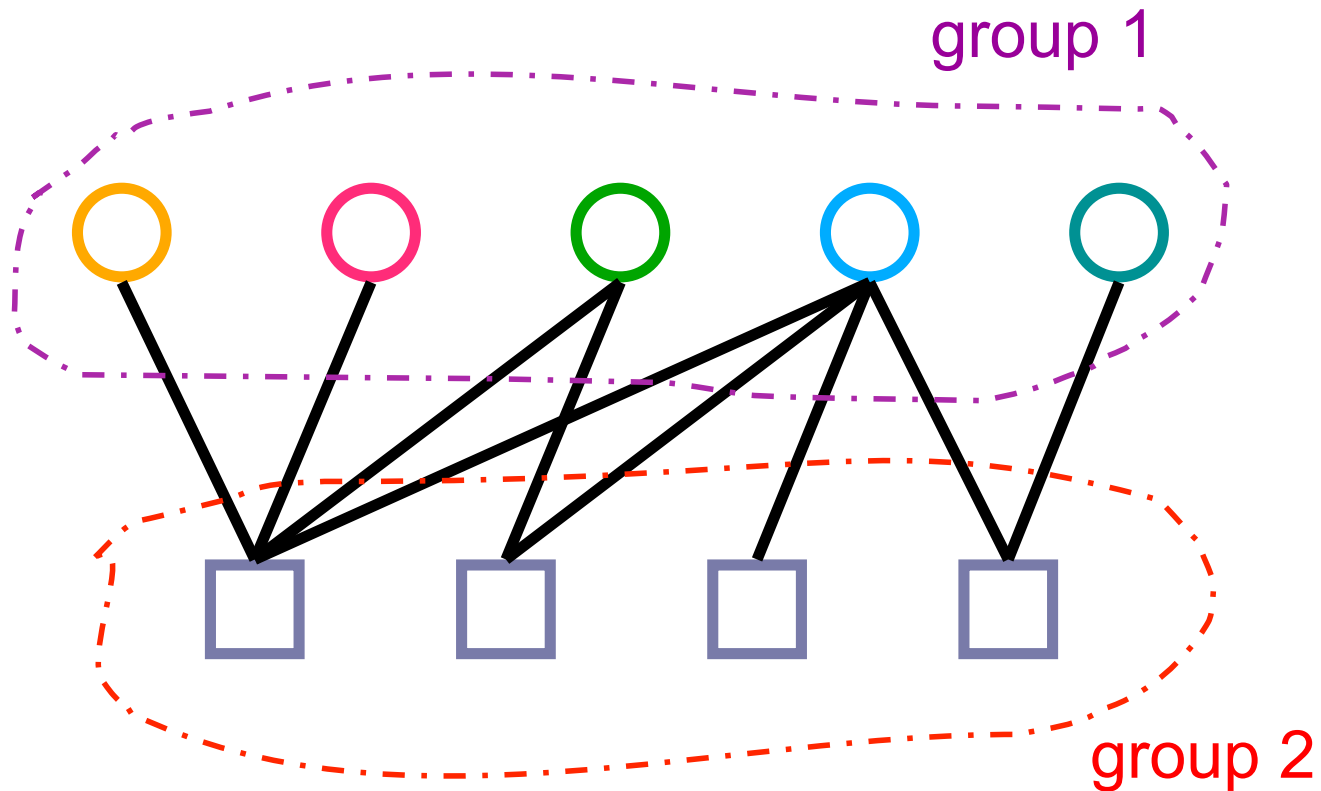


Other important network paradigms: Bipartite networks, trees, cliques, and cores



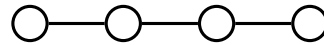
Other important network paradigms

- Bipartite networks
- Hypergraphs
- Trees
- Planar graphs
- Cliques

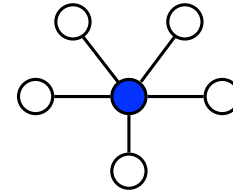
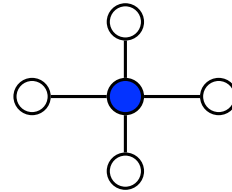
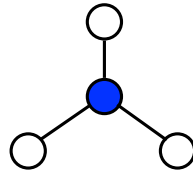
This content largely from Adamic's lectures

