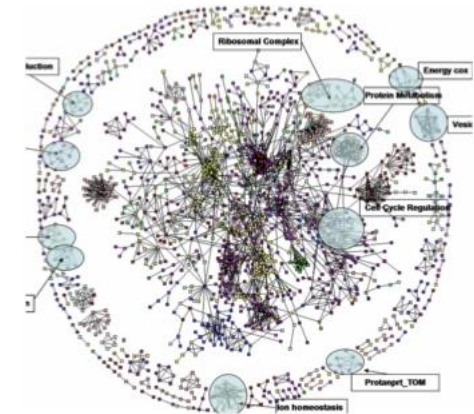
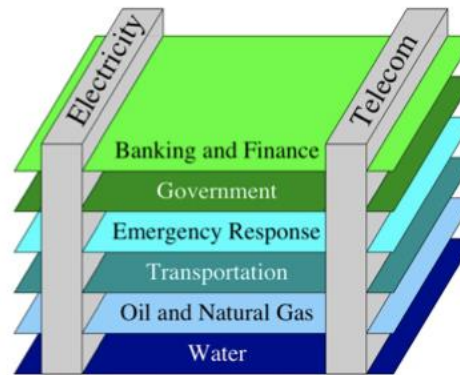
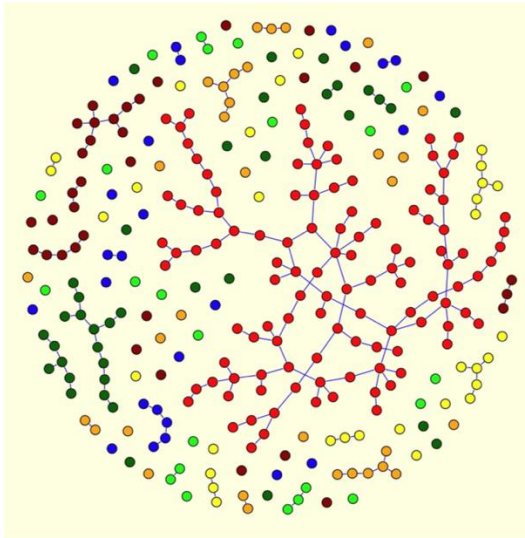


