

Weighted and Signed Networks

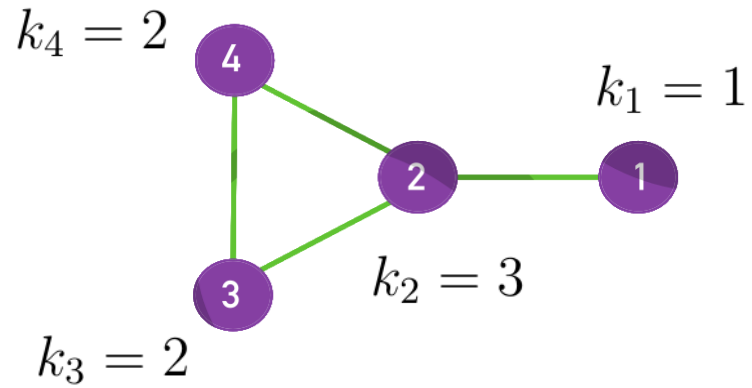


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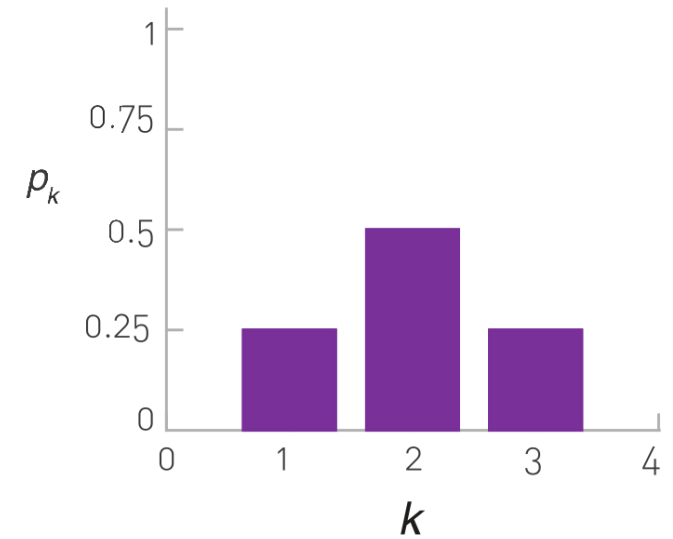
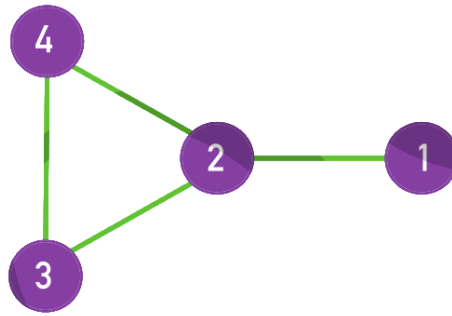
Recap

Degree k :
number of neighbors



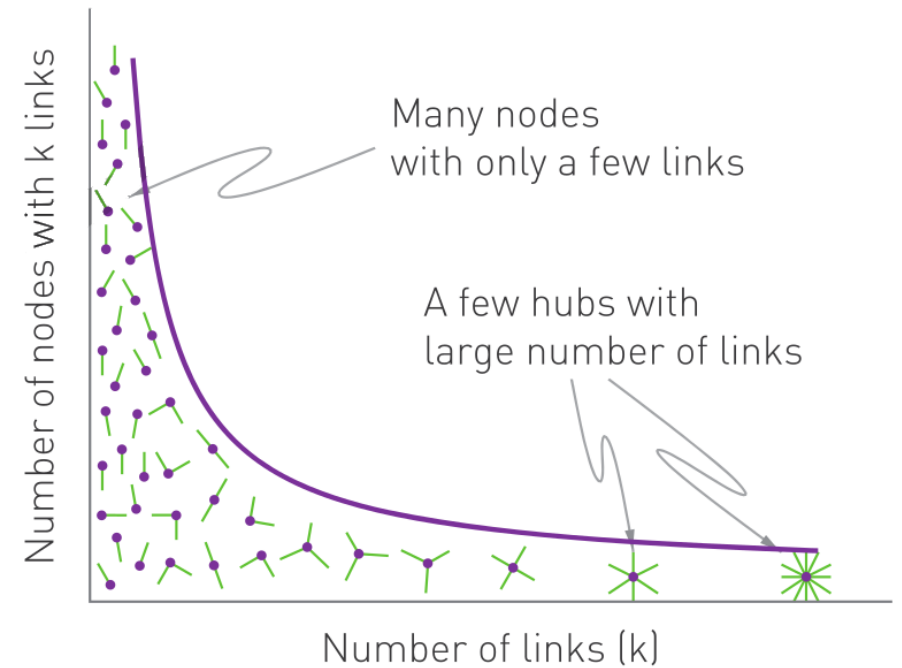
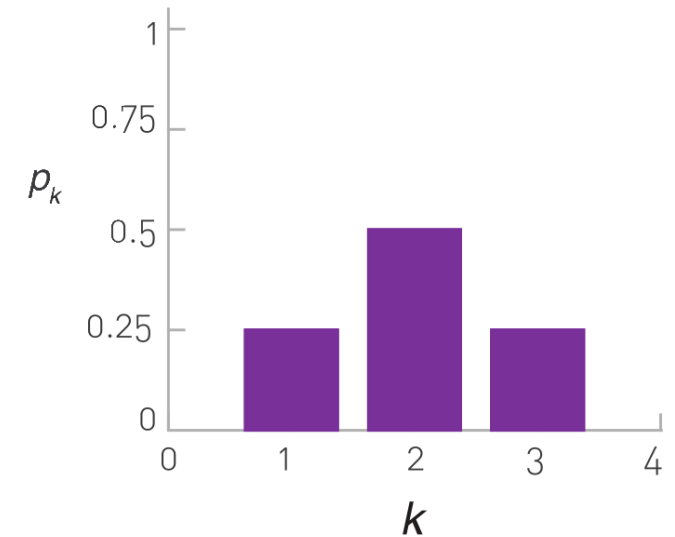
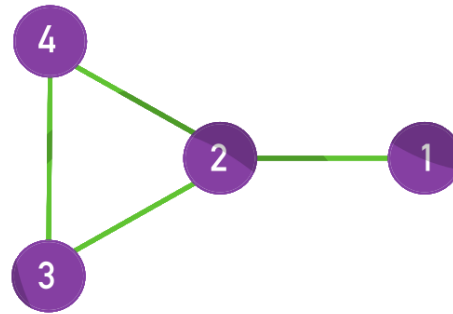
Recap

Degree distribution p_k :
probability that a randomly
selected node has degree k

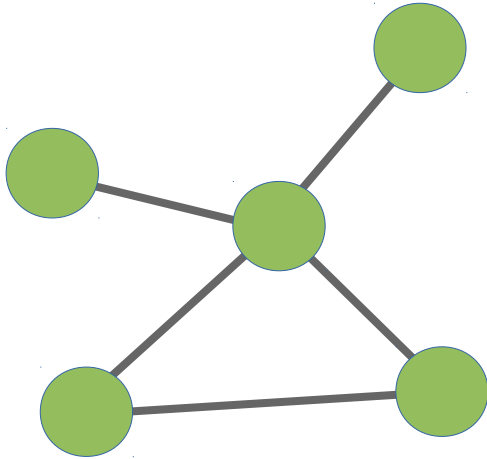


Recap

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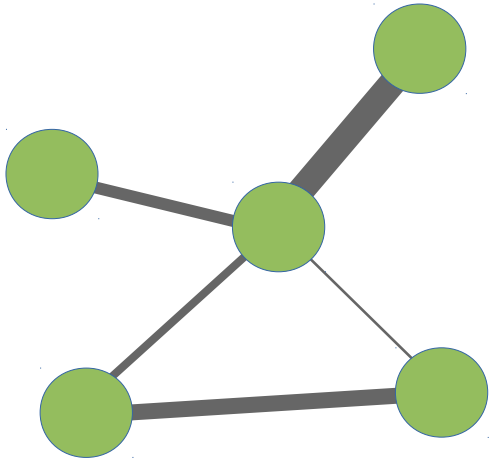
Today



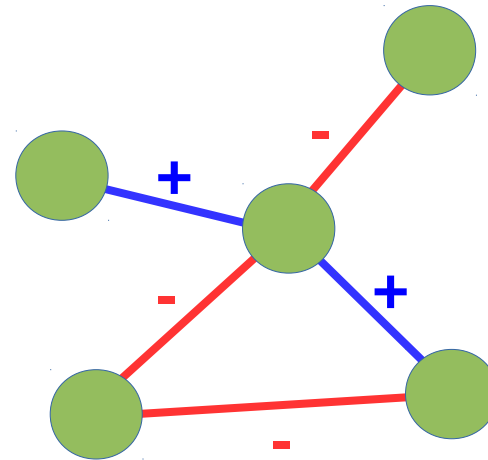
So far:

Links indicate if interaction between node pairs exist.