Some Basic Types of Graphs

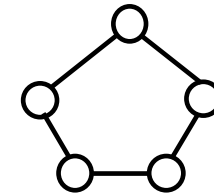
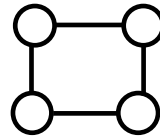
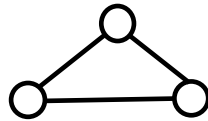
Paths



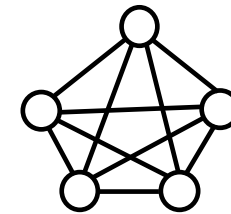
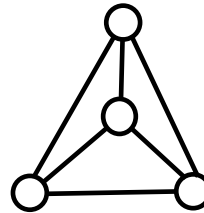
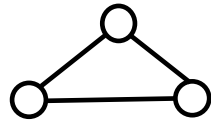
Stars



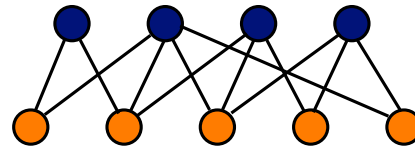
Cycles



Complete Graphs

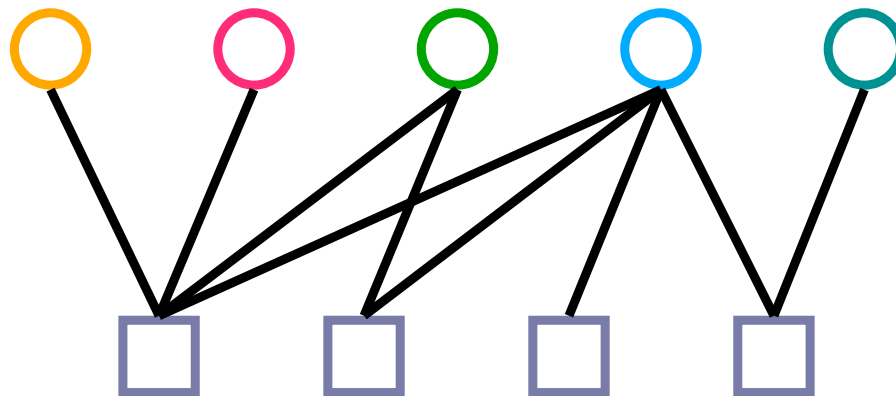


Bipartite Graphs



Bipartite (two-mode) networks

- edges occur only between two groups of nodes, not within those groups
- for example, we may have individuals and *events*
 - directors and boards of directors
 - customers and the items they purchase
 - metabolites and the reactions they participate in

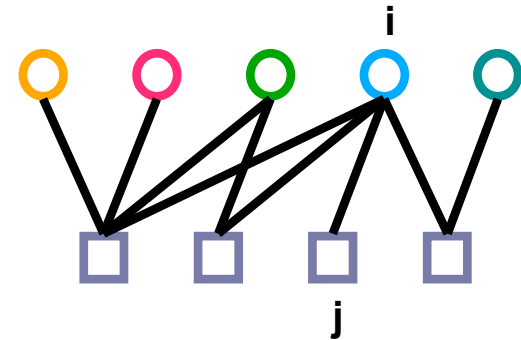


in matrix notation

■ B_{ij}

■ = 1 if node i from the first group
links to node j from the second group

■ = 0 otherwise



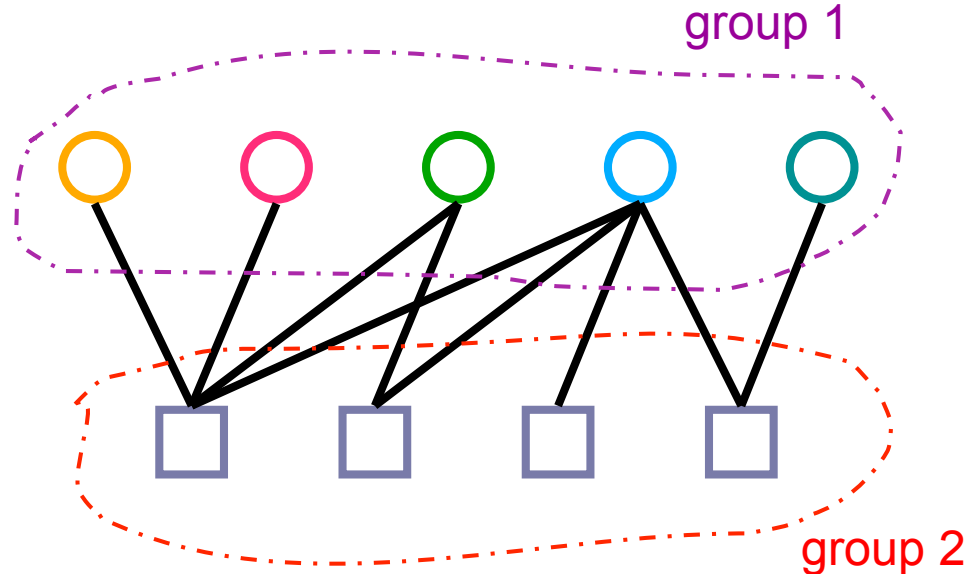
■ B is usually not a square matrix!