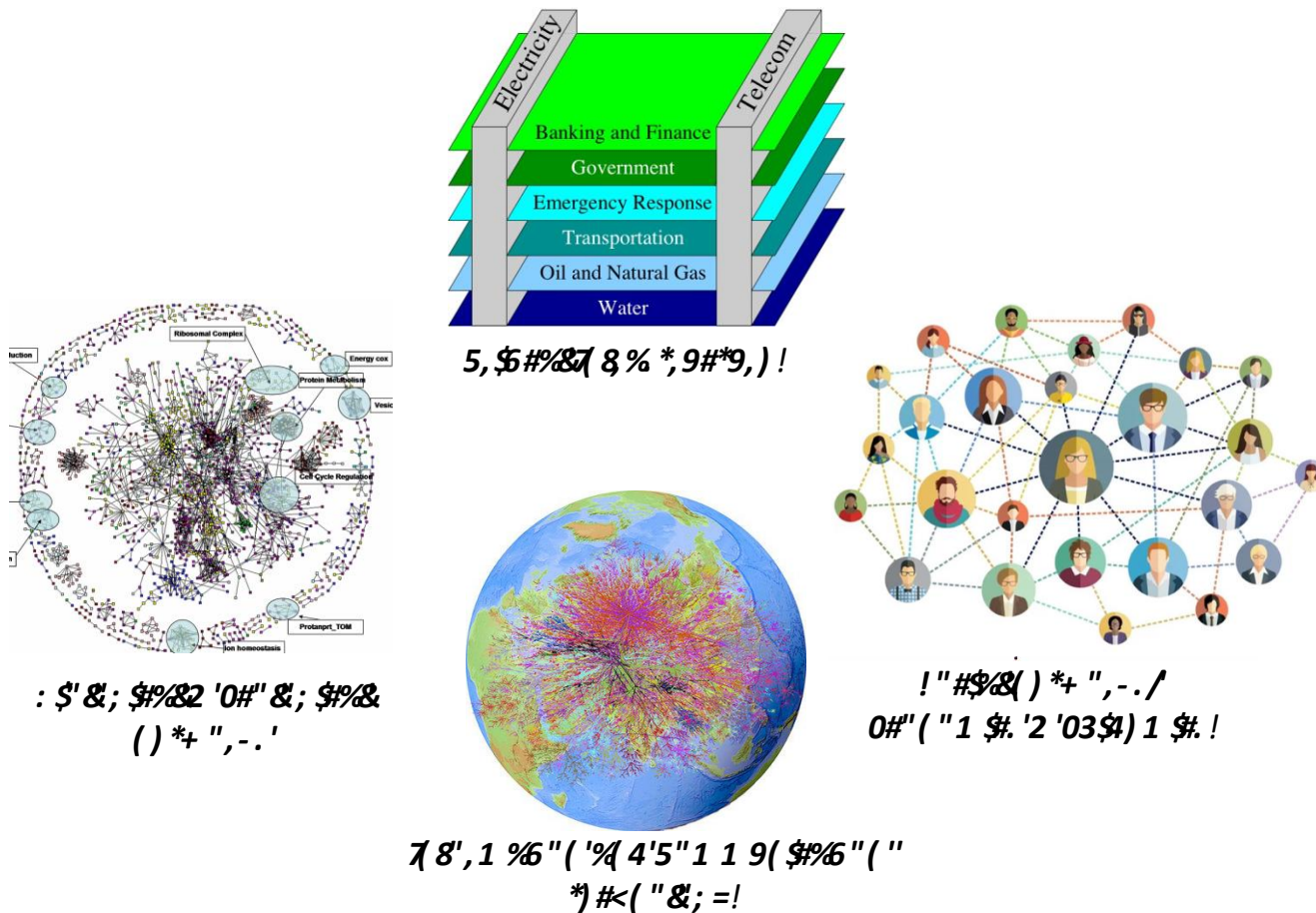
Network Theory & Applications

Takeaways



ECS / MAE 253, Spring 2025
June 2, 2025

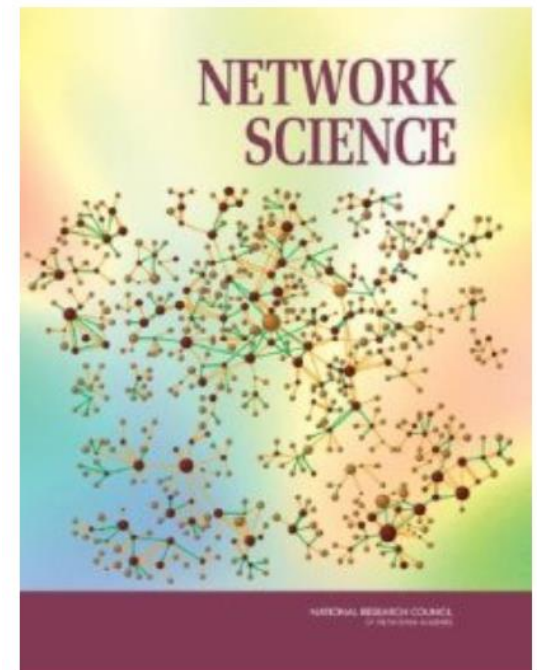
Networks are pervasive



Each network can be a complex system with emergent behaviors

Brief history of network science

- Networks are pervasive in the natural and engineered world
- Previously studied in domain-specific silos
- In the late 1990's a collection of events lead to the field of network science
 - Data on broad scale degree distributions in the internet
 - The Watts-Strogatz “small-world” network
 - The Barabasi-Albert “scale free” network
 - The unique properties of power law degree distributions:
 - The Robust yet Fragile paradigm
 - Epidemic threshold can approach zero



2005

The past decade, a “Science of Networks”:
(Physical, Biological, Social)

- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** /spontaneously arising versus **engineered** /built.
- **Directed** versus **undirected** edges.
- Each network **optimizes** something unique.
- Identifying **similarities** and fundamental **differences**
- Interplay of **topology** and **function** ?

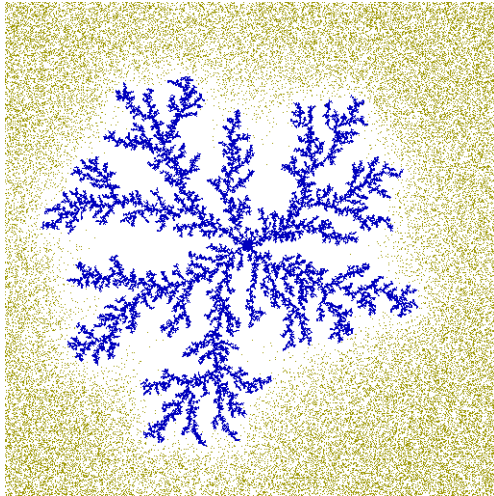
What is a Network?

- **Topology** (i.e., structure: nodes/vertices and edges/links)
Measures of topology
- **Activity** (i.e., function, processes on networks, dynamics of nodes and edges)

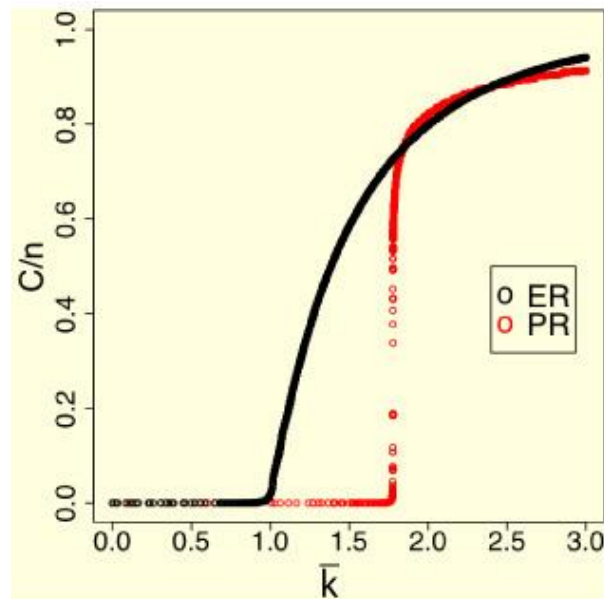
The term “**complex network**” is generally used to refer to a network with a non-trivial degree distribution and/or one with emergent behaviors.

What are emergent collective behaviors?

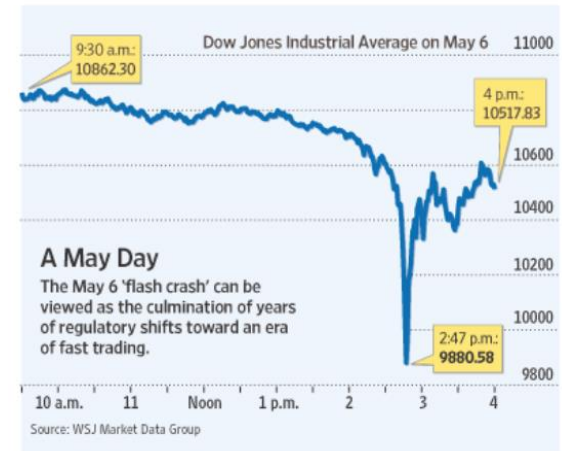
Behaviors not predicted a priori from the constituent equations of motion.



Synchronization and pattern formation.



Phase transitions
“Tipping points”



Cascading failures.

Structural properties

- **Degree distribution**
 - Many real-world networks across domains have broad scale degree distributions
 - Most real-world networks do NOT have exactly power law degree distributions (i.e., they are not “scale-free”).
 - Power laws provide some theoretical tools for analysis.
- **Diameter**
- **Assortative/disassortative**
- **Component sizes and size distribution**
- **Community structure / modularity**
- **Motifs**
- **Centrality measures** (betweenness, eigenvector, degree, etc.)

