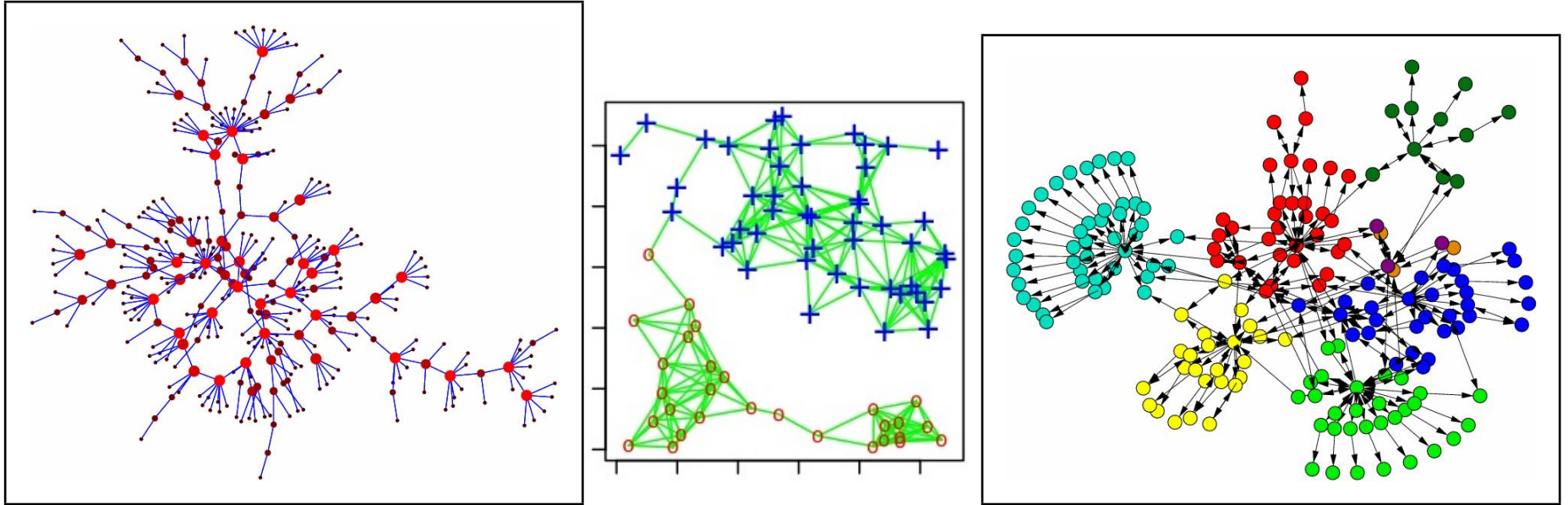


Overview of Network Theory, I



ECS 253 / MAE 253, Spring 2025, Lecture 1

Prof. Raissa D'Souza
University of California, Davis

Class composition

What graduate programs represented?

- Computer Science
- MAE
- Physics
- Applied Math
- Statistics
- Animal behavior
- Political Science
- Ecology

ECS/MAE 253 Network theory and applications

Course description:

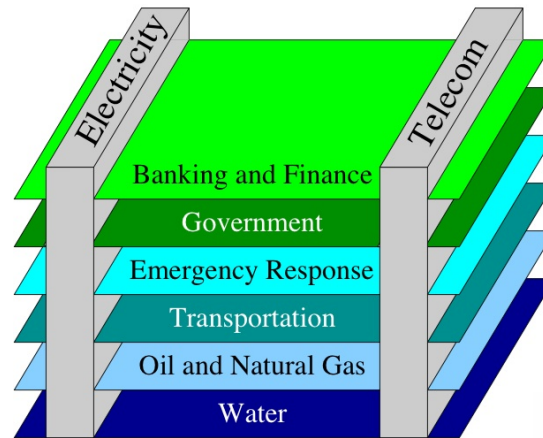
Develops the mathematical theory underlying growth, structure and function of networks with applications to physical, social, biological and engineered systems. Topics include network growth, resilience, epidemiology, phase transitions, software and algorithms, routing and search control, cascading failures.

(This is not a computer networking course. Instead see ECE 273 Networking Architecture and Resource Management.)

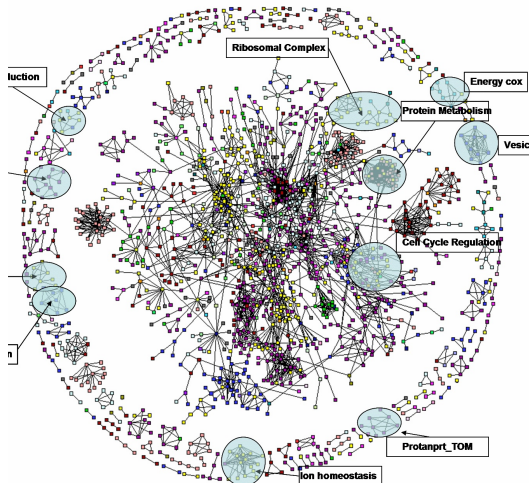
Raissa's background:

- 1999, PhD, Physics, Massachusetts Inst of Tech:
 - Joint appointment: Statistical Physics and Lab for Computer Science
- 2000-2002, Postdoctoral Research Fellow, Bell Laboratories:
 - Joint appointment: Fundamental Mathematics and Theoretical Physics Research Groups.
- 2002-2005, Postdoctoral Research Fellow, Microsoft Research:
 - “Theory Group” (Physics and Theoretical Computer Science)
- Fall 2005-present, UC Davis:
 - Dept of Computer Science, Dept of Mech and Aerospace Eng., Grad Group Applied Math, Physics Grad Group, Biosystems Eng Grad Group.
- 2007-present, External Faculty Member, Santa Fe Institute.
2018-onwards, Science Board Member.
- July 2022-present, UC Davis:
 - Associate Dean of Research, COE

