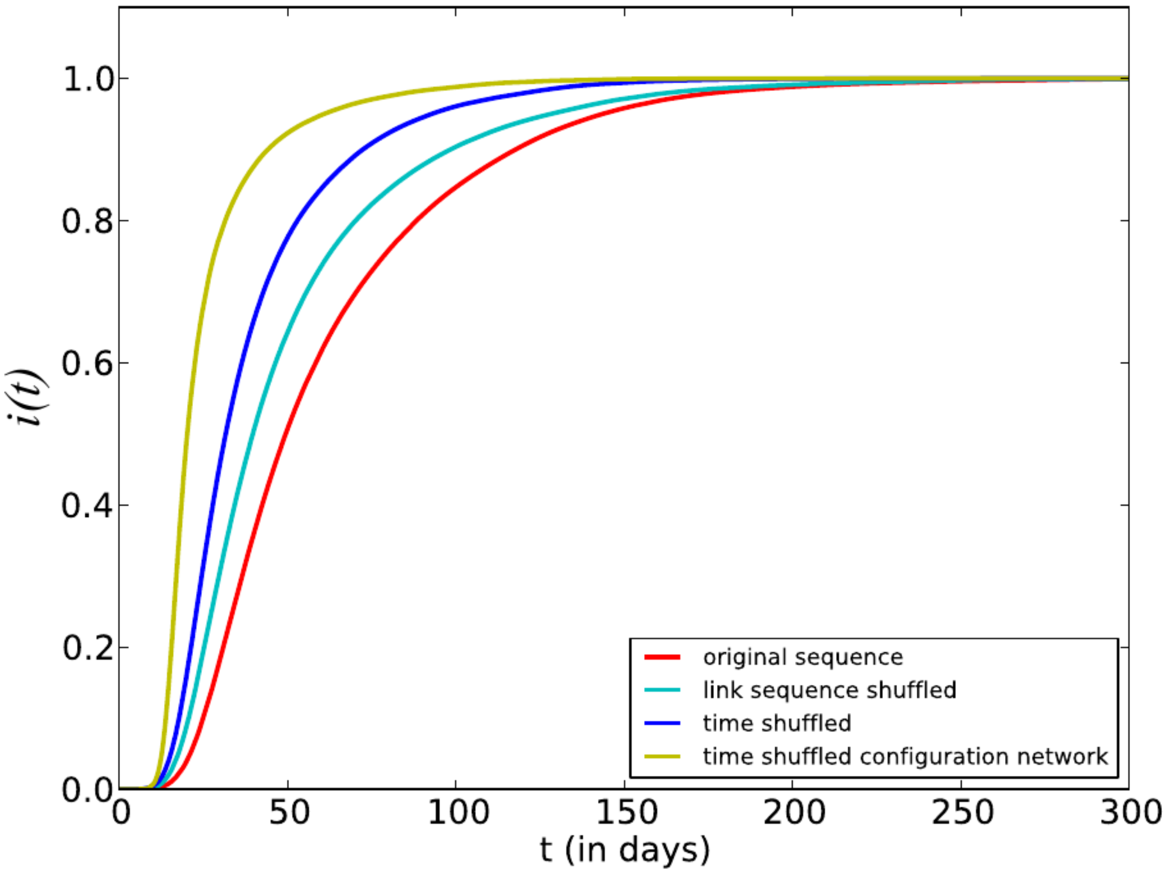
Randomization 3: time seq. shuffled

- Shuffle event sequences to remove **WT** and **LL**
- **CS** and **BD** remain.