Analysis tools:

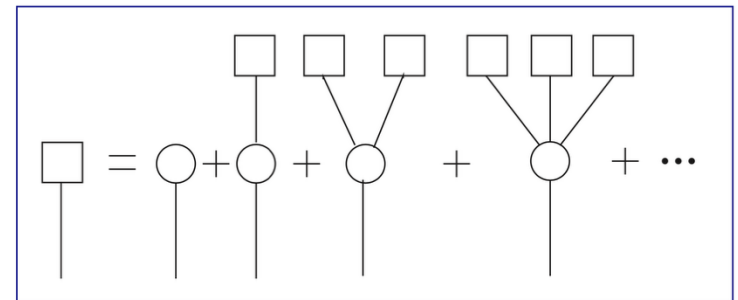
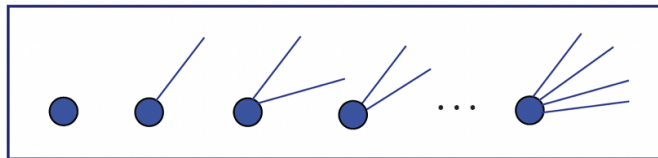
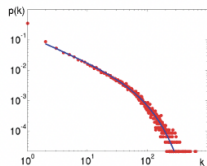
Network growth models:

- Provide a theoretical framework for analysis
- Erdos-Renyi
- PA and variants
- Optimization models (can give rise to PA)
- Duplication-mutation-complementation

Mathematical tools:

- Rate equations / kinetic theory
- Configuration models
- Generating functions

$$x_k(t+1) = \frac{k-1}{n}x_{k-1}(t) - \frac{k}{n}x_k(t)$$



- Ensemble of random graphs
- Thermodynamic limit $N \rightarrow \infty$
- Equilibrium / Steady-state behavior

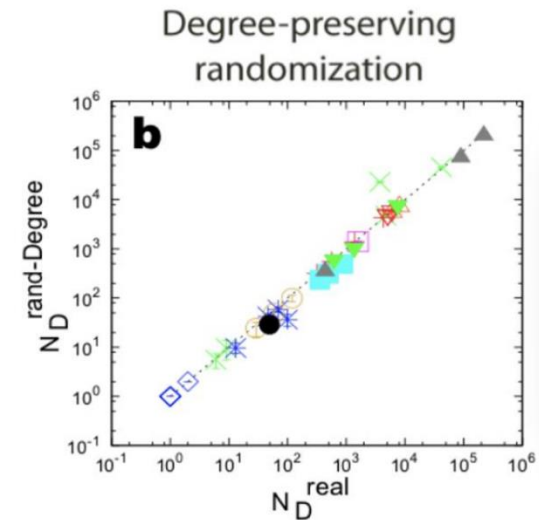
Things we can analyze:

- Achievements:
 - Percolation & extent of connectivity
 - Epidemic thresholds
 - Diffusion and spreading
 - Opinion dynamics
 - Voter models
 - Search on networks
 - Network control theory

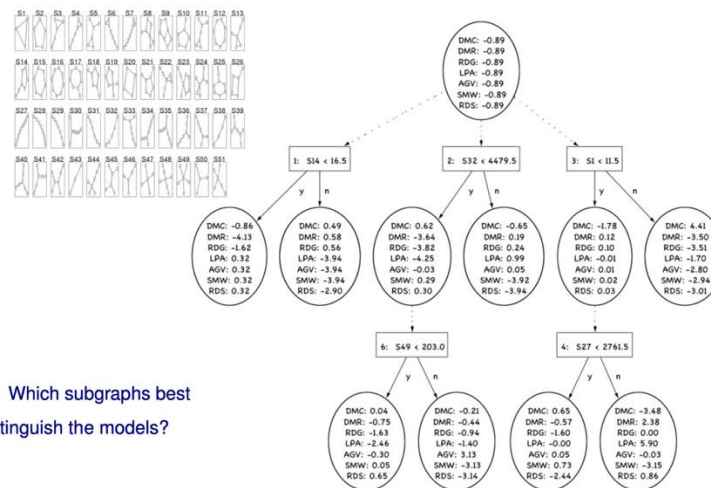
Empirical networks :

How to disentangle whether the details of the network matter for dynamics unfolding on the network