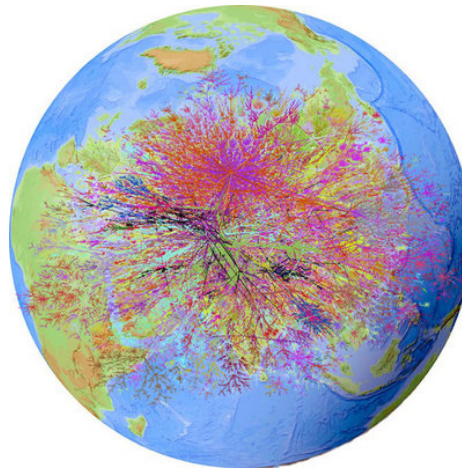
Complex networks are ubiquitous:



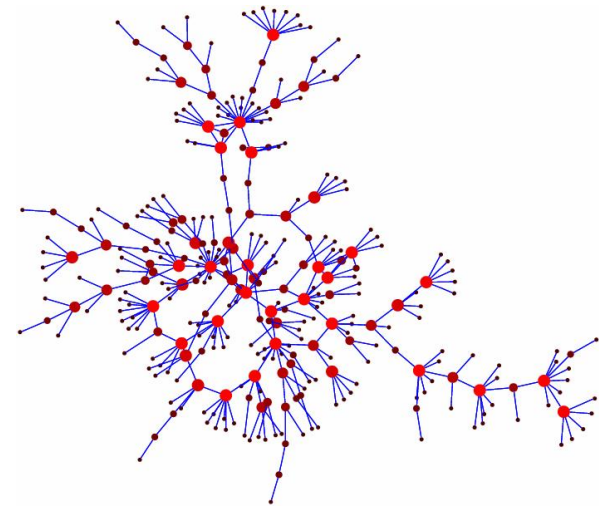
Critical Infrastructure



Biological & Ecological networks



Information and Communication technology



***Social networks:
Economics & Epidemics***

What is a Network?

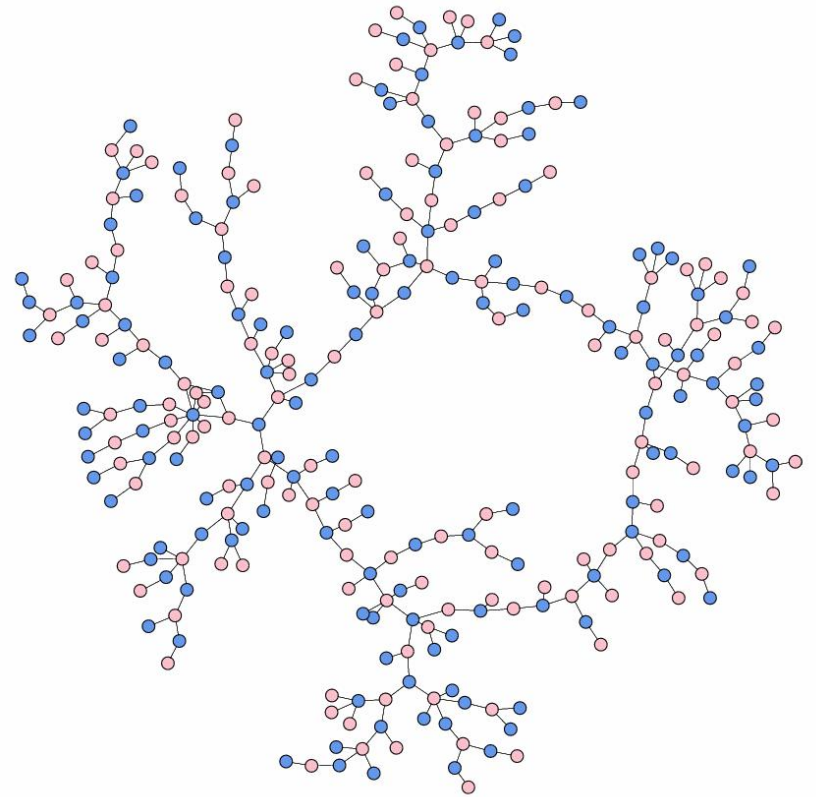
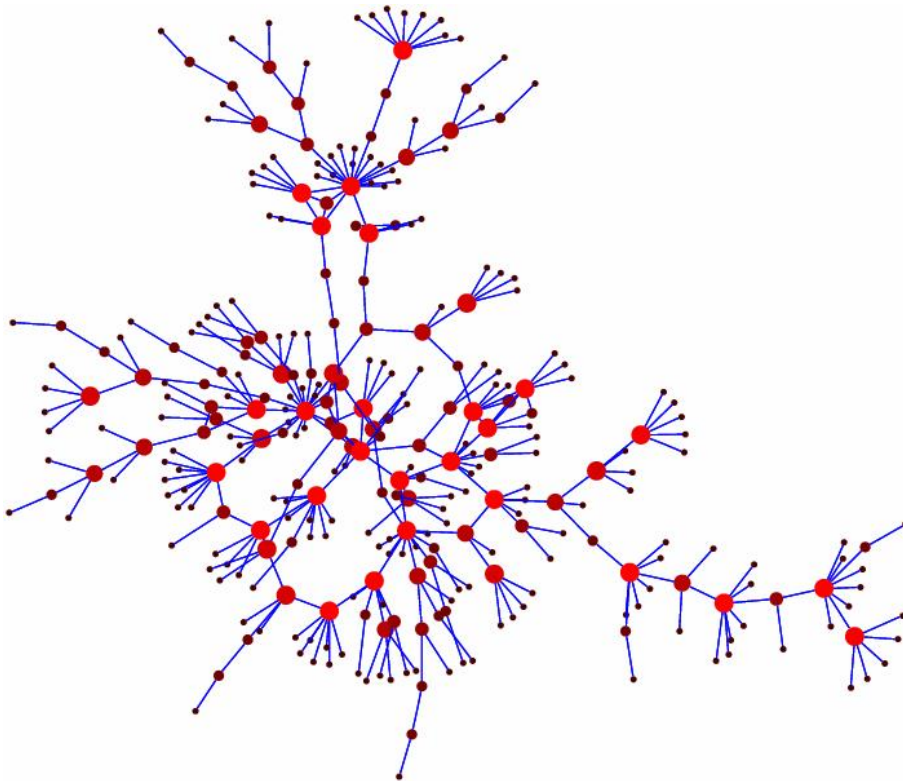
- **Topology** (i.e., structure: nodes/vertices and edges/links)
Measures of topology
- **Activity** (i.e., function, processes on networks, dynamics of nodes and edges)