Intensity or nature of interaction?



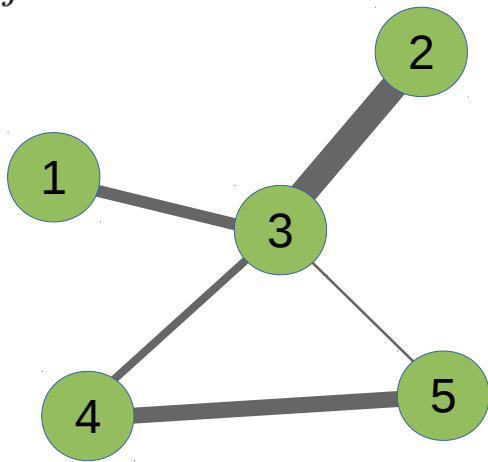
Weighted network



Signed network

Weighted Networks

$$w_{ij} > 0$$



Link weight w_{ij} represents the strength or intensity of the interaction between nodes.

Examples:

- Social networks: frequency or intimacy (friends vs acquaintances)
- Airline network: number of passengers
- Scientific collaboration: #papers co-authored

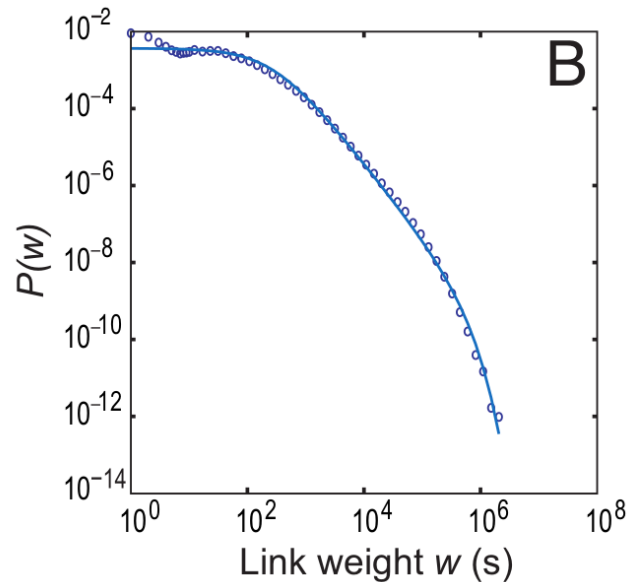
Weight distribution

$p(w)$: probability that a random link has weight w

Link weight distribution is broad!

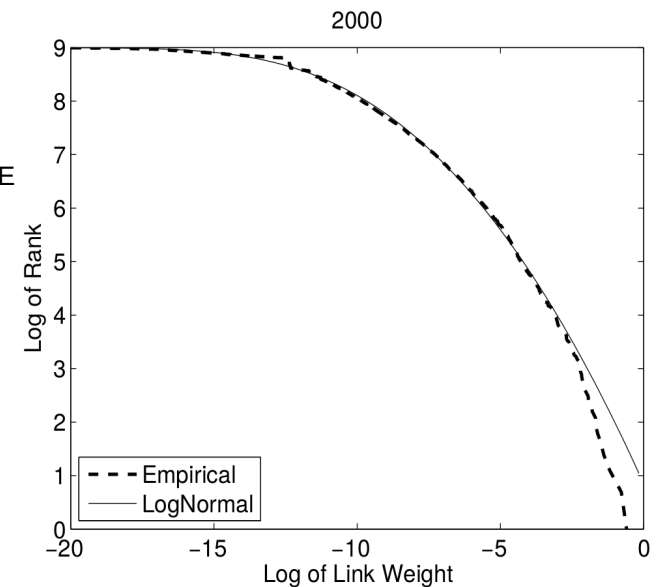
Social ties:

Onnela, J-P., et al., PNAS
104.18 (2007): 7332-7336.



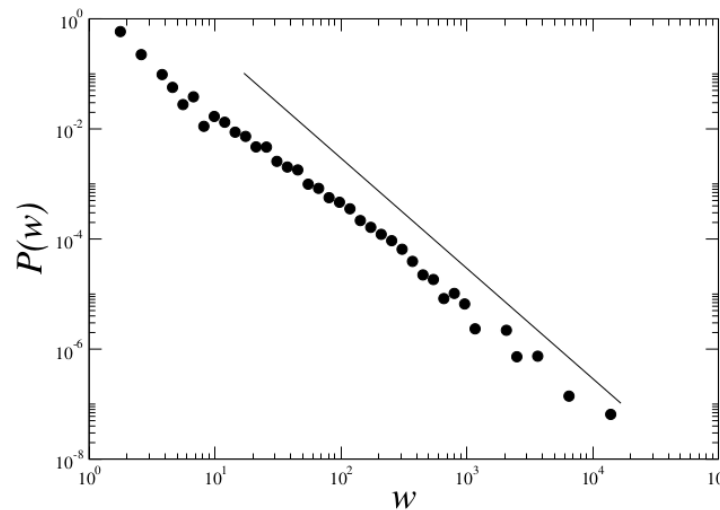
World Trade Network:

Fagiolo et al. Physical Review E
79.3 (2009): 036115.



Inter-urban traffic:

De Montis, et al., Environment and
Planning B: Planning and Design
34.5 (2007): 905-924.



Node strength

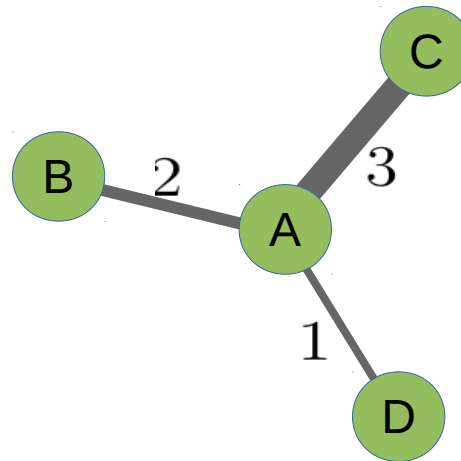
Unweighted network
Degree: #adjacent links