- Degree preserving rewiring
- Temporal shuffling for time varying networks



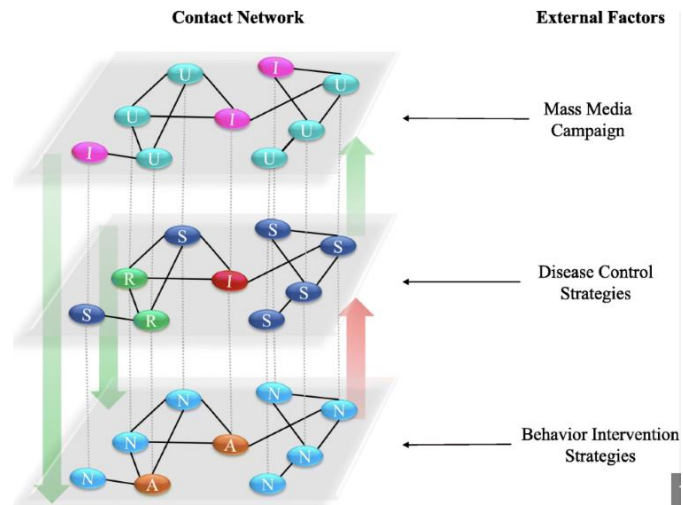
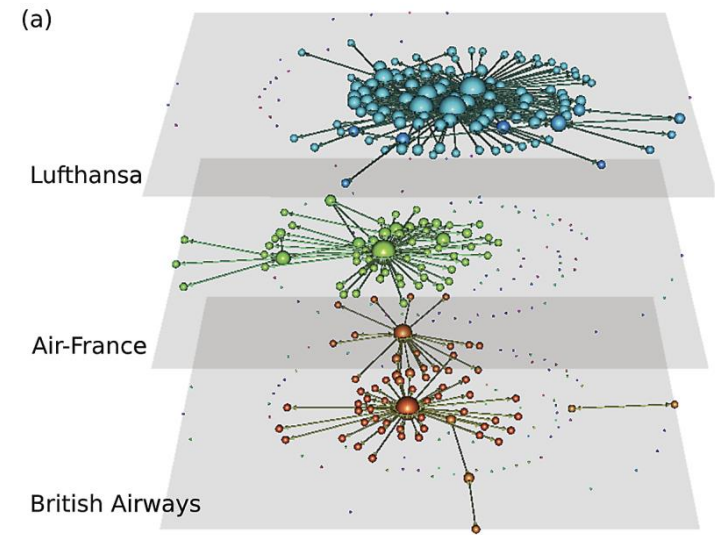
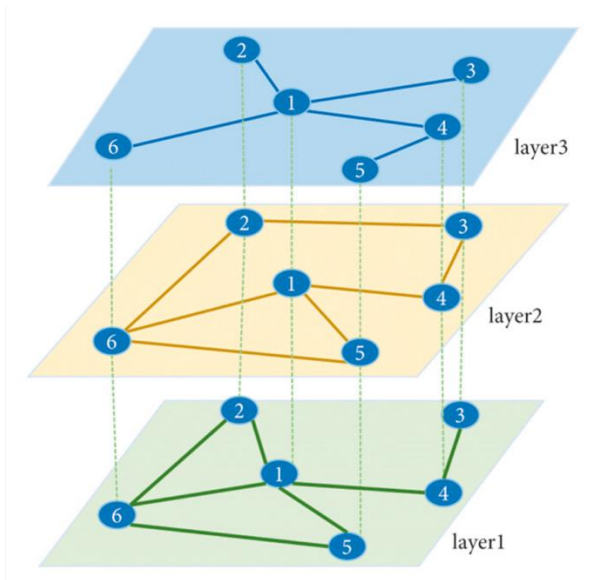
Can **build classifiers** to select which theoretical model best fits the empirical network but this is NOT model validation.



Important future directions:

- Multiplex and multilayered networks
 - Higher order interactions beyond dyadic
 - Temporally varying networks
-
- Can neglect temporal nature if there is separation of timescales:
 - Fast dynamics and slow network changes -> topology essentially static
 - Slow dynamics and fast network changes -> topology changes rapidly; mean-field theory

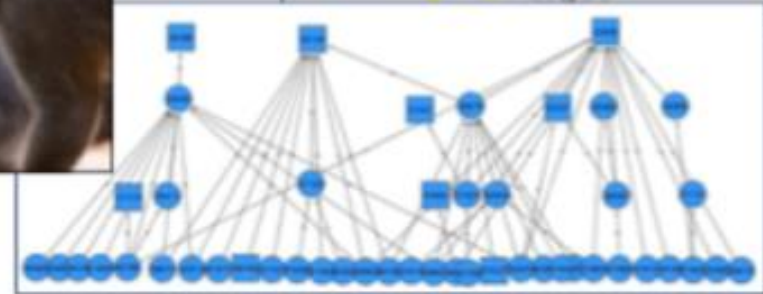
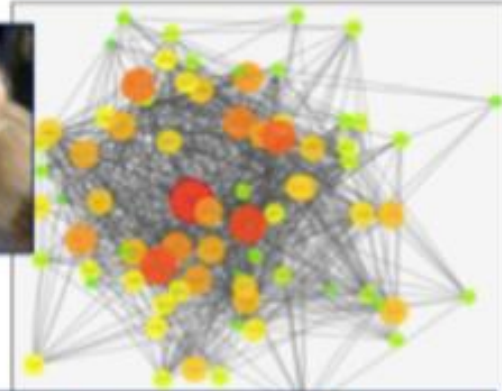
Multiplex networks



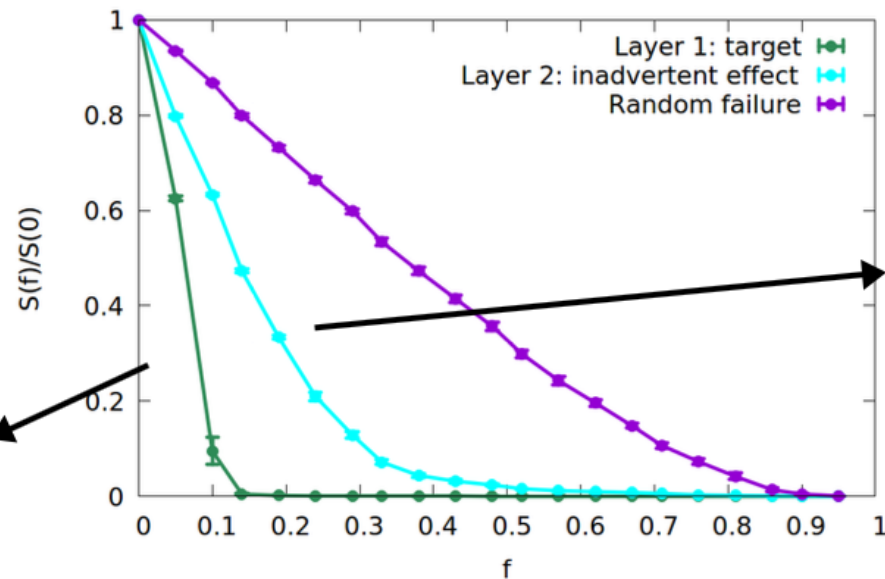
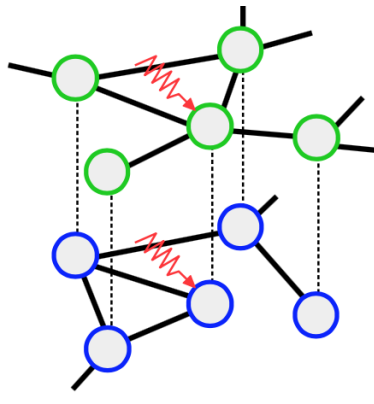
Primate Behavior -- Multiplex Networks

Affiliative versus competitive interactions

- Grooming
- Huddling
- Alliances
- Aggression
- Policing
- Submission



Simple targeted removal in multiplex networks



Attacking
Layer 1.