Modeling networks

- Network growth
- Phase transitions
- Algorithms: analysis, growth/formation, searching and spreading
- Processes on networks

Example social networks

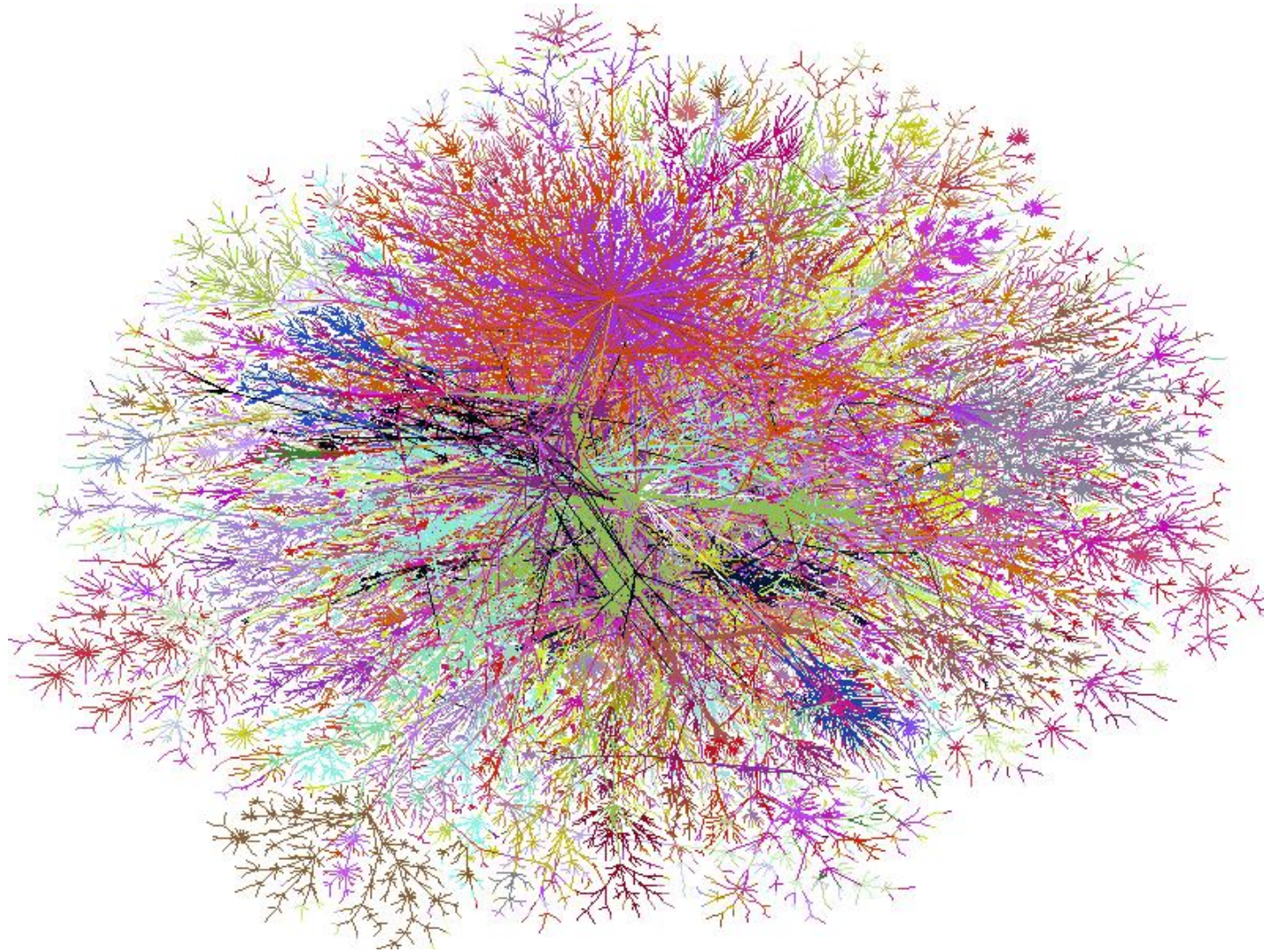
(Immunology; viral marketing; alliances/policy)