$$k_A = 3$$

Weighted network:
Strength: sum of weight of adjacent links

$$s_i = \sum_{(i,j) \in E} w_{ij}$$

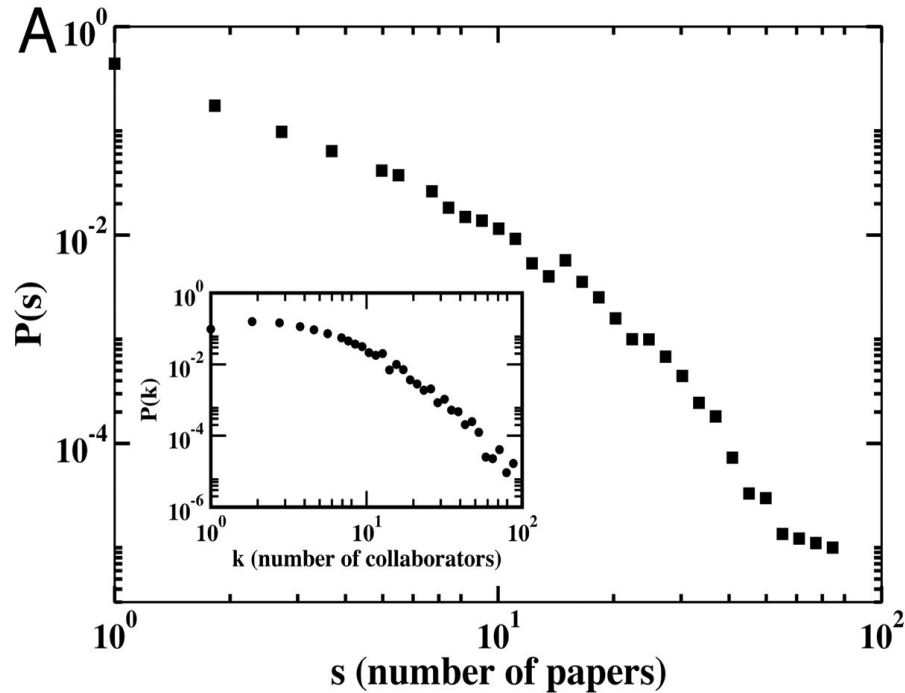
$$s_A = 1 + 2 + 3 = 6$$



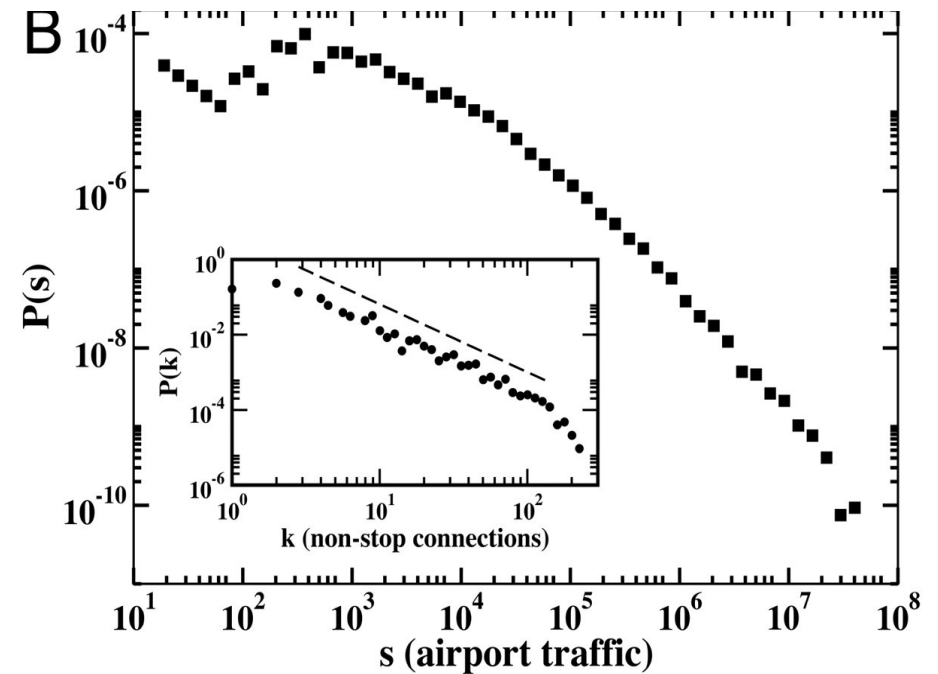
Strength distribution

Similarly to the degree distribution, strength distribution is **heavy-tailed**.

Scientific collaboration:



Airtraffic:



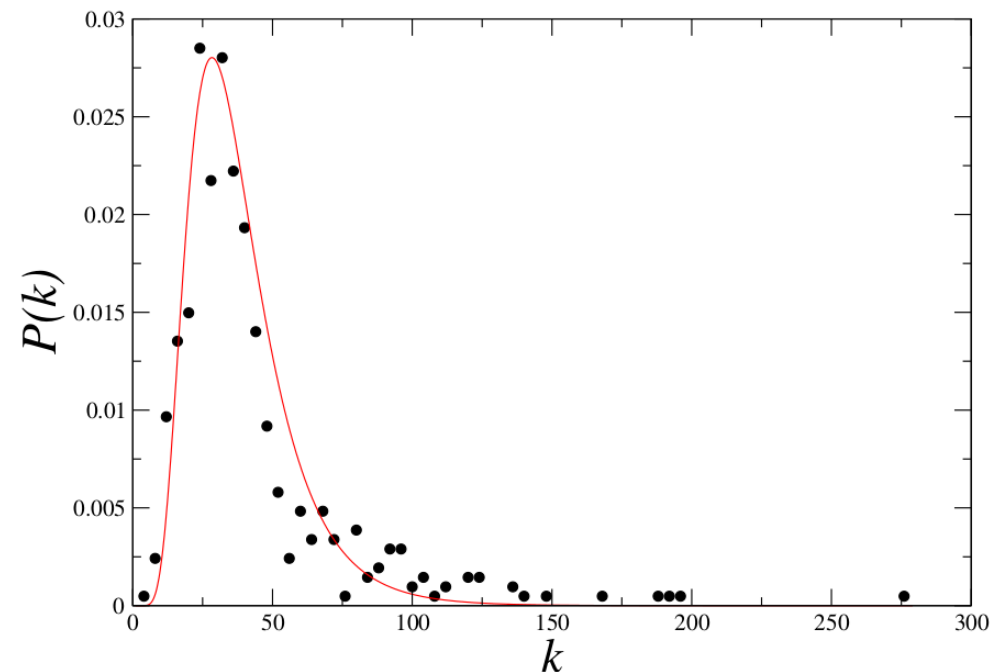
A. Barrat et al. PNAS 2004;101:3747-3752

Strength distribution strength

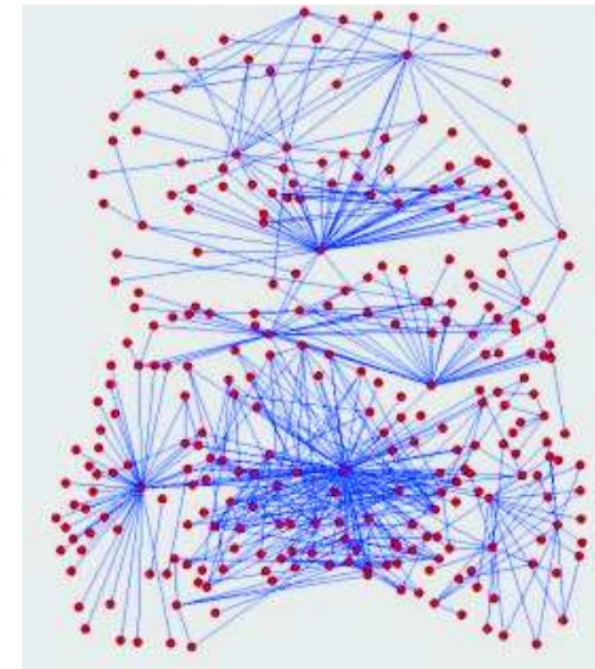
Inter-urban network:

- 375 municipalities in the Italian region of Sardinia
- Link weight: number of commuters

Degree distribution



Light(ish)-tailed
(Log-normal fit shown, tail exponent >3)



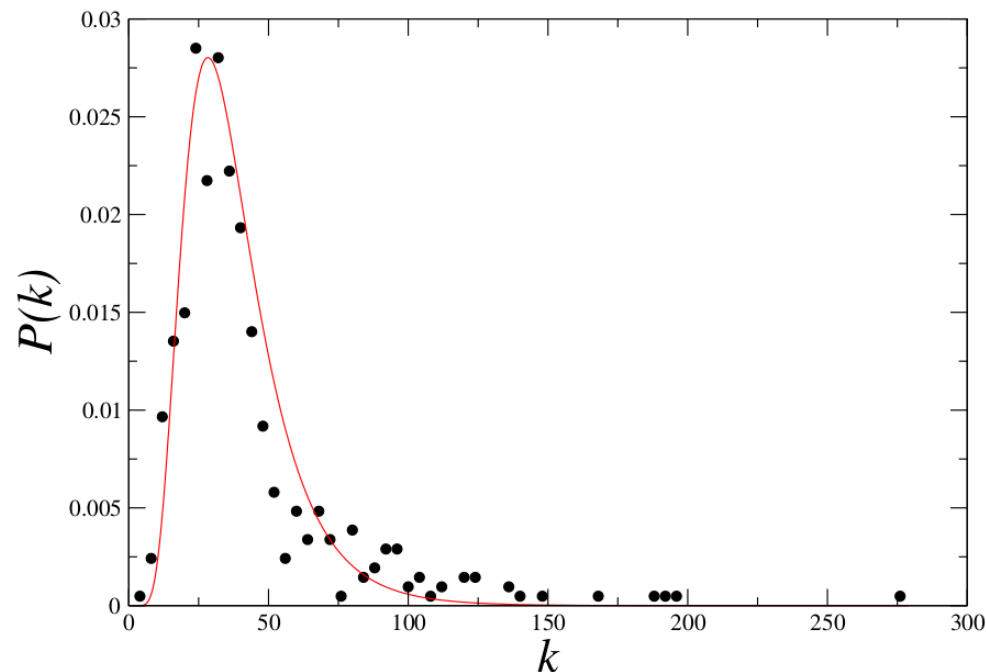
Links shown: >50
commuters

Strength distribution strength

Inter-urban network:

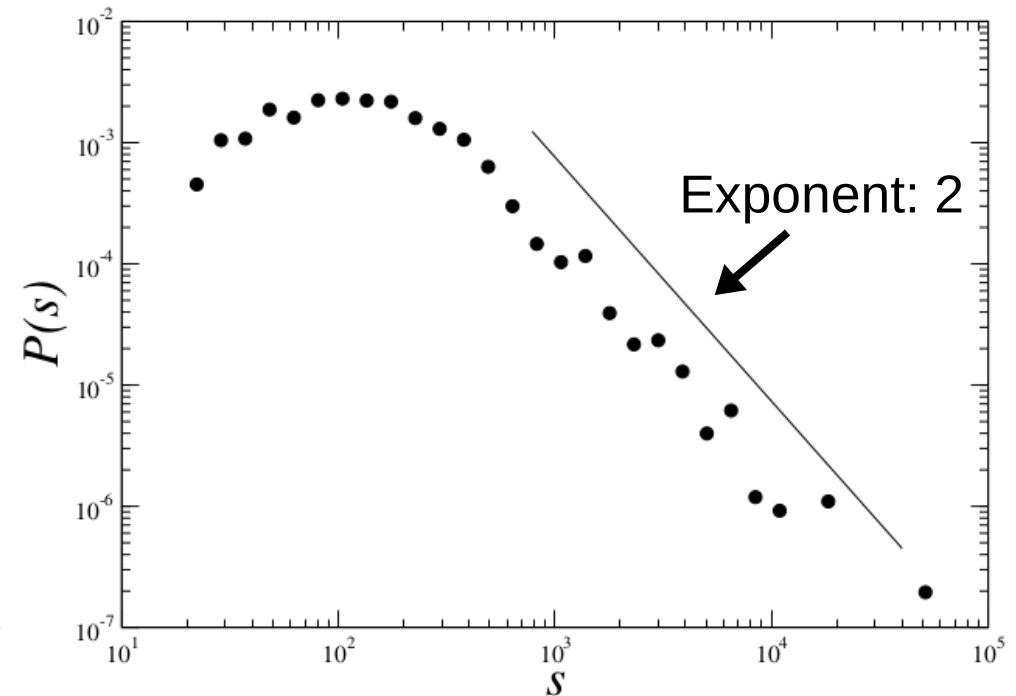
- 375 municipalities in the Italian region of Sardinia
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Degree distribution



Light(ish)-tailed
(Log-normal fit shown, tail exponent > 3)

Strength distribution



Heavy-tailed

Strength of weak ties

Are links the large weights the important ones?

- Yes, e.g. epidemic between cities spreads faster if there is lot of traffic.
- Not always: **Strength of weak ties**

Strength of weak ties

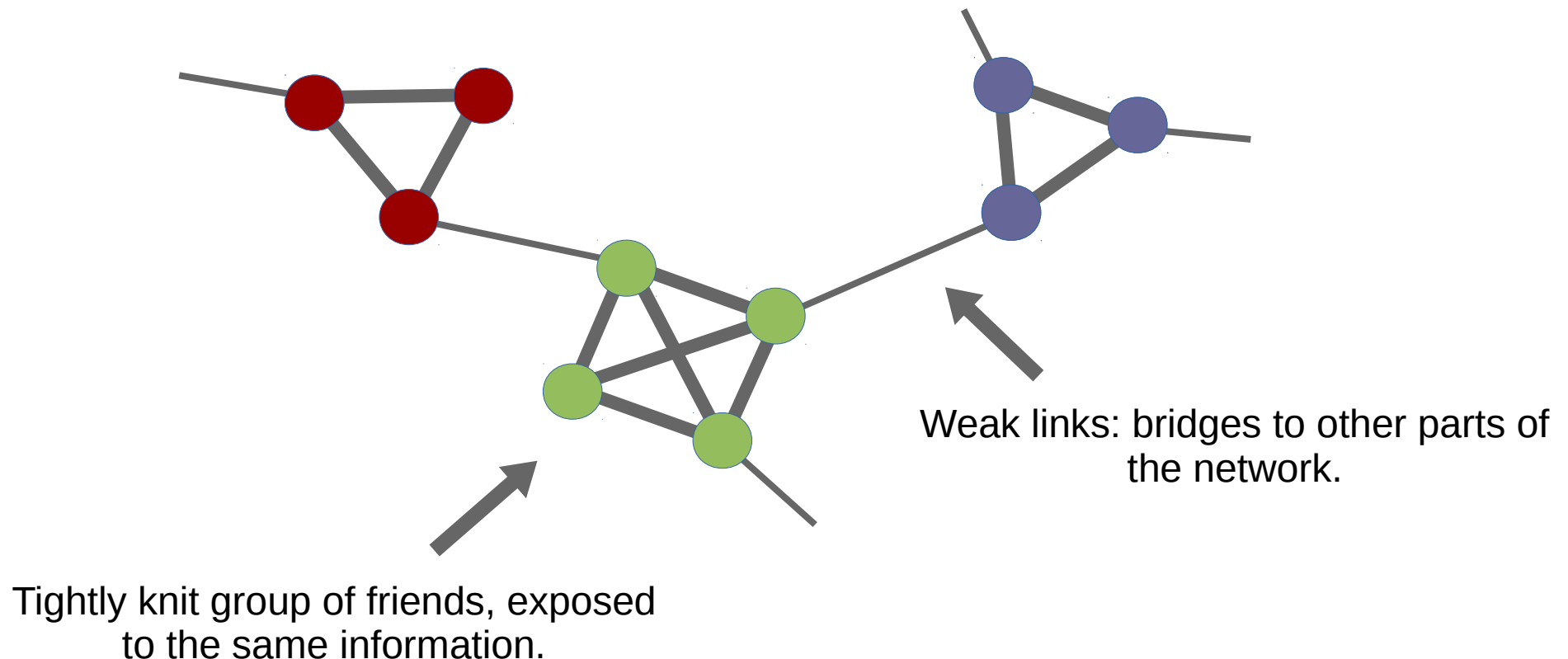
Granovetter, 1969:

- Wanted to show: people find jobs through social networks
- Surveyed ~100 people, asked if they found a job through a friend
- Common answer: not friend, but acquaintance

Strength of weak ties

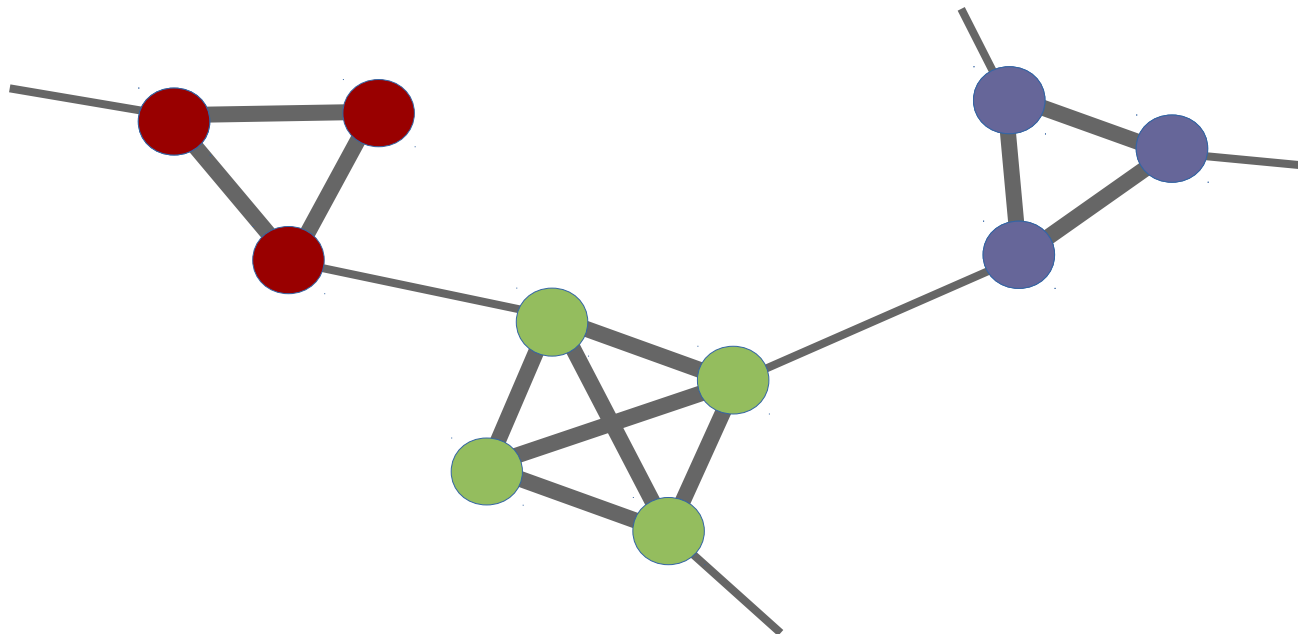
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Validating Strength of weak ties

- Granovetter has no access to entire social network
- Now: phone call network (Onella et al., 2007)
- Hypothesis:
 - 1) Strong links inside communities – weak ones outside.
 - 2) Weak links are bridges that hold the network together.