Inadvertently
destroying Layer 2.

E.g., Goal: destroy aggressive layers, but preserve affiliative layers

Pareto front: tradeoff both objectives

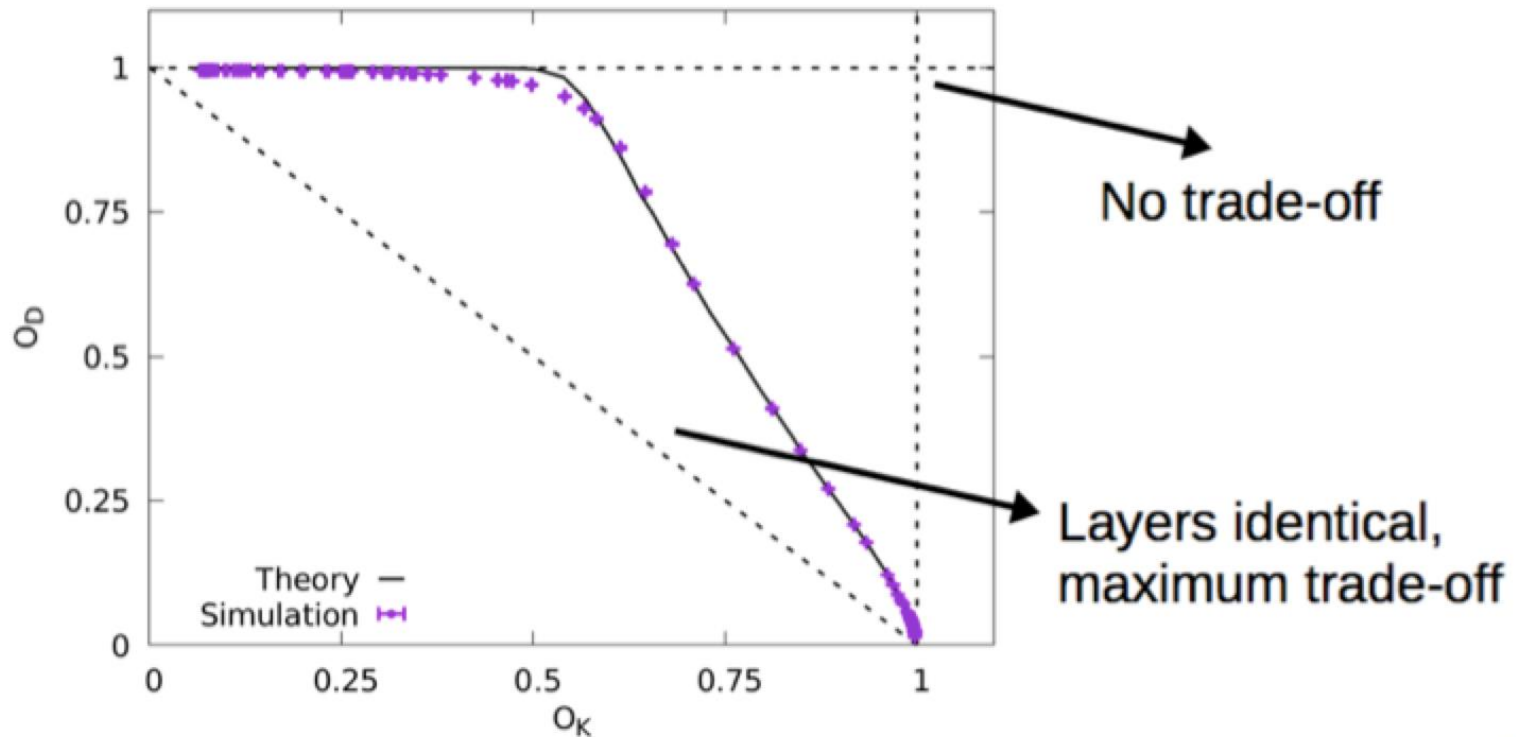
Quantify how close we are to optimal for each objective:

$$O_D = \frac{1}{|\mathcal{L}_D|} \sum_{\alpha \in \mathcal{L}_D} \frac{S_\alpha - S_\alpha^{\min}}{S_\alpha^{\max} - S_\alpha^{\min}}$$

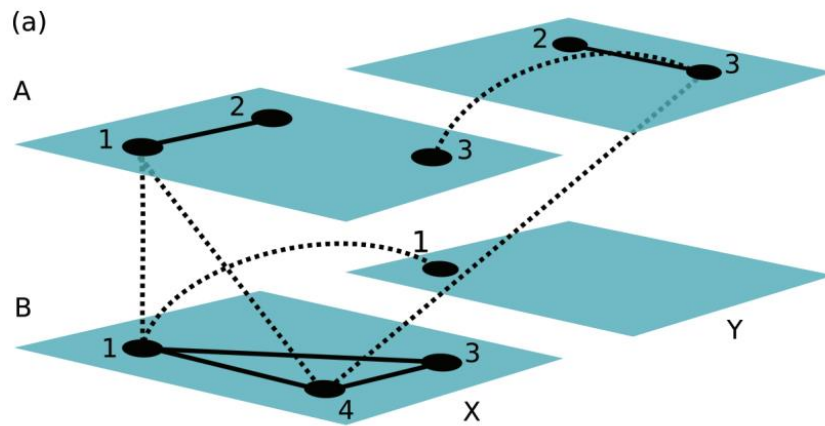
How much actually destroyed in relation maximum possible

$$O_K = \frac{1}{|\mathcal{L}_K|} \sum_{\alpha \in \mathcal{L}_K} \frac{S_\alpha^{\max} - S_\alpha}{S_\alpha^{\max} - S_\alpha^{\min}}$$