M. E. J. Newman

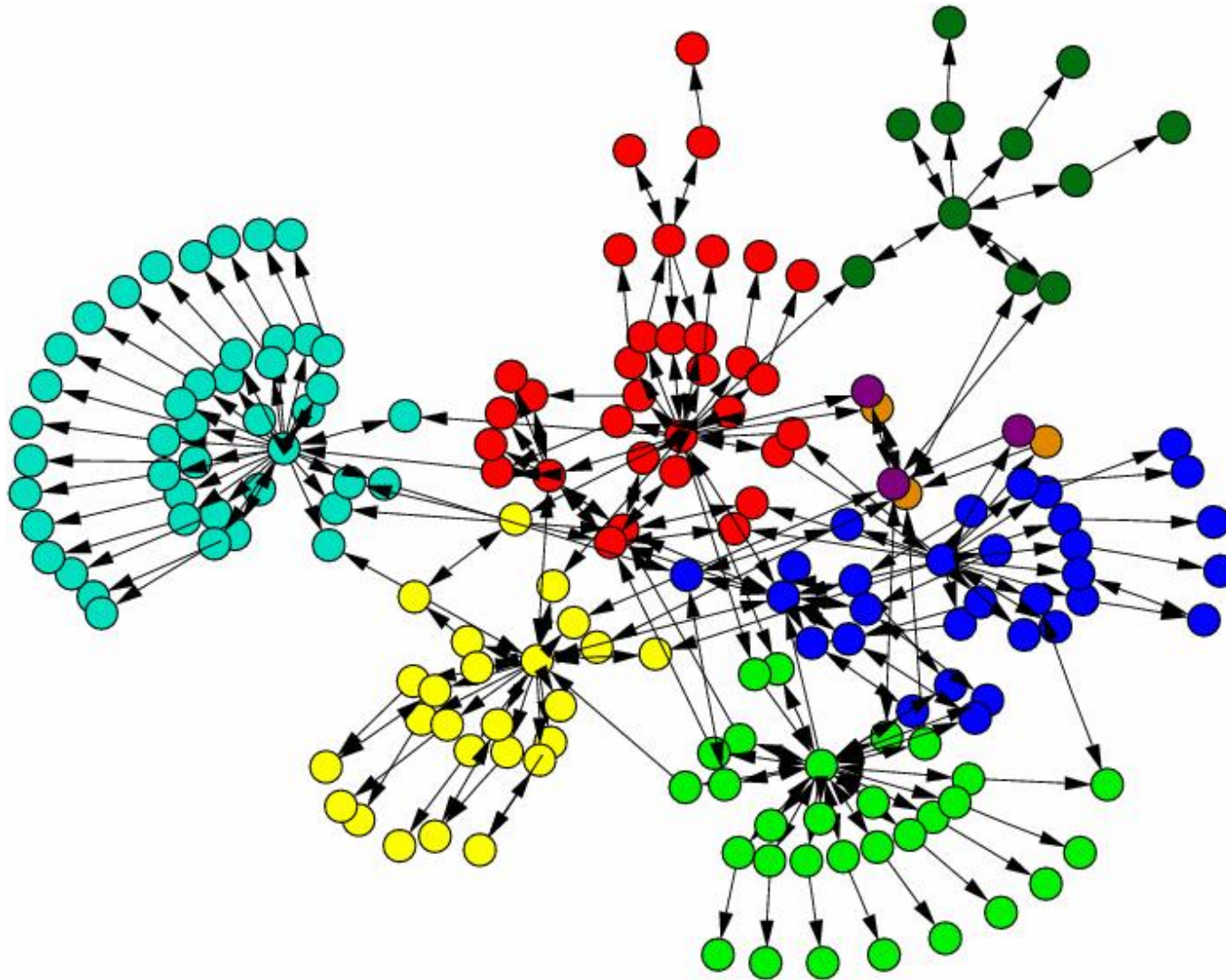
The Internet

(Robustness to failure; optimizing future growth; testing protocols on sample topologies)