■ for example: we have n customers and m products

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

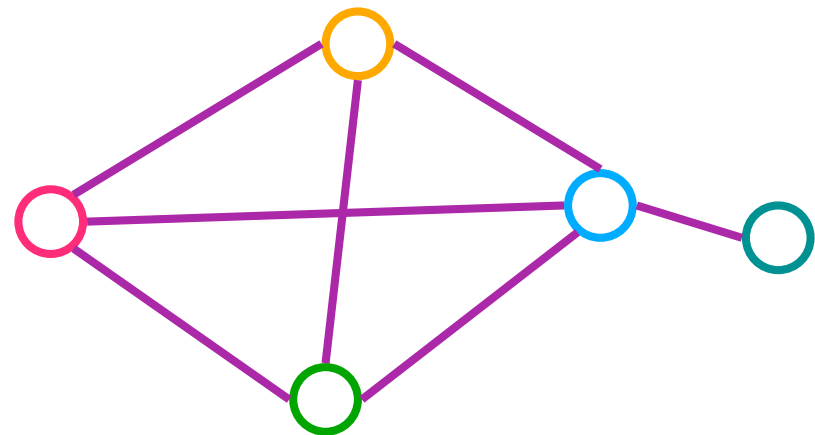
going from a bipartite to a one-mode graph

■ Two-mode network



■ One mode projection

- two nodes from the first group are connected if they link to the same node in the second group
- naturally high occurrence of cliques
- some loss of information
- Can use weighted edges to preserve group occurrences

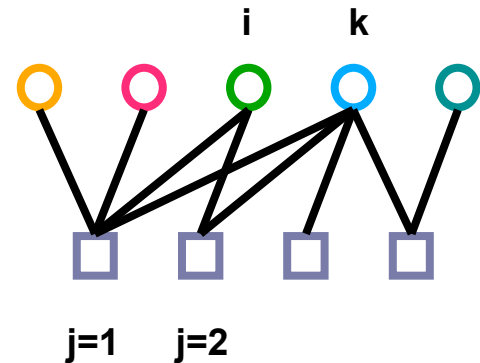


Our SWN examples are from bipartite collapse

- The Oracle of Bacon – The path to Kevin Bacon
<http://oracleofbacon.org/>
- SixDegrees.org – connecting causes and celebrities
<http://www.sixdegrees.org/>
- Six degrees of Kevin Garnett
http://www.slate.com/articles/sports/slate_labs/2013/10/six_degrees_of_kevin_garnett_connect_any_two_athletes_who_ve_ever_played.html
- Six degrees of NBA separation
<http://harvardsportsanalysis.wordpress.com/2011/03/04/six-degrees-of-nba-separation/>
(Blog post explaining use of Dijkstra's algorithm)

Collapsing to a one-mode network

- i and k are linked if they both link to j
- $P_{ij} = \sum_k B_{ki} B_{kj}$
- $P' = B B^T$
 - the transpose of a matrix swaps B_{xy} and B_{yx}
 - if B is an $n \times m$ matrix, B^T is an $m \times n$ matrix



$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

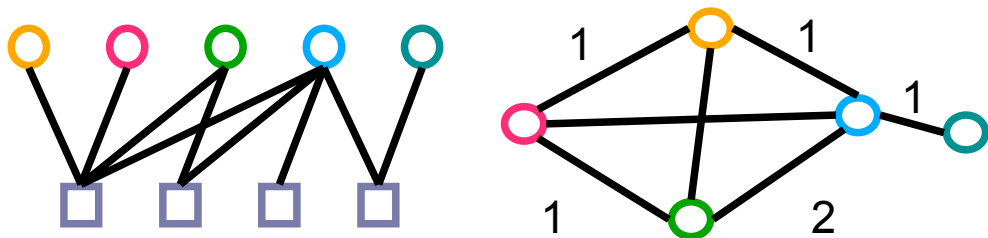
$$B^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Matrix multiplication