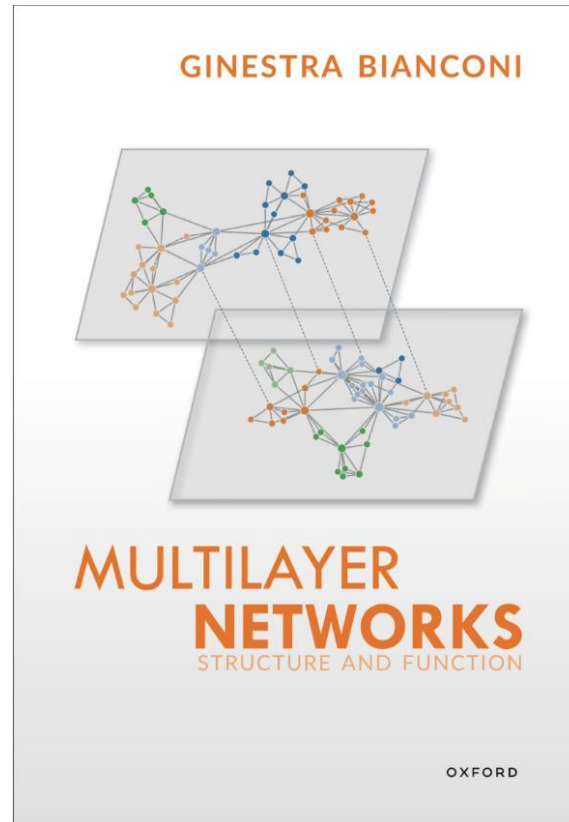
How much actually preserved in relation maximum possible



Multilayer networks

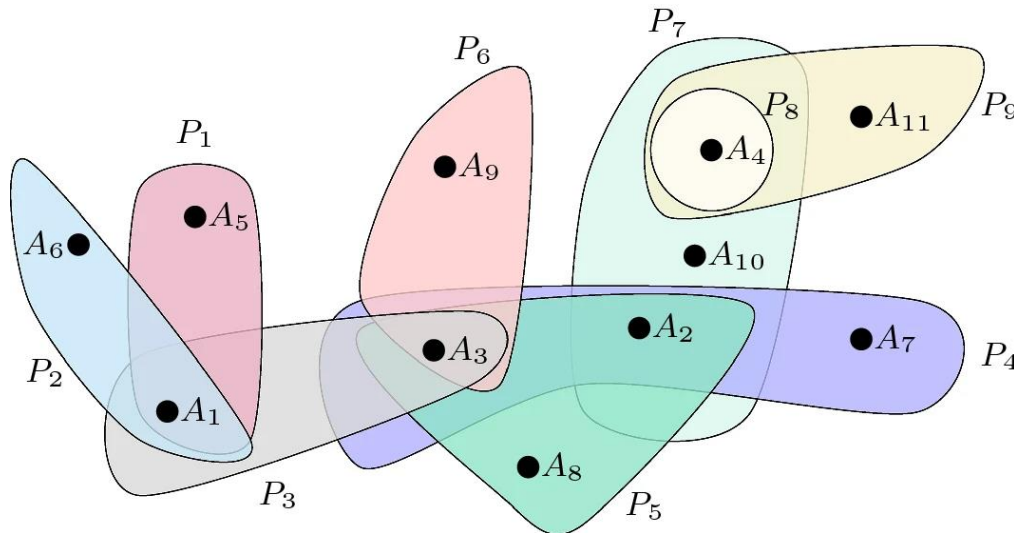


Multilayer networks



2018

Higher order interactions -- Hypergraphs



Example higher-order interactions:

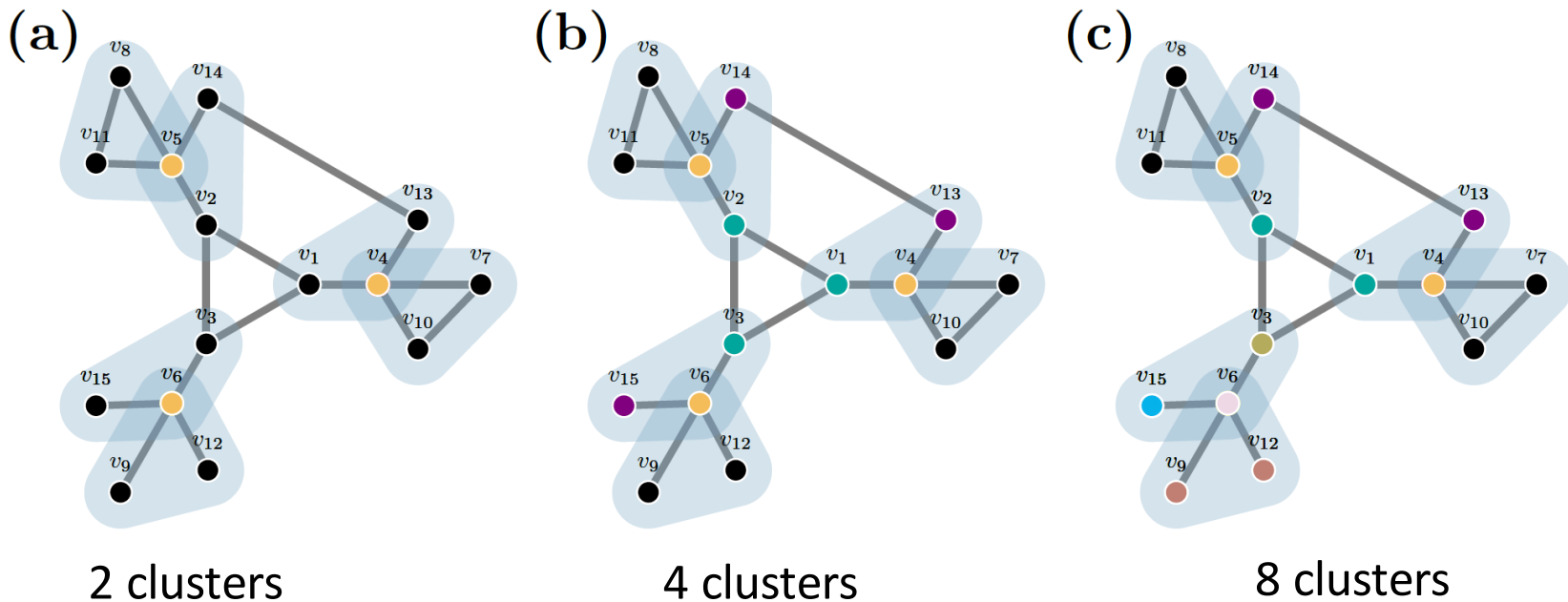
- Chemical reactions
- Co-authorship networks