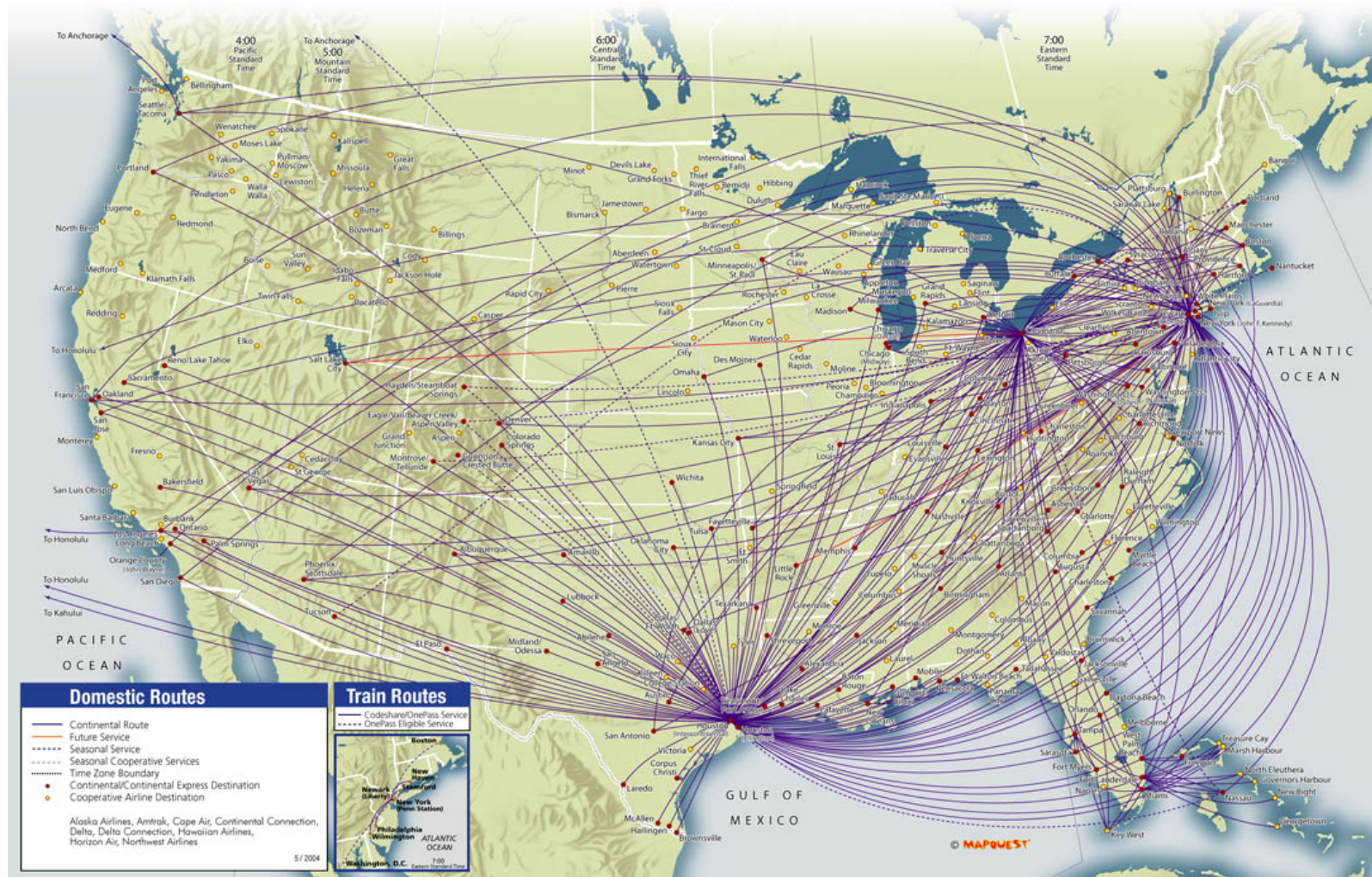


H. Burch and B. Cheswick

**A typical web domain
(Web search/organization and growth
centralized vs. decentralized protocols)**

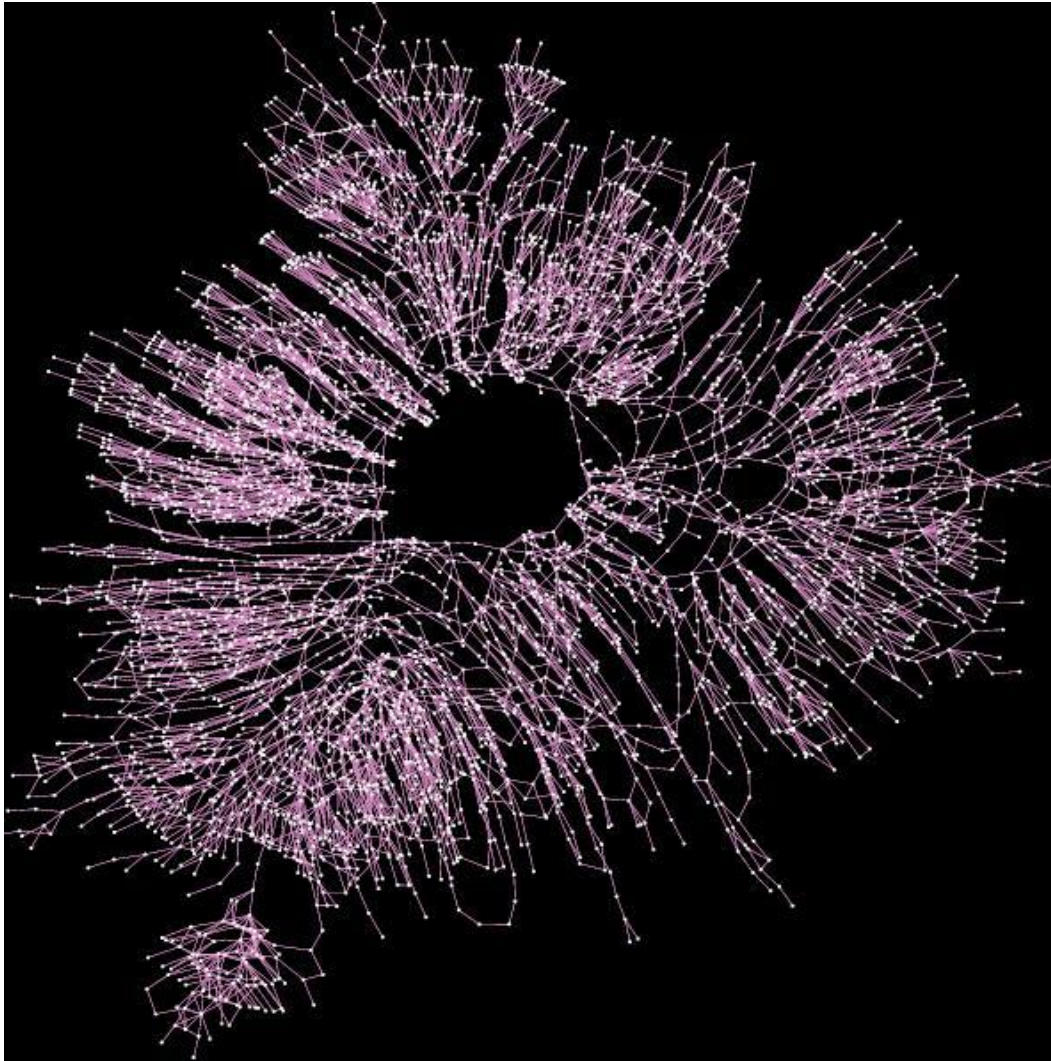


The airline network (Optimization; dynamic external demands)



Continental Airlines

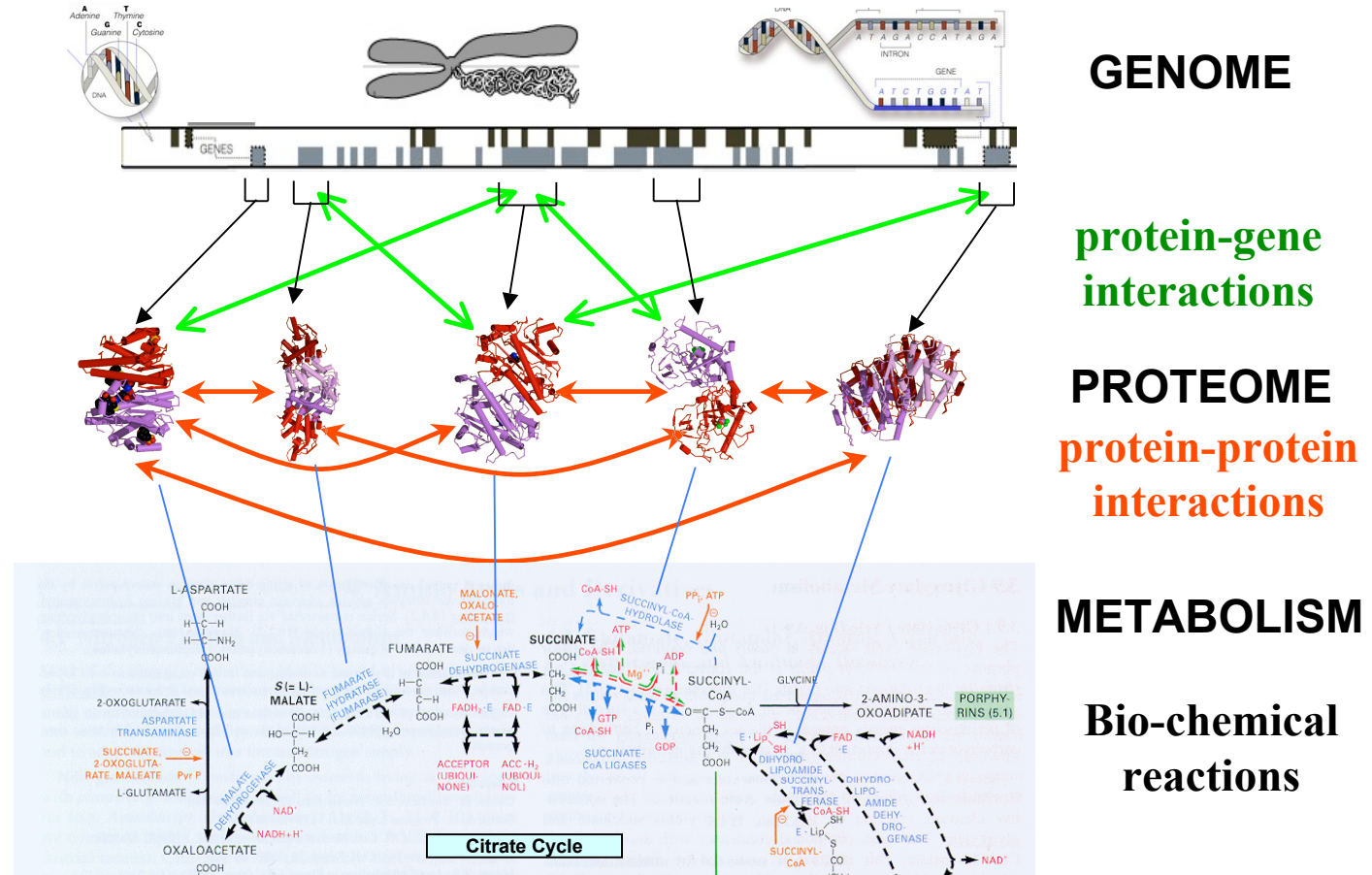
The power grid (Mitigating failure; Distributed sources)