- general formula for matrix multiplication $Z_{ij} = \sum_k X_{ik} Y_{kj}$
- let $Z = P'$, $X = B$, $Y = B^T$

$$P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

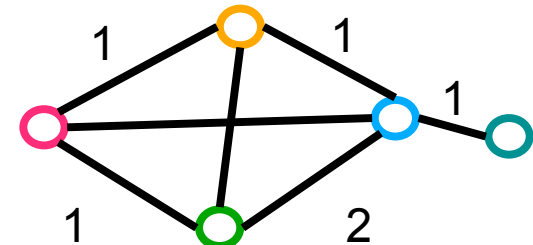
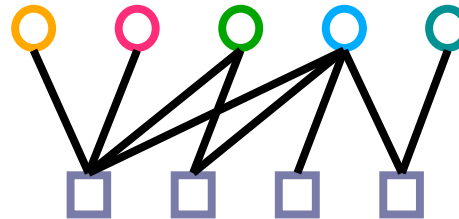
$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 1*1 + 1*1 + 1*0 + 1*0 = 2$$



Collapsing a two-mode network to a one mode-network

- Assume the nodes in group 1 are people and the nodes in group 2 are movies
- P' is symmetric
- The diagonal entries of P' give the number of movies each person has seen
- The off-diagonal elements of P' give the number of movies that both people have seen

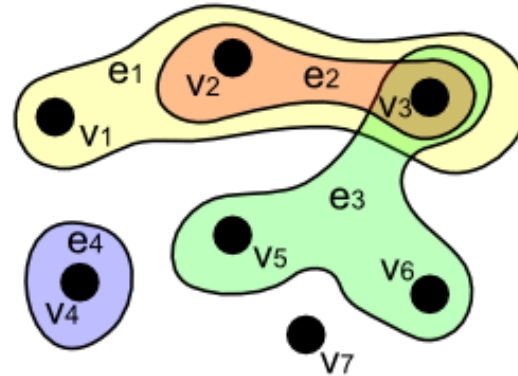
$$P' = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$



HyperGraphs

- Edges join more than two nodes at a time (*hyperEdge*)

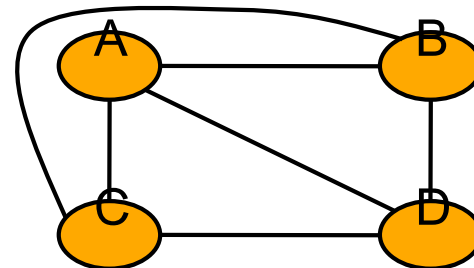
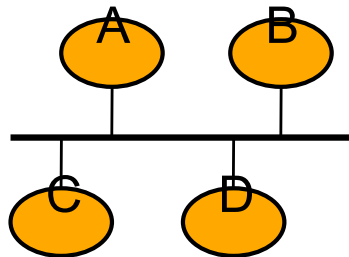
- Affiliation networks



- Examples

- Families

- Subnetworks



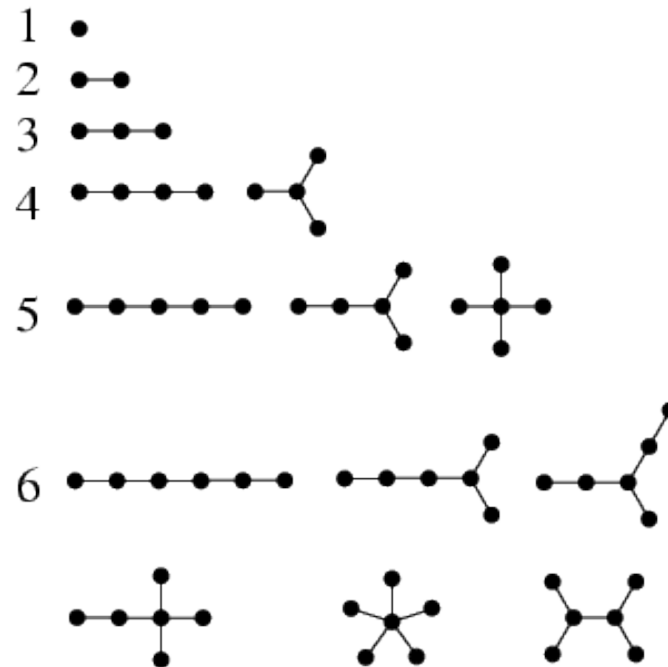
Hypergraphs — beyond dyadic interactions

In ways, good models of social networks.