- **Hypergraph coupling:** Hyperedges representing higher-order interactions (dyadic, triadic, etc.)
- **Challenge:** Hyperedges of all order contribute to the dynamics and the stability calculations.

Cluster synchronization on hypergraphs

Anastasiya Salova, R.D., arXiv:2101.05464

Example hypergraph and admissible cluster synchronization states:

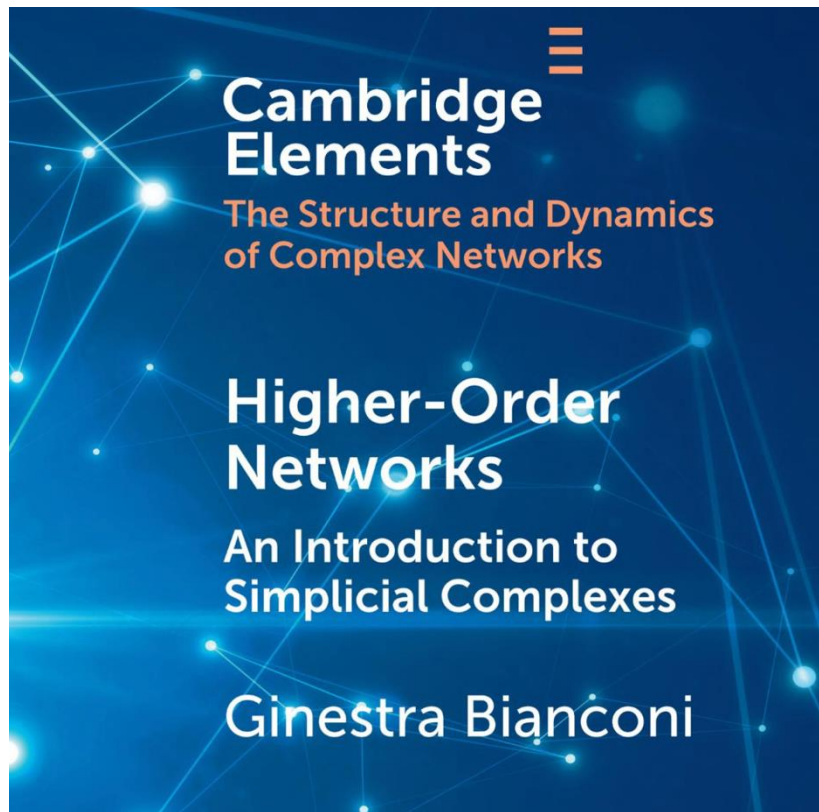


Node evolution:

$$\dot{x}_i = F(x_i) + \sum_{m=2}^d \sigma^{(m)} \sum_{e \in \mathcal{E}_i^{(m)}} [I^{(m)}]_{i,e} G^{(m)}(x_i, x_e)$$

Higher order interactions – simplicial complexes

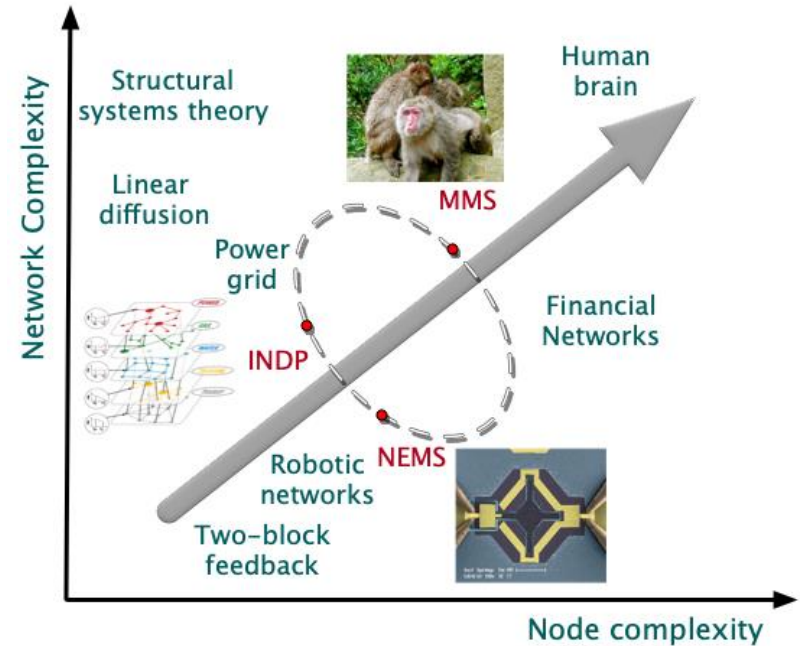
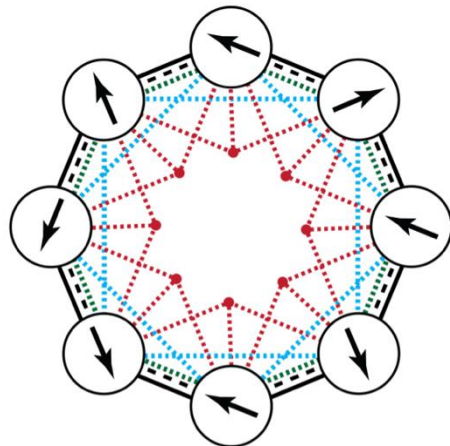
More constrained than hypergraphs are simplicial complexes



2021

Key issue: Dynamics of and on networks

- Network structure
 - Dynamic vs static
- Dynamical processes on networks
 - Diffusion/spreading processes
 - SI/SIS/SIR dynamics
- Interplay of structure and function
- Nodal dynamics and network structure can interact to give new emergent properties!