M. E. J. Newman

Biology: Networks at many levels

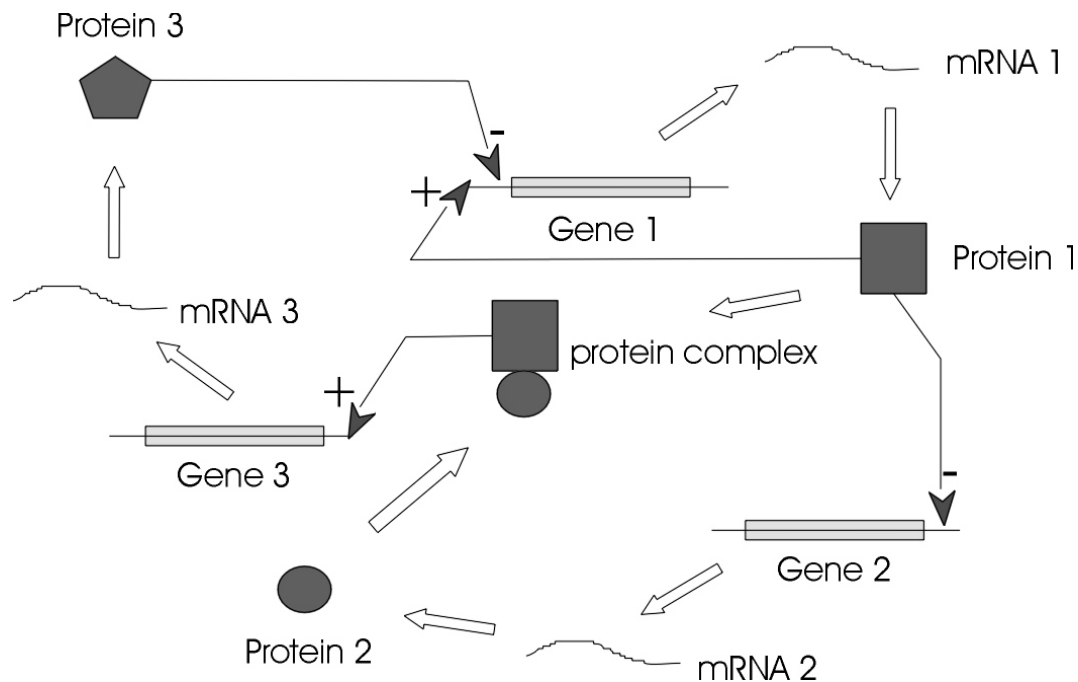
Control mechanisms / drug design/ gene therapy / biomarkers of disease



research in biological networks

■ gene regulatory networks

- humans have only 30,000 genes, 98% shared with chimps
- the complexity is in the interaction of genes
- can we predict what result of the inhibition of one gene will be?



Slide from Adamic's course

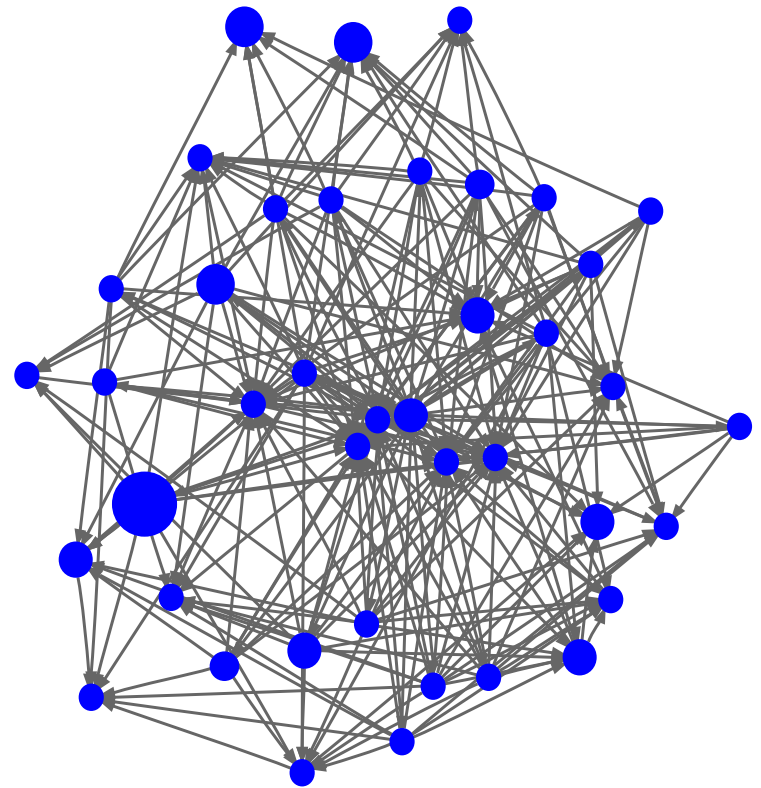
Software systems

(Highly evolveable, modular, robust to mutation,
exhibit punctuated eqm)

Open-source software as a “systems” paradigm.