- “Complex Networks as Hypergraphs”
Ernesto Estrada, Juan A. Rodriguez-Velazquez
arXiv:physics/0505137, 2005.
- “Random hypergraphs and their applications”,
G Ghoshal, V Zlatić, G Caldarelli, MEJ Newman,
Physical Review E 79 (6), 2009.
- Ramanathan, R., et al. “Beyond graphs: Capturing groups in networks.”
NetSciCom, 2011 IEEE Conference, 2011.
- “Information Flows: A Critique of Transfer Entropies”
Ryan G. James, Nix Barnett, James P. Crutchfield
Accepted to Physical Review Letters, April 12, 2016.
- See also hypergraph mining, hypergraph learning algorithms, overlapping communities, link prediction...

Trees

- Trees are undirected graphs that contain no cycles



- For n nodes, number of edges $m = n - 1$
- Any node can be dedicated as the root

examples of trees

■ In nature

- trees
- river networks
- arteries (or veins, but not both)

■ Man made

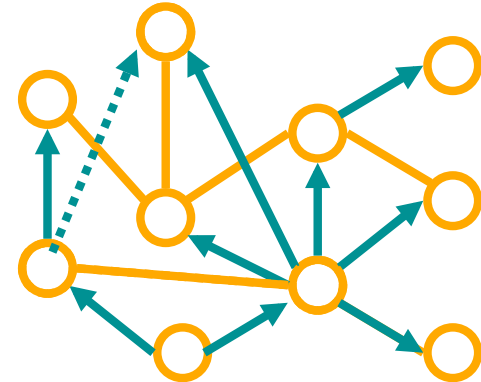
- sewer system

■ Computer science

- binary search trees
- decision trees (AI)

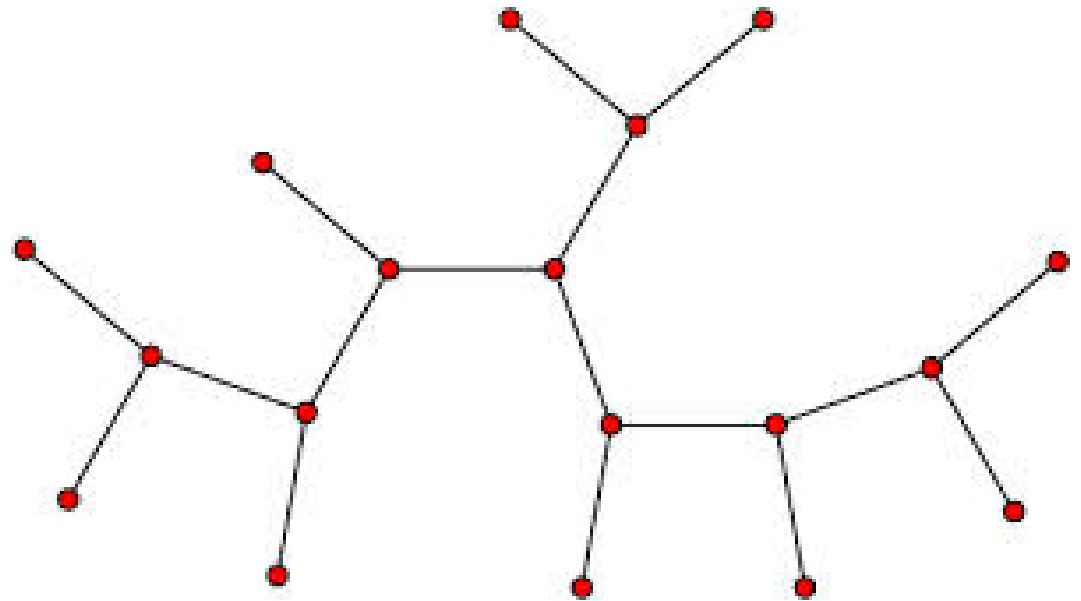
■ Network analysis

- minimum spanning trees
 - from one node – how to reach all other nodes most quickly
 - may not be unique, because shortest paths are not always unique
 - depends on weight of edges