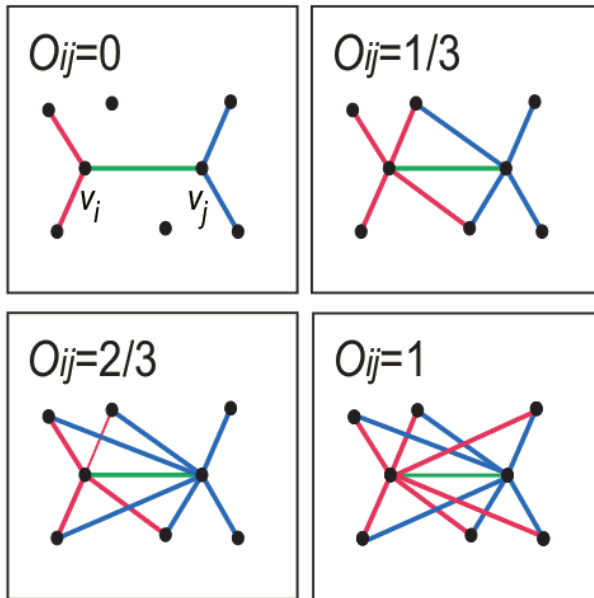


1) Strong links inside communities

- Topological overlap:

$$O_{ij} = \frac{c_{ij}}{k_i - 1 + k_j - 1 + c_{ij}}$$

#common neighbors

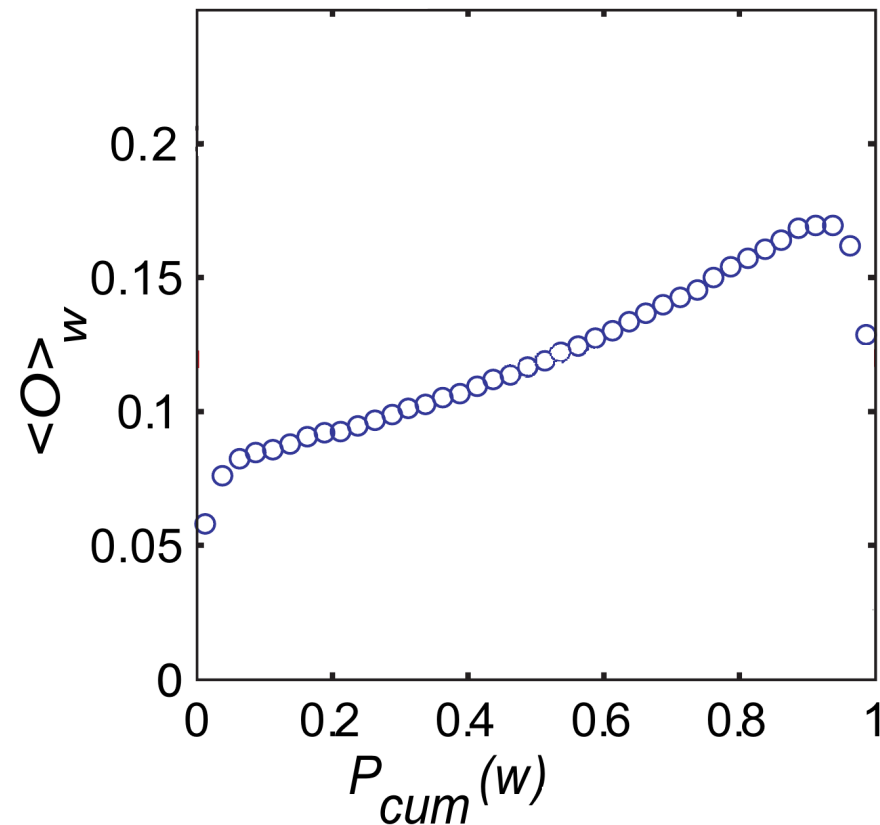
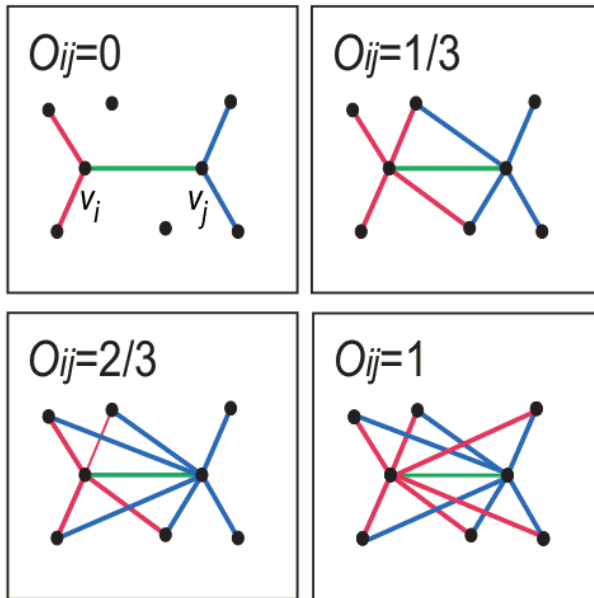


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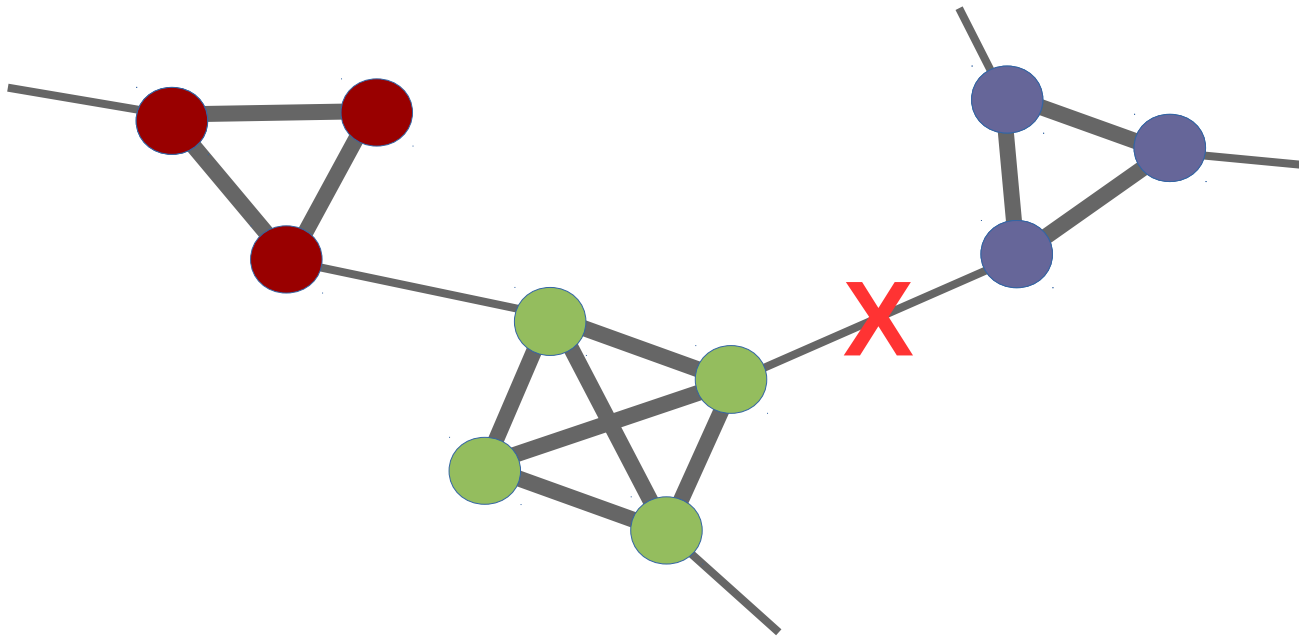
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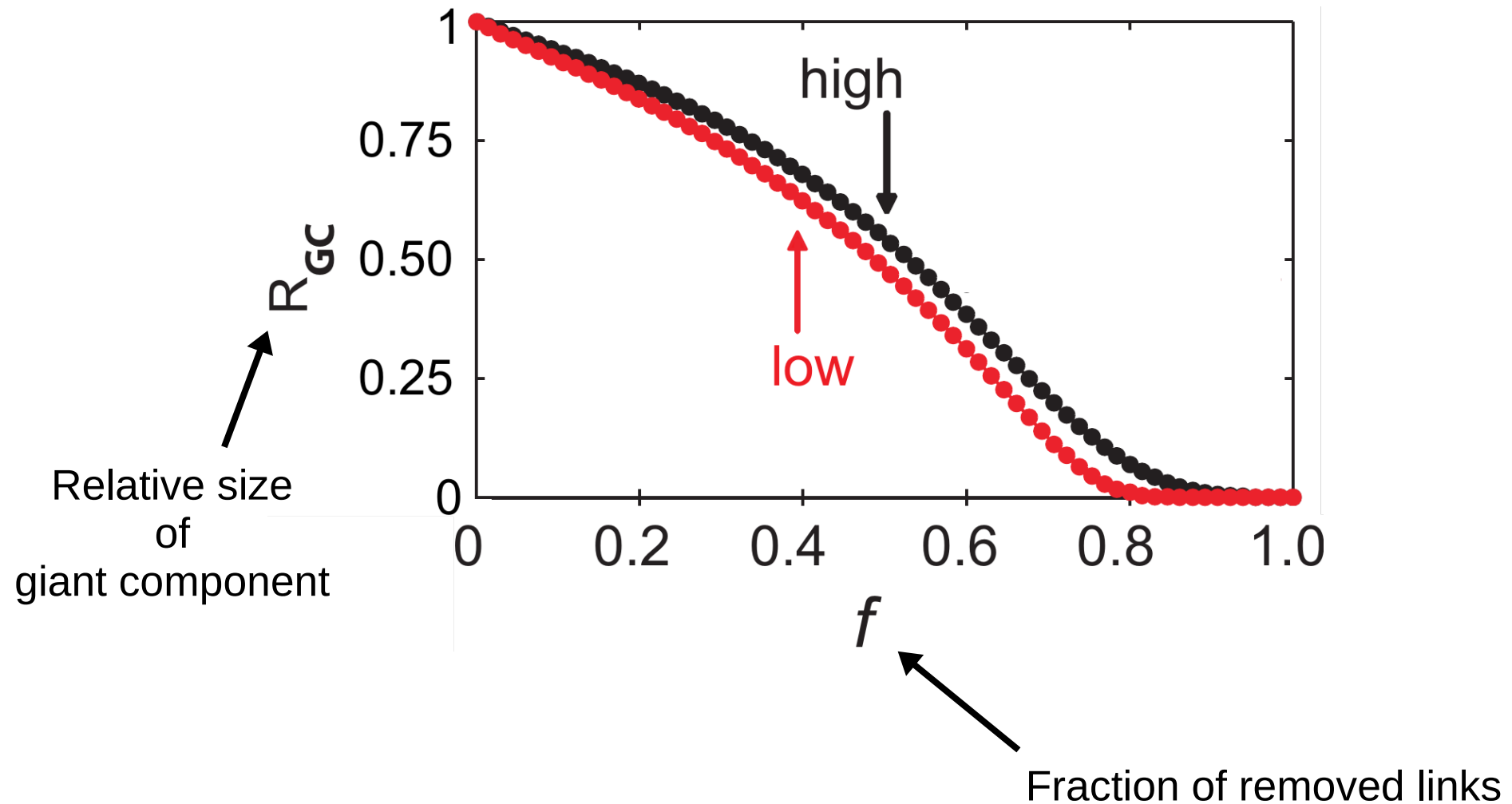
2) Weak links are bridges