Networks:

- Function calls
- Email communication
- Socio-Technical congruence



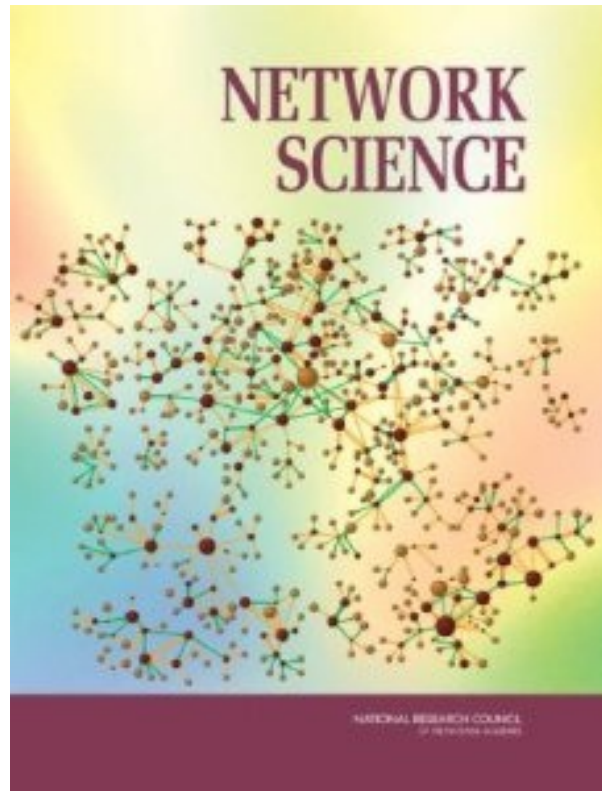
The past decade, a “Science of Networks”: (Physical, Biological, Social)

- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** /spontaneously arising versus **engineered** /built.
- **Directed** versus **undirected** edges.
- Each network **optimizes** something unique.
- Identifying **similarities** and fundamental **differences**
- Interplay of **topology** and **function** ?
- Unifying features: – **Broad heterogeneity in node degree.**
– **Small Worlds** (Diameter $\sim \log(N)$).

Explosion of work and **tools**

- R, Graphviz, Pajek, igraph, Network Workbench, NetworkX, Netdraw, UCInet, Bioconductor, Ubigraph....

Natn Acam Sciences/Natn Research Council Study (2005)



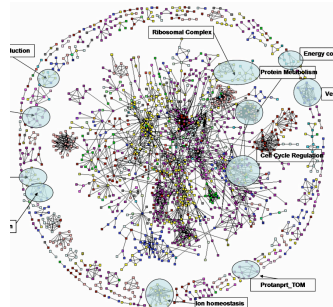
“all our modern critical infrastructure relies on networks... too much emphasis on specific applications/jargon/disciplinary stovepipes... need a cross-cutting science of networks...
Research for the 21st century”

In reality a collection of interacting networks:

Networks:



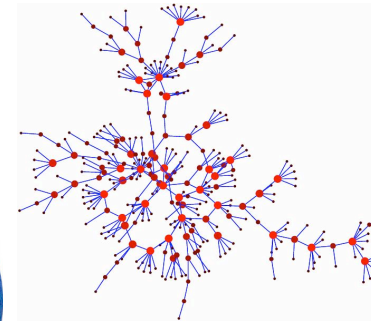
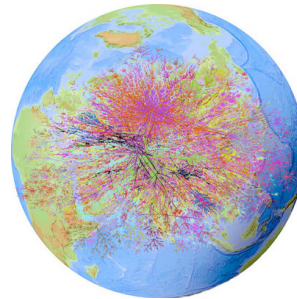
**Transportation
Networks/
Power grid**
(distribution/
collection networks)



Biological networks

- protein interaction
- genetic regulation
- drug design

Computer networks



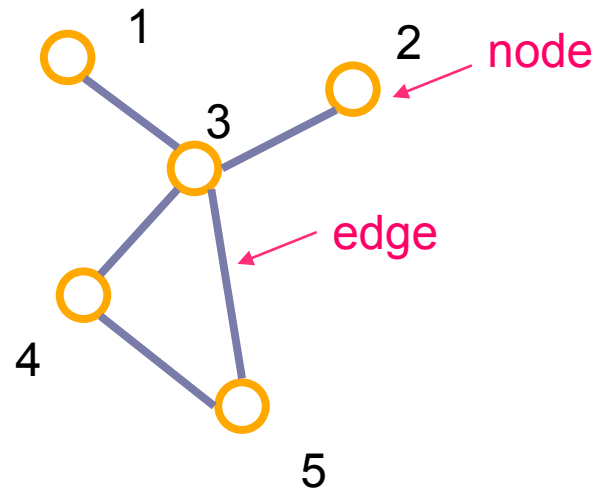
Social networks

- Immunology
- Information
- Commerce

- E-commerce → WWW → Internet → Power grid → River networks.
- Biological virus → Social contact network → Transportation networks → Communication networks → Power grid → River networks.
(Historical progression: Spatial waves (Black plague) Regional outbreaks (ships) Global pandemics (airplanes))

What are networks?

- Networks are collections of points joined by lines.