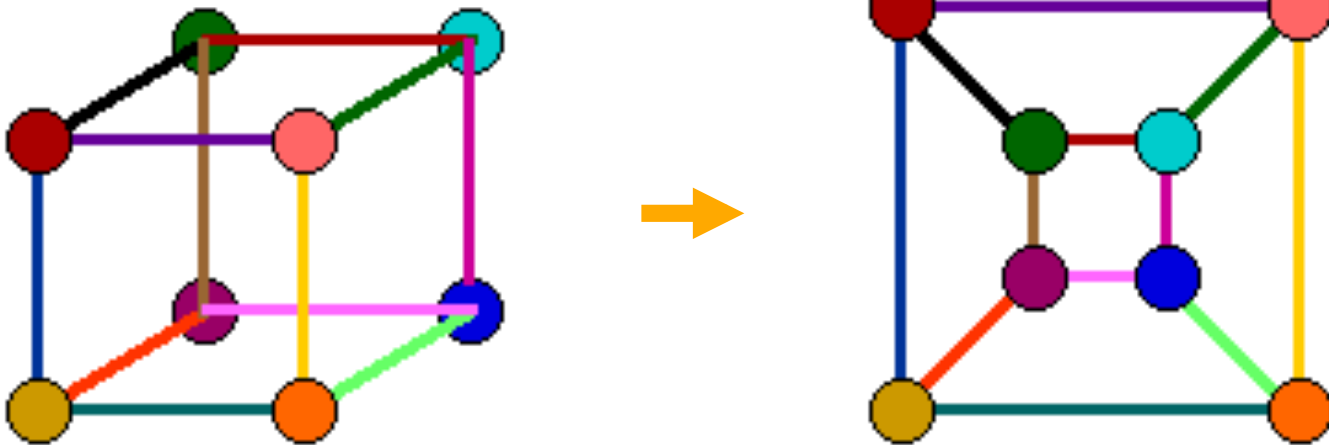
Searching on a tree

- Breadth first search :
explore all the neighbors first
- Depth first search :
take a step out in hop-count each iteration