Further Resources

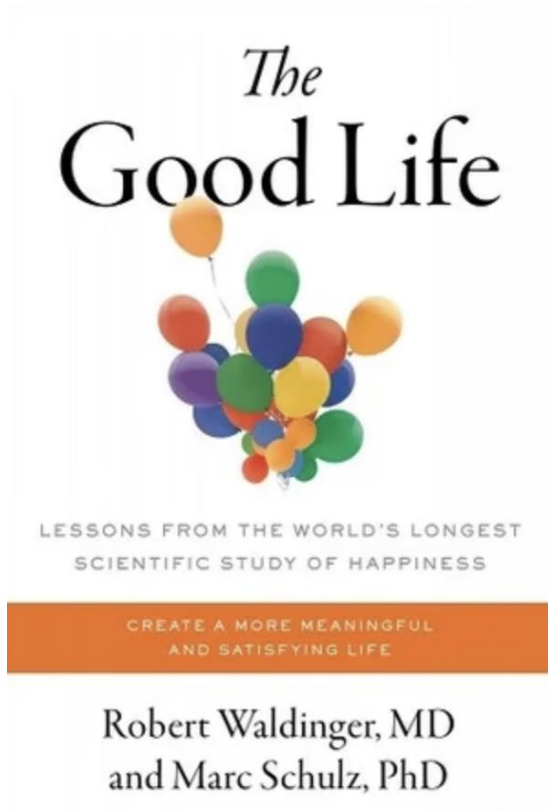
Societies and conferences:

- The Network Science Society
<https://netscisociety.net>
- The Cambridge Networks Network (CNN)
<https://www.cnn.group.cam.ac.uk/>
- The Complex Systems Society
<https://cssociety.org>

Journals:

- Network Science, Cambridge Univ Press
- Journal of Complex Networks, Oxford Univ Press
- Applied Network Science, Springer
- Journal of Physics, Complexity, IoP

Your social networks are essential

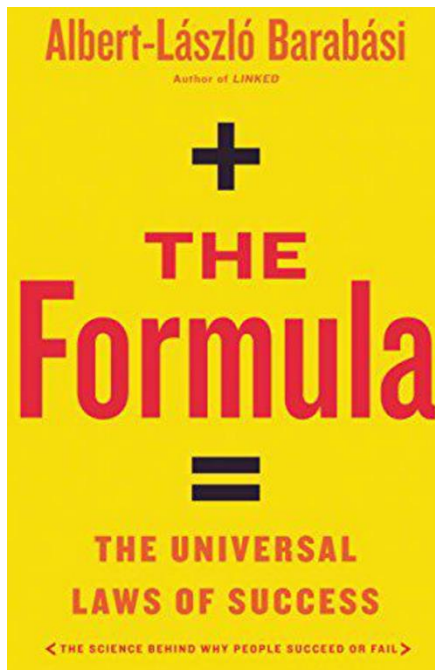


WHAT THE LONGEST STUDY ON HUMAN HAPPINESS FOUND IS THE KEY TO A GOOD LIFE

The Harvard Study of Adult Development has established a strong correlation between deep relationships and well-being. The question is, how does a person nurture those deep relationships?

- Better mental health
- Better physical health

Network theory for personal success



SOCIAL SCIENCE

Unlocking the science of success

A complexity expert reveals how social networks create recognition and acclaim

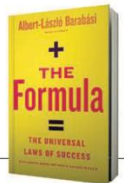
By **Raissa M. D'Souza**

Want to master your professional and social networks to maximize recognition? Want to learn how to build productive teams that create lasting impact? In his new

matter most. And even when performance can be measured, for the highest achievers, just a small increase in performance can lead to an exponential increase in how we perceive their value and in the amount of recognition they receive. Performance is ultimately limited by our personal abili-

The Formula
The Universal Laws
of Success

Albert-László Barabási
Little, Brown, 2018. 316 pp.



Final details – peer assessment form

	A	B	C	D	E	F
1						
2		Name of Group Member you are rating:				
3						
4		Performance Dimensions & Definitions				
5		Strongly Disagree: 1	Disagree: 2	Neutral: 3	Agree: 4	Agree Strongly: 5
6						
7		<i>Attendance and Punctuality: Group member was present for group meetings, conference calls, internet chats, or other scheduled meetings/conversations for working on the project. Group member arrived at scheduled meetings on time.</i>				
8		Rating:				
9						
10		<i>Level of Contribution: Group member's contributions were on-time, thorough, and accurate. Group member took responsibility for completing integral portions of the project.</i>				
11		Rating:				
12						
13		<i>Interpersonal Relations: Group member positively contributed to group performance (e.g., helped group move ahead, constructively resolved conflicts, was not destructive to group functioning, showed respect for other group members, <u>assisted in ensuring everyone had an opportunity to contribute equally</u>, etc.)</i>				
14		Rating:				
15						
16		Comments:				
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◀ ▶

Instructions

SELF

NAME 2

NAME 3

NAME 4

NAME 5

NAME 6

+

Final details – peer assessment form

⋮		HW5 (Final project) Available until Jun 10 at 11:59pm Due Jun 10 at 11:59pm 50 pts			⋮
⋮		HW5 -- peer evaluation for team members Available until Jun 10 at 11:59pm Due Jun 10 at 11:59pm 30 pts			⋮