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	×	×	×	×	16,4
Time	✓	×	×	✓	22,9
Link	×	✓	×	✓	27,5



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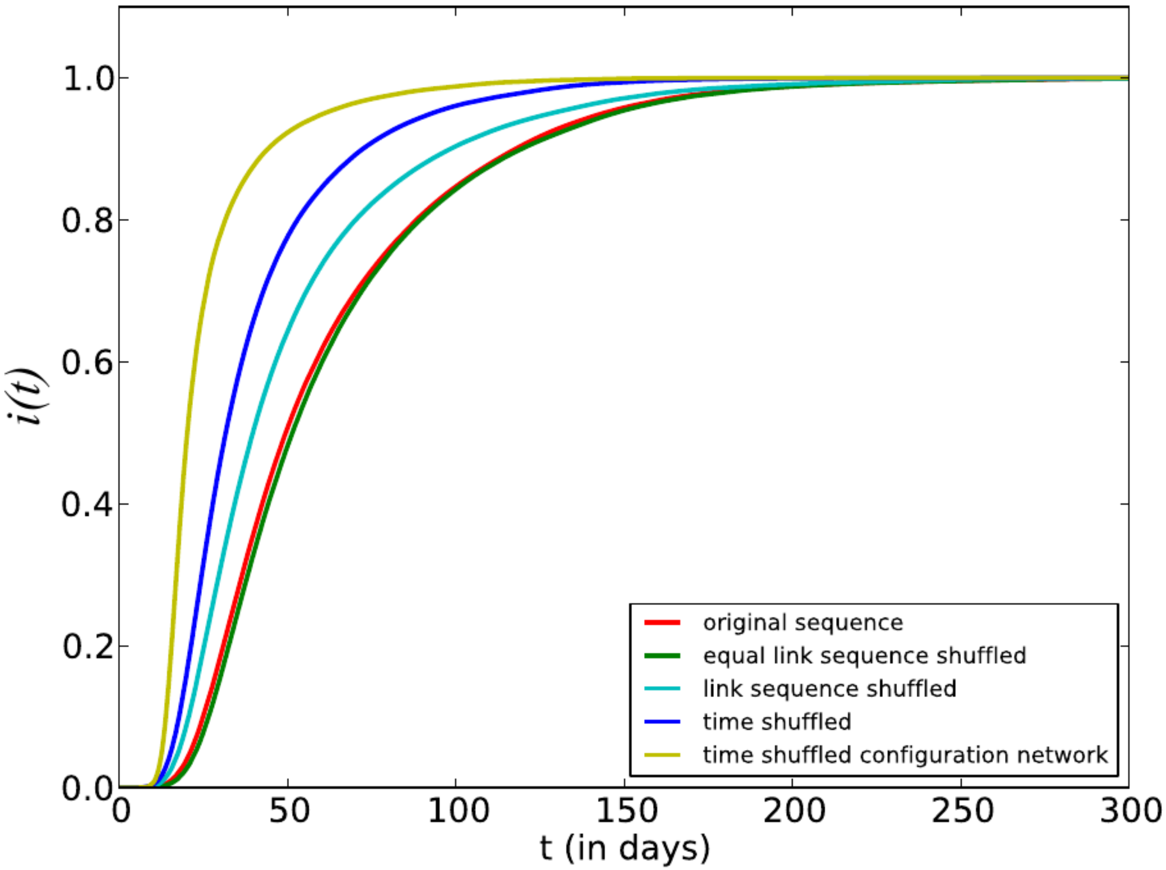
Rand. 4: equal link time seq. shuffled

- Shuffle event sequences if they have the same weight to remove **LL**
- **CS**, **WT** and **BD** remain.