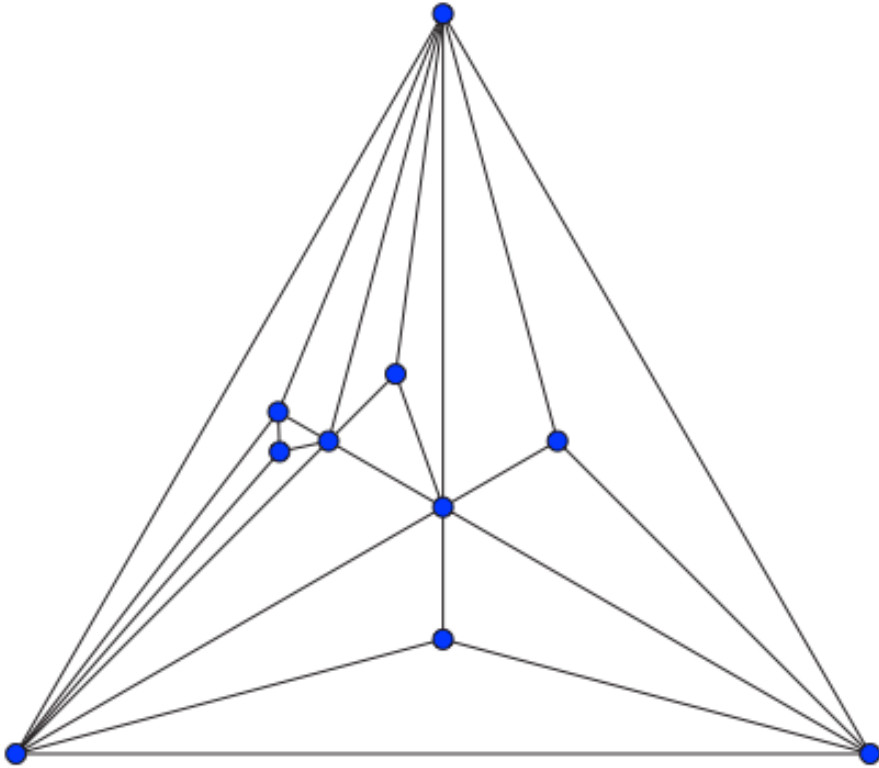


Planar graphs

- A graph is planar if it can be drawn on a plane without any edges crossing



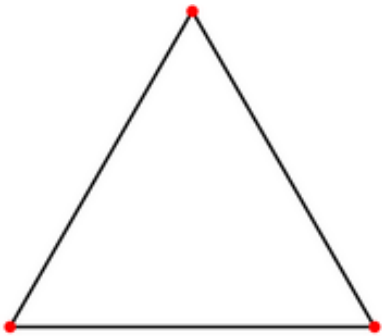
Apollonian network



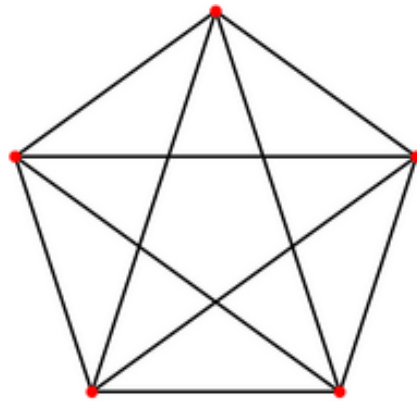
- An undirected graph formed by a process of recursively subdividing a randomly selected triangle into three smaller triangles.
- A planar graph with power law degree distribution, and small world property.

Cliques and complete graphs

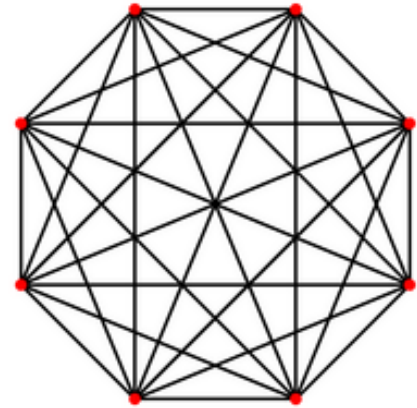
- K_n is the complete graph (clique) with n vertices
 - each vertex is connected to every other vertex
 - there are $n(n-1)/2$ undirected edges