- Removing bridges breaks the network in parts.



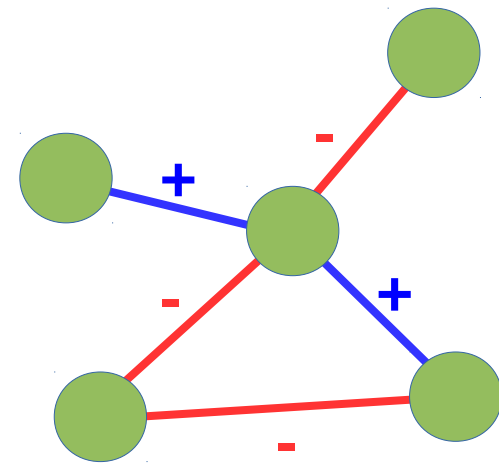
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Signed Networks

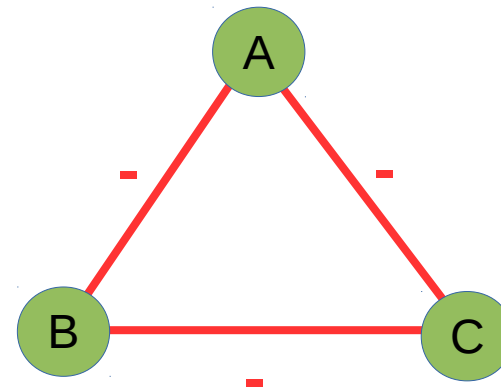
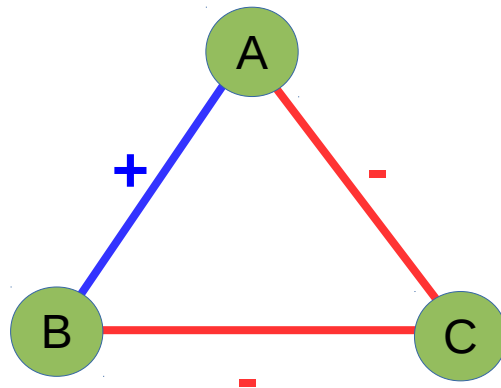
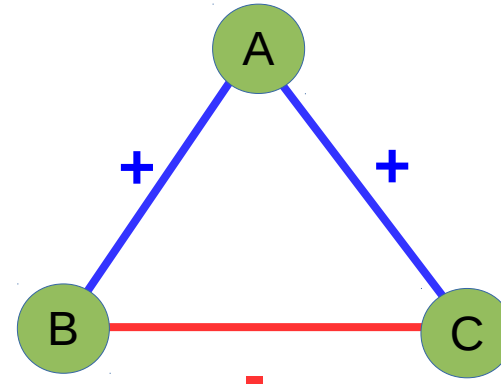
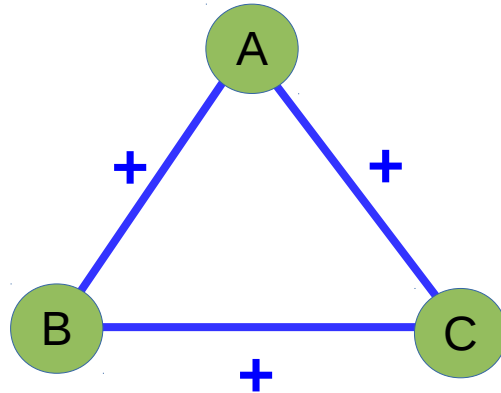
- Links represent positive or negative relations
- Examples:
 - Social network: friendship – hostility
 - International relations: alliance – conflict
 - Cellular biology: activation – inhibition
 - Ecology: mutualism – competition
- Today: social networks



Signed network

Social Balance Theory

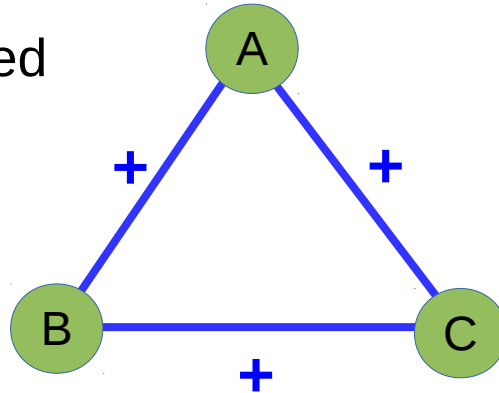
- Heider (1940s): triads



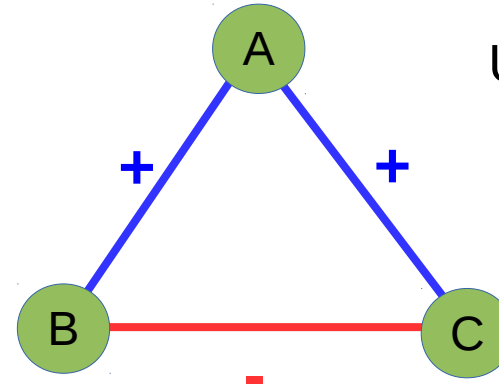
Social Balance Theory

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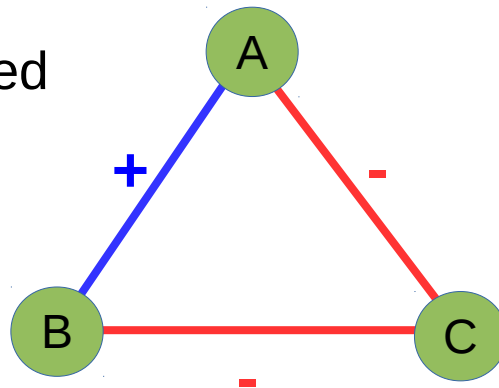
Balanced