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	×	×	×	×	16,4
Time	✓	×	×	✓	22,9
Link	×	✓	×	✓	27,5
Equal link	✓	✓	×	✓	35,3



→ **Multi-link processes slightly accelerate the spread.**

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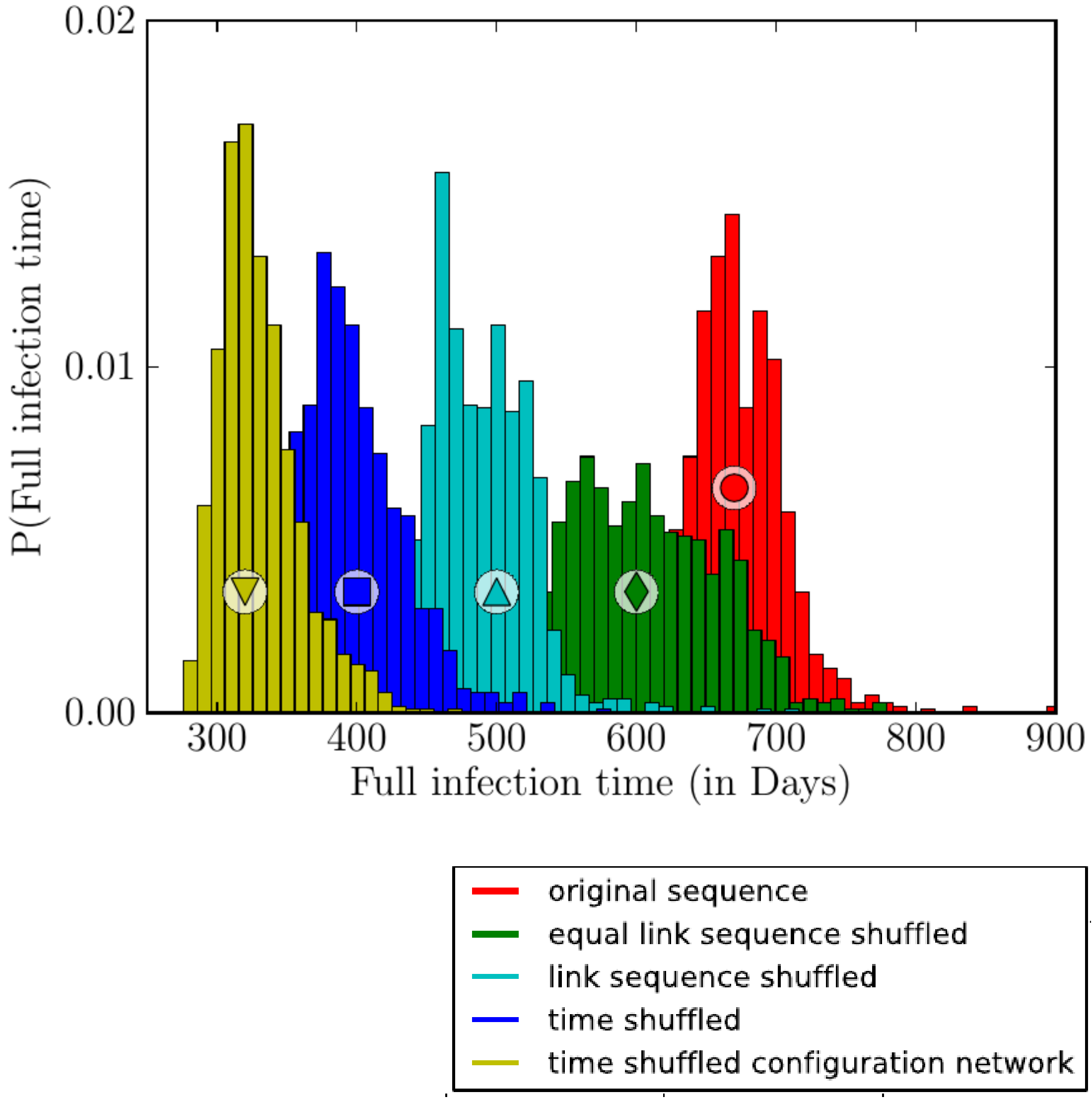
Long time behavior

- Distribution of complete infection time
- Evidence of effect of correlations in the late time stage.
- Multi-link correlations have contrary effect compared to early stage
- **WT** and **BD** are the main factors in slowing down