“Network” $\equiv$ “Graph”

points	lines	
vertices	edges, arcs	math
nodes	links	computer science
sites	bonds	physics
actors	ties, relations	sociology

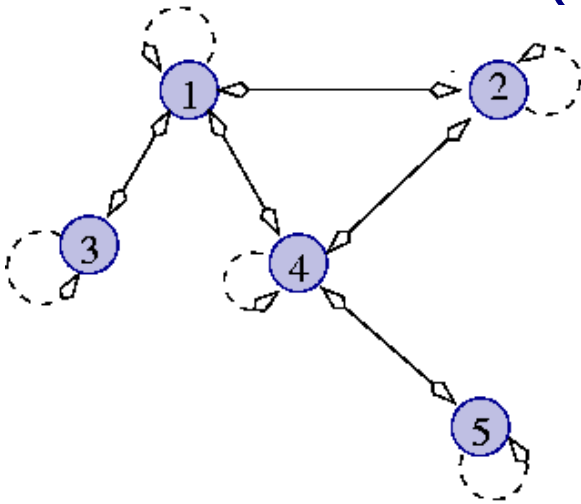
Slide from Adamic's course

How do we represent a simple individual network as a mathematical object?

NETWORK TOPOLOGY

Connectivity matrix, M :

$$M_{ij} = \begin{cases} 1 & \text{if edge exists between } i \text{ and } j \\ 0 & \text{otherwise.} \end{cases}$$

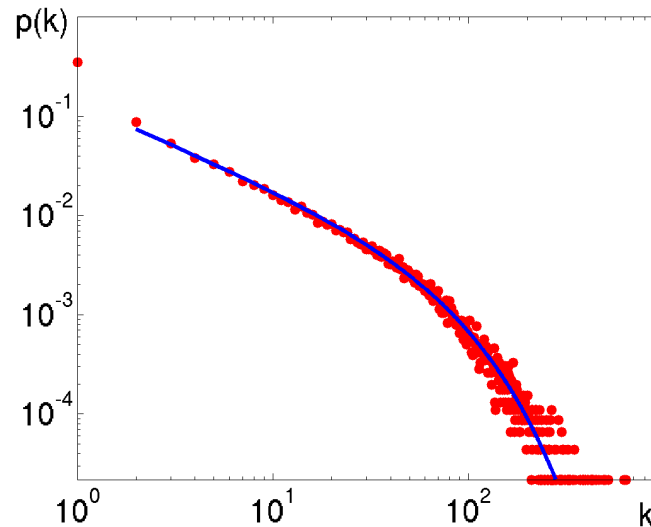


$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = M$$

Node **degree** is number of links.

Typical measures of network topology

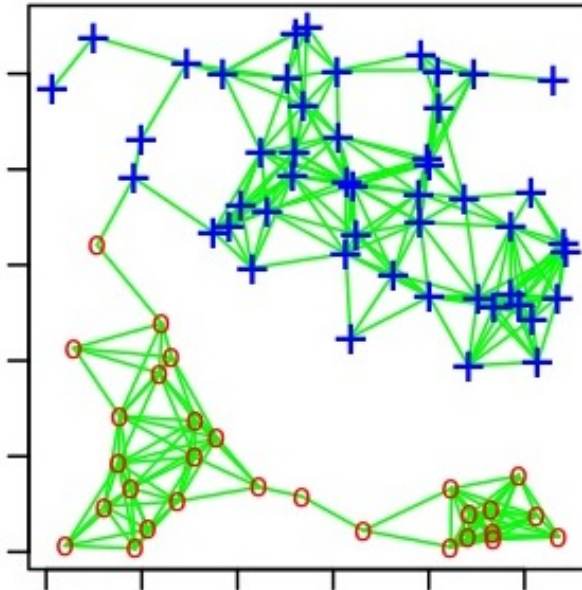
- **Degree distribution** (fraction of nodes with degree k , for all k)



- **Clustering coefficient** (fraction of triangles in the graph/transitivity: Are my friends friends with each other?)
Also a local measure, for each node c_i is number of connections existing between neighbors/total number of possible connections.

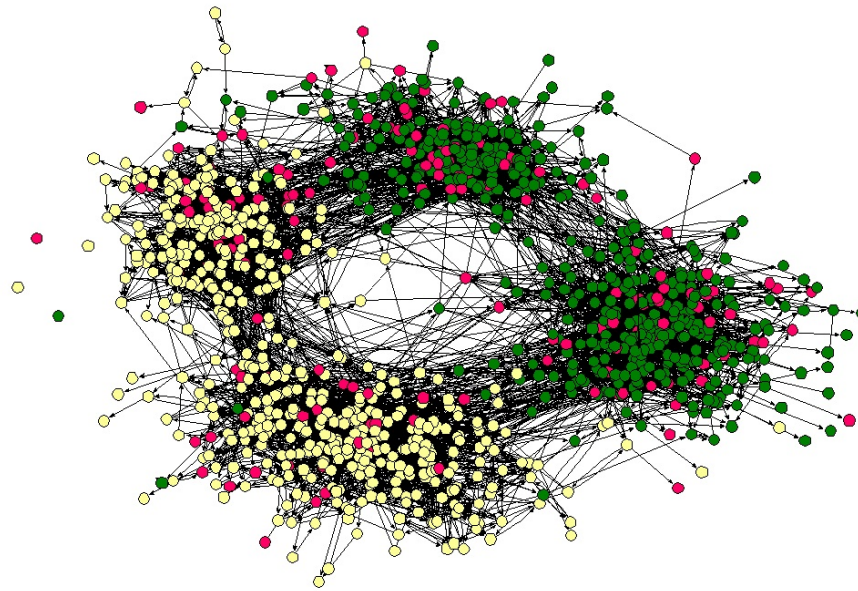
Typical measures of network topology, cont

- **Diameter** (Greatest distance between any two connected nodes)
“Small world” if $d \sim \log N$ and strong clustering.
(Watts Stogatz, *Nature* **393**, 1998.)
- **Betweenness centrality** (Fraction of shortest paths passing through a node, i.e., is a node a bottleneck for flow?)



Typical measures of network topology, cont

- **Assortative/dissortative mixing** (Are nodes with similar attributes more or less likely to link to each other? Mixing by node degree common. Also, in social networks mixing by gender and race.)



(Example of assortative mixing by race. Friendship network of HS students: White, African American and Other.)