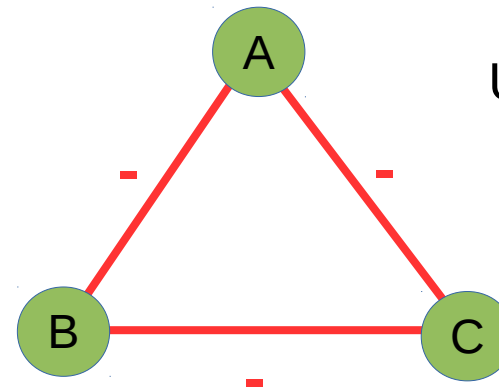
Unbalanced



Balanced

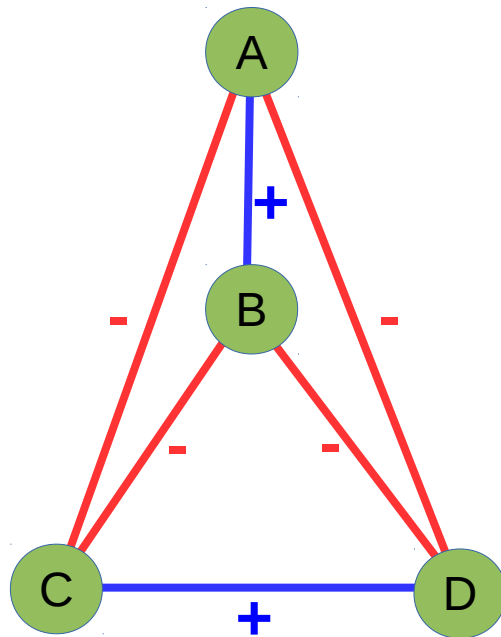


Unbalanced

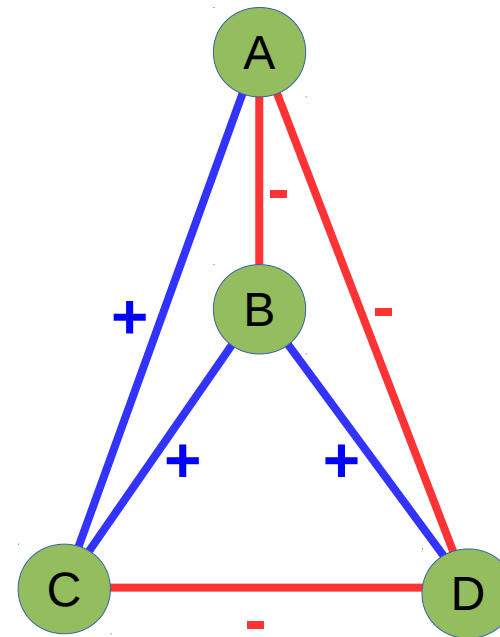


Social Balance Theory: $N > 3$

- Cliques (= all-to-all networks)
- Structural balance: all triangles balanced



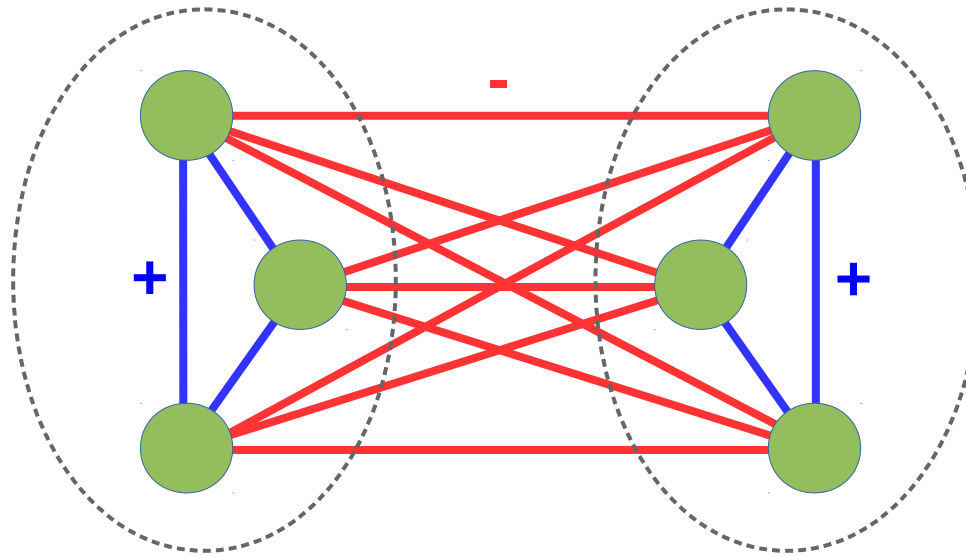
Balanced



Unbalanced

Social Balance Theory: $N > 3$

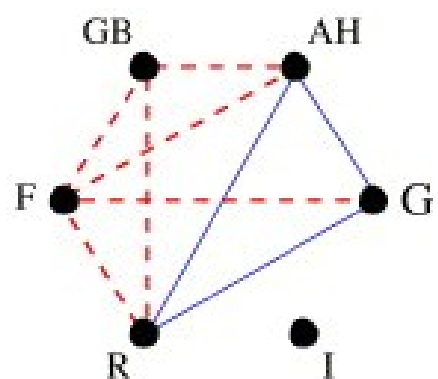
- Structural balance theorem (Harary, 1953):
If complete graph is balanced $\rightarrow$
 - All nodes friends; or
 - Two groups, inside groups all nodes friends, between groups all enemies



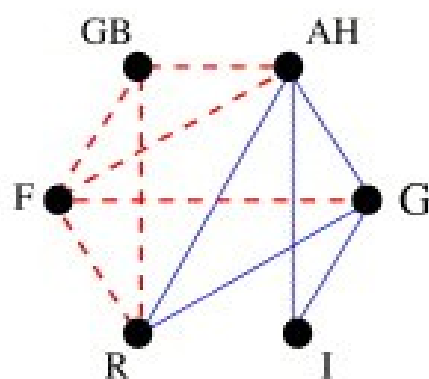
- Local rule $\rightarrow$ macroscopic behavior
- Proof

Complete networks application

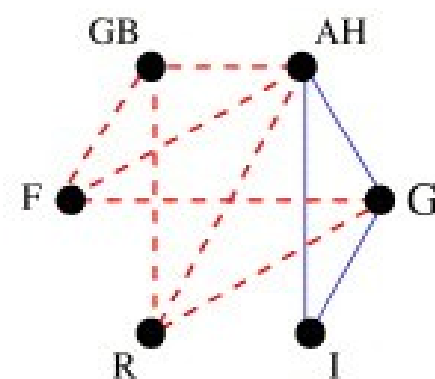
- All-to-all networks relevant to small groups
- Alliances leading up to WWI



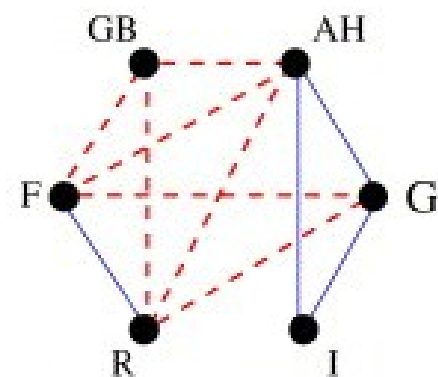
3 Emperor's league 1872-81



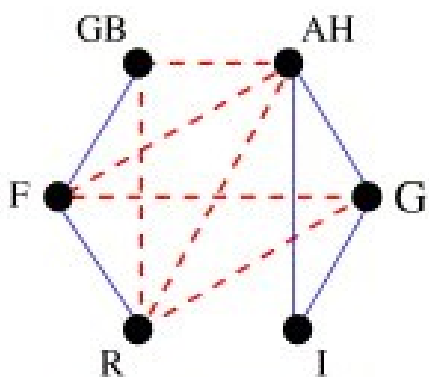
Triple Alliance 1882



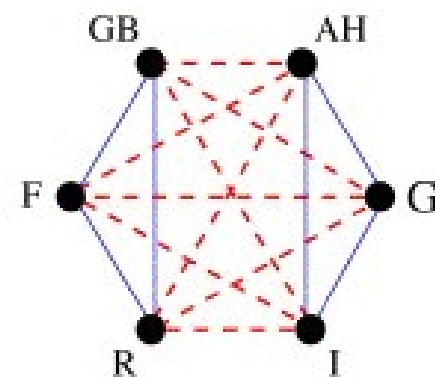
German-Russian Lapse 1890



French-Russian Alliance 1891-94



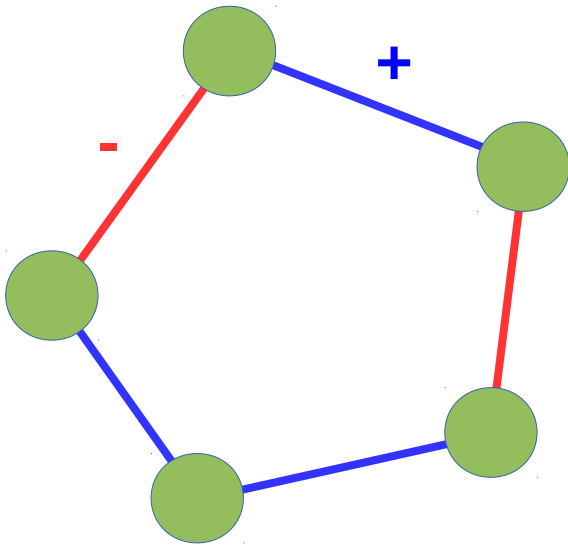
Entente Cordiale 1904



British-Russian Alliance 1907

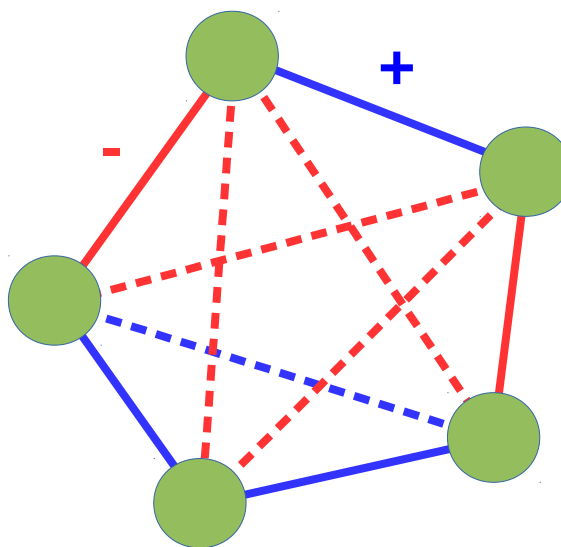
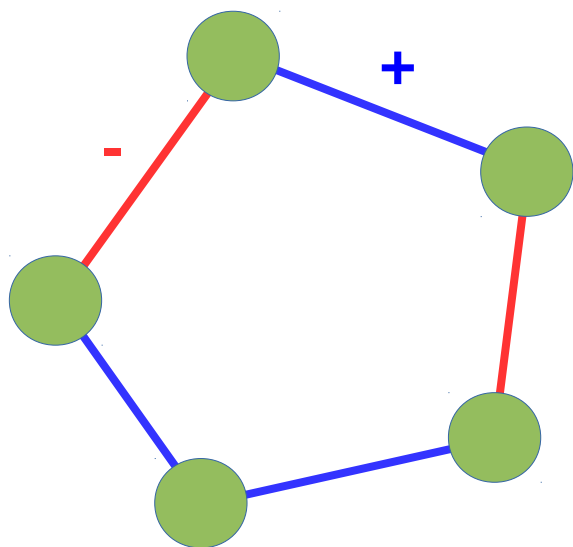
Sparse networks

- Not everyone has an opinion about everyone



Sparse networks

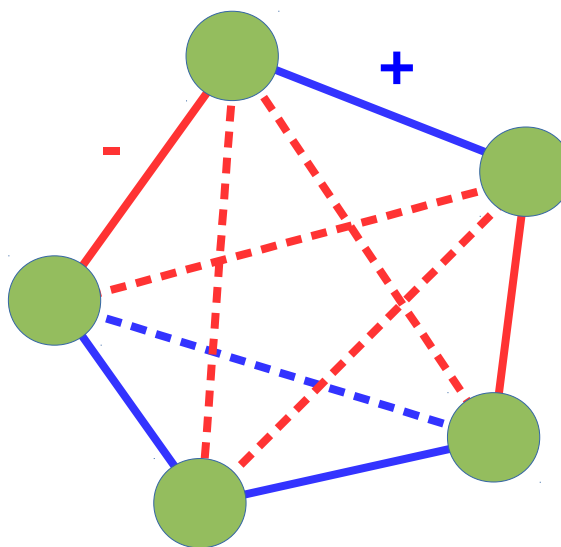
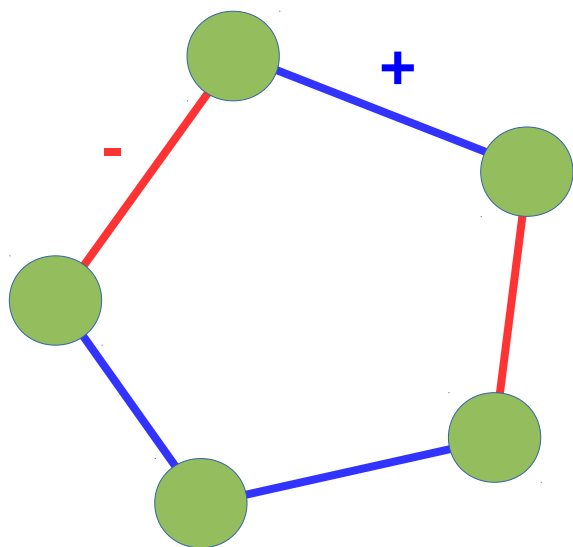
- Not everyone has an opinion about everyone
- Two ways to generalize the definition:
 1. Locally: Can we fill out the missing links to create a balanced complete network?



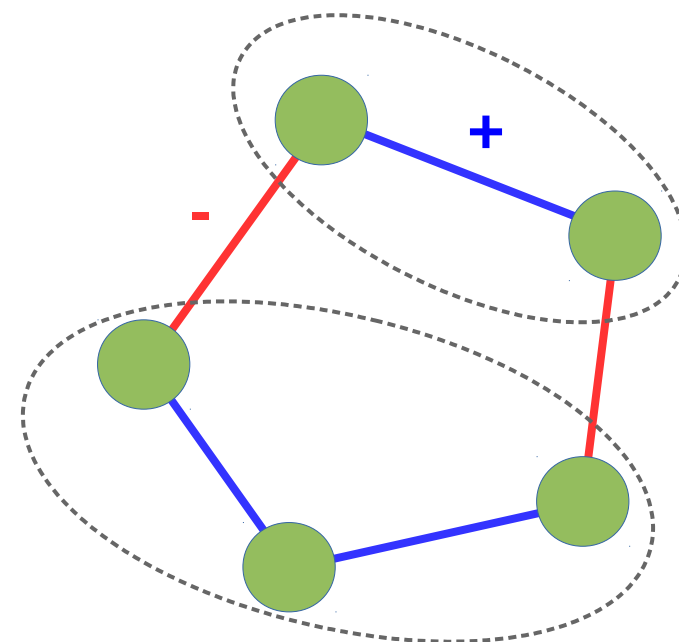
Def 1: Balanced

Sparse networks

- Not everyone has an opinion about everyone
- Two ways to generalize the definition:
 1. Locally: Can we fill out the missing links to create a balanced complete network?
 2. Globally: Can we divide nodes into to groups such that inside groups only + links, between groups only - links?



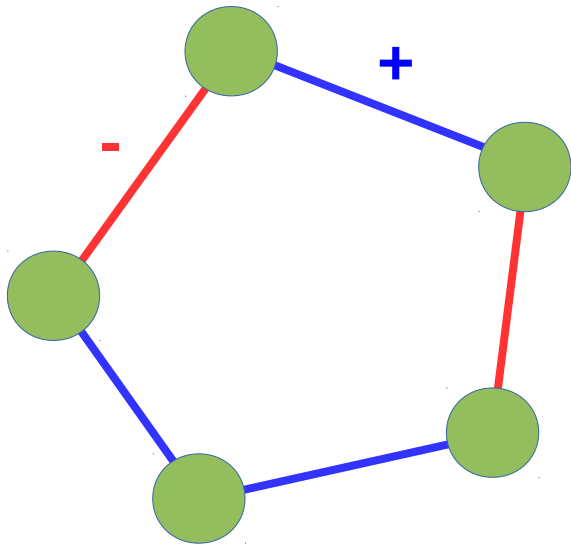
Def 1: Balanced



Def 2: Balanced