	WT	BD	LL	CS	25%m
Original	✓	✓	✓	✓	33,7
TimeConf	×	×	×	×	16,4
Time	✓	×	×	✓	22,9
Link	×	✓	×	✓	27,5
Equal link	✓	✓	×	✓	35,3



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Summary

- Timescale of dynamics and changes in network structure comparable

  → Temporal networks

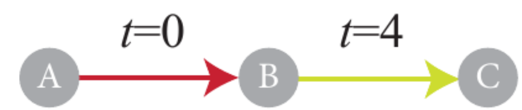
- Time respecting paths profound effect on spreading
- Temporal inhomogeneities: circadian rhythm and burstiness
- Measures more involved, computationally more difficult

Temporal motifs

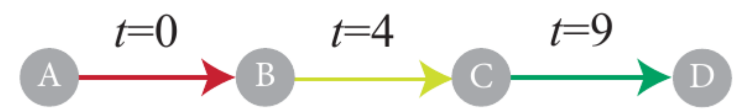
Definitions

Δt=5

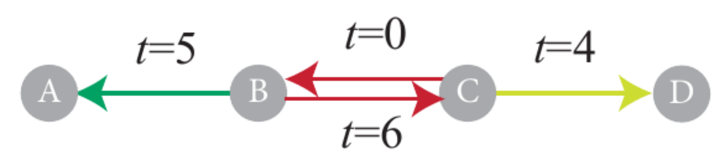
- **Δt adjacent** are two events if they share at least one node and are performed in *Δt*



- **Δt connected** are two events if there exists a sequence of events $e_i = e_{k0} e_{k1} e_{k2} \ldots e_{kn} = e_j$ such that all pairs of consecutive events are *Δt* adjacent



- **Connected temporal subgraph** consists of set of events, which are all *Δt* connected



- **Valid temporal subgraph** are connected temporal subgraphs where all *Δt* connected events of each node are consecutive

- **Maximal temporal subgraph** for an event e_i is a unique maximal subgraph E^*_{max} that contains e_i and in which all event pairs are *Δt* connected

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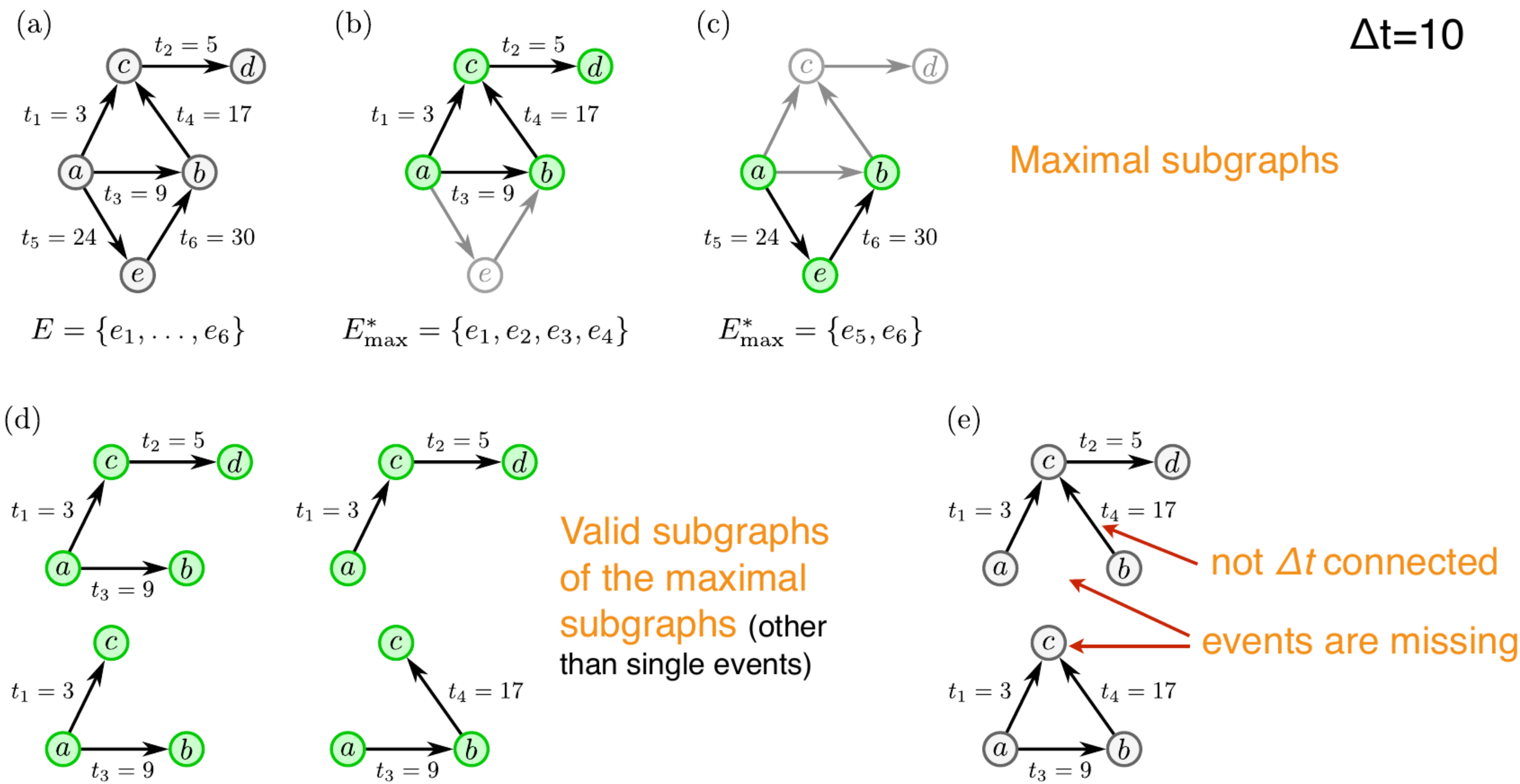
Detection

Mezoscopic correlated and casual temporal structures with topological and temporal order isomorphism



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Algorithm

Mezoscopic correlated and casual temporal structures with topological and temporal order isomorphism

- To detect them we need to group events into equivalent classes where timing not but direction and ordering matters

1. Find all maximum connected subgraphs E^{*}_{max}
   - start from an event e_i
   - iterate forward and backward to find all Δt adjacent events
   - repeat it for all new events

2. Find all valid subgraphs E^{*}
   (this can be reduced to find all induced subgraphs of a static graph)

3. Identify the motifs for all E^{*} subgraphs
   (map to directed coloured graphs and find isomorphic structures with equivalent ordering, e.g. using the bliss algorithm (Junttila and Kaski (2007)))

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What to compare to?

Candidates null models

1. Time-shuffled reference: randomly redistribute event times between events
   - Destroys all temporal correlations and casual correlations

a	b	t_1	→	a	b	t_3
a	c	t_2		a	c	t_4
e	f	t_3		e	f	t_1
f	a	t_4		f	a	t_2
	…				…	

2. Time-reversed reference: read the event sequence in a reversed order
   - Destroys all casual correlations but keeps all temporal correlations

a	b	t_1	→	f	a	t_1
a	c	t_2		e	f	t_2
e	f	t_3		a	c	t_3
f	a	t_4		a	b	t_4
	…				…	

3. Self reference: compare different periods of the sequence to each other
   - Highlights seasonal dependencies

a	b	t_1	↔	d	w	t_{91}
a	c	t_2		y	a	t_{92}
e	f	t_3		a	b	t_{93}
f	a	t_4		q	s	t_{94}
	…				…	

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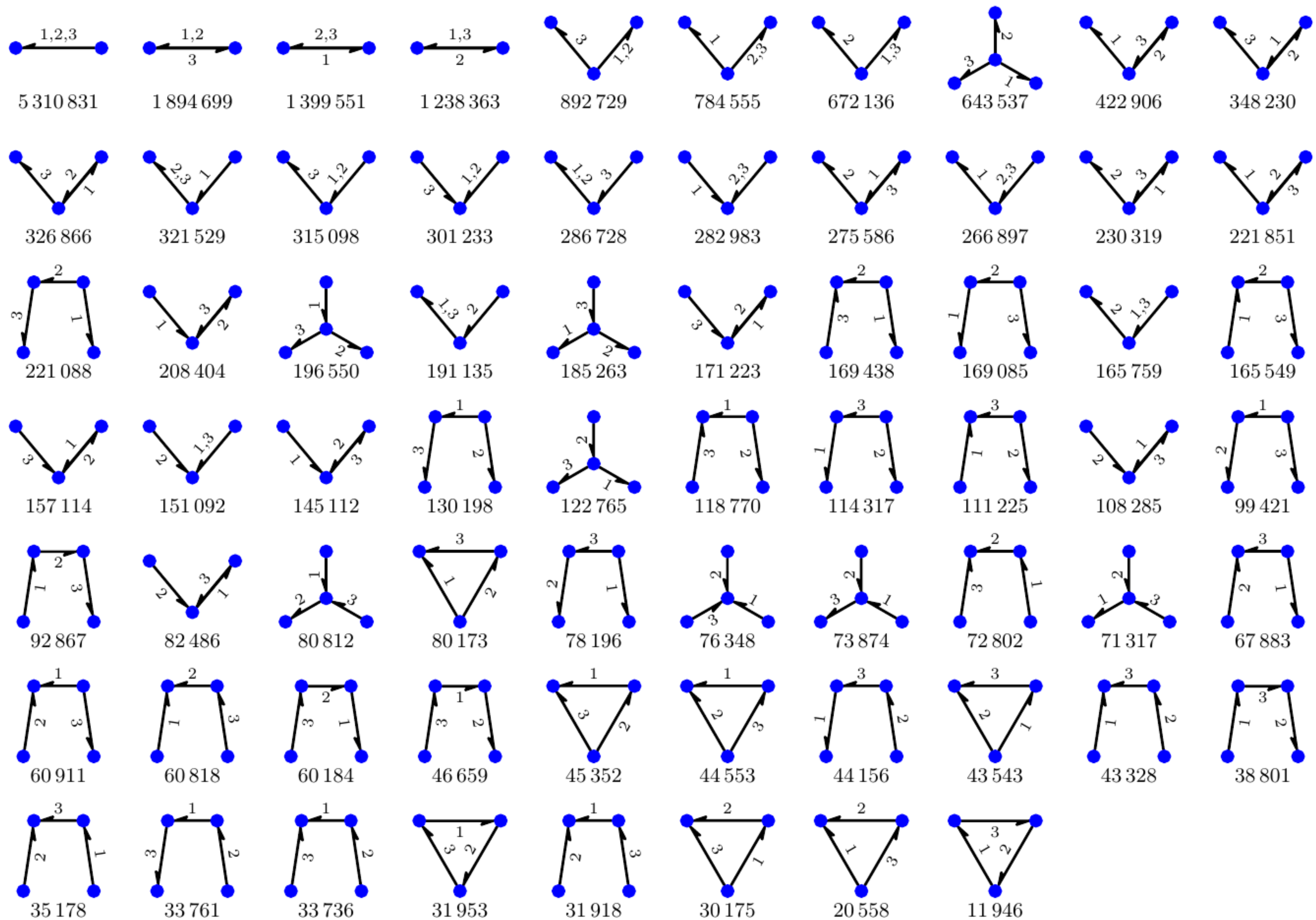
Phone call network

All 3-call motifs



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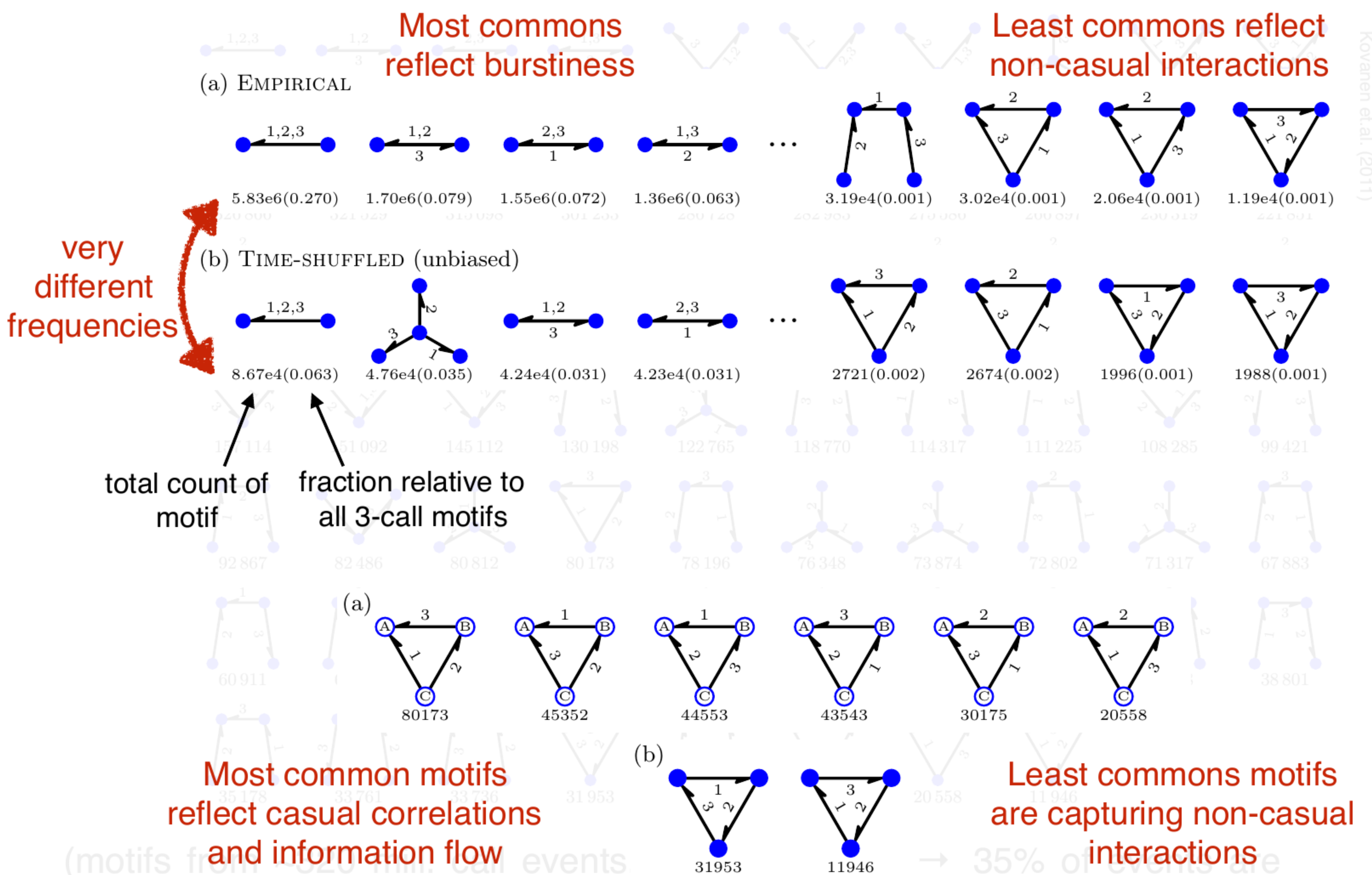
Phone call network

All 3-call motifs



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