K_3

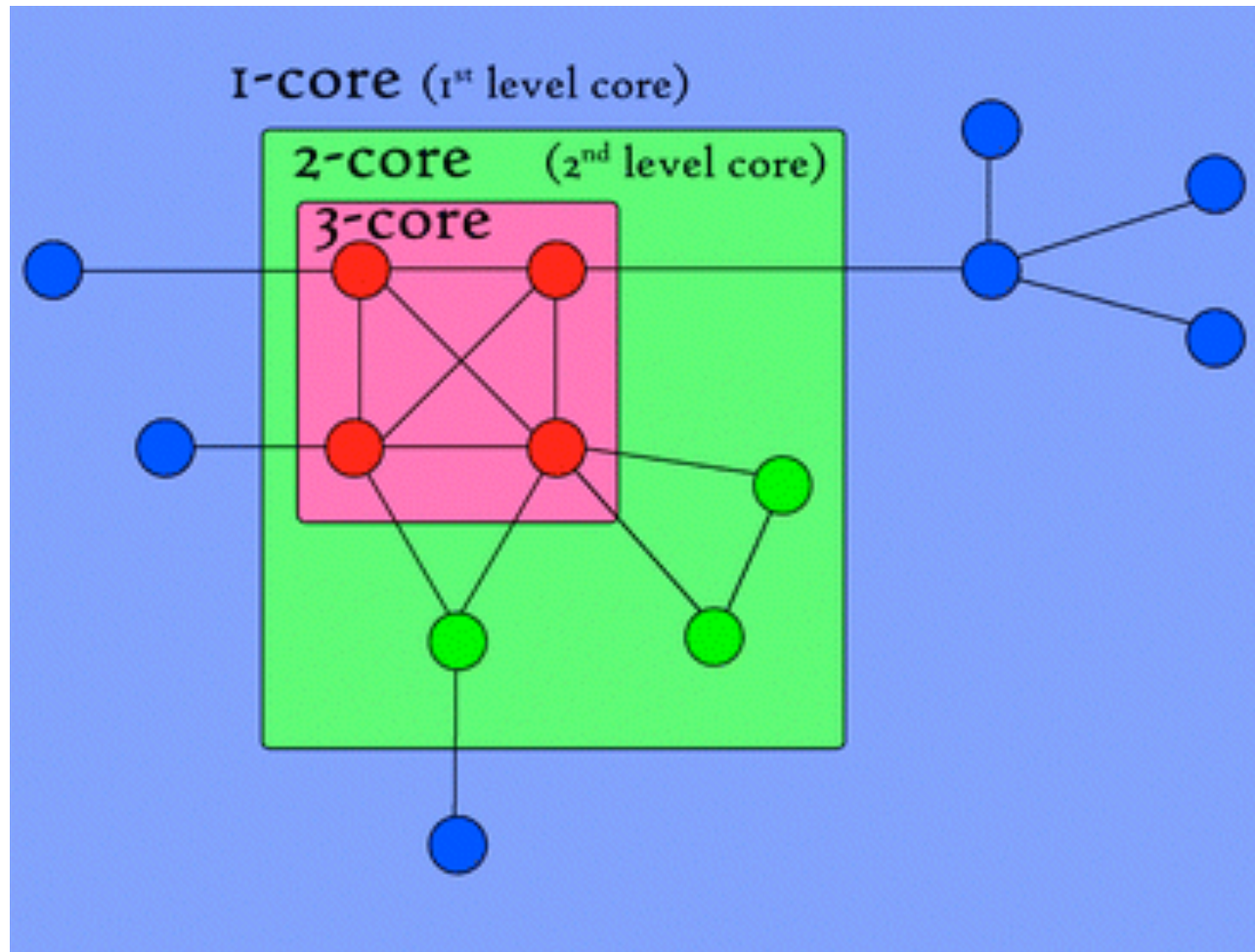


K_5



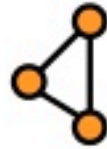
K_8

The k-core and k-shell



- k-clique

k=3 (triangle)



k=4



k=5

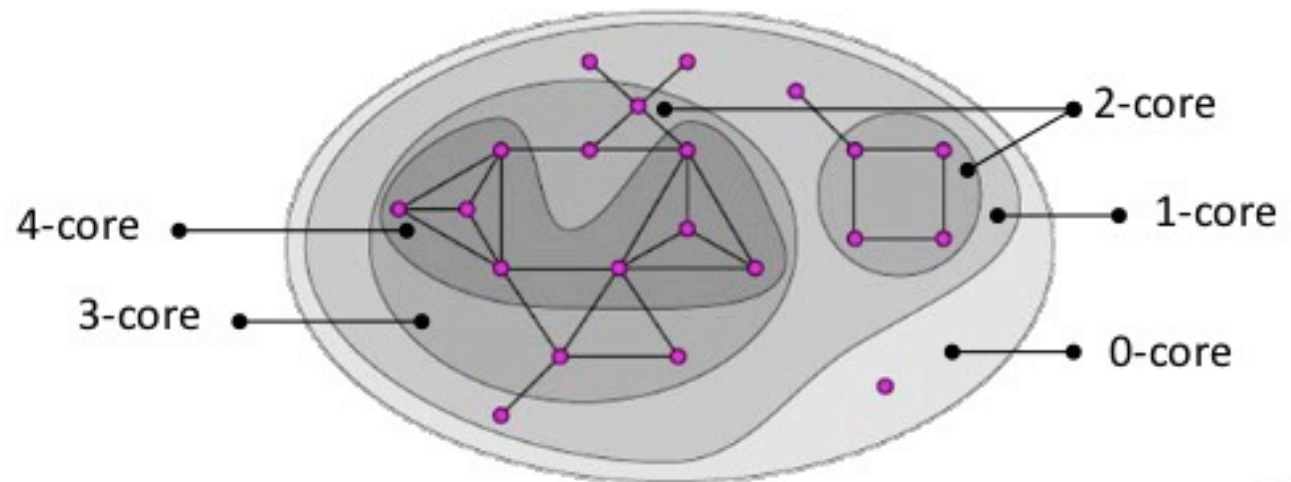


- N-clique



N=2 (star)

- k-core



k-core decomposition

- For visualization
- k-core decomposition of the Internet
 - router level
 - AS level
 - e.g. Carmi et. al. PNAS 2007.

“A model of Internet topology using k-shell decomposition”
A nucleus, a fractal layer, and tendrils.
- in random graphs and statistical physics:
 - “K-core organization of complex networks”
SN Dorogovtsev, AV Goltsev, JFF Mendes
Physical review letters, 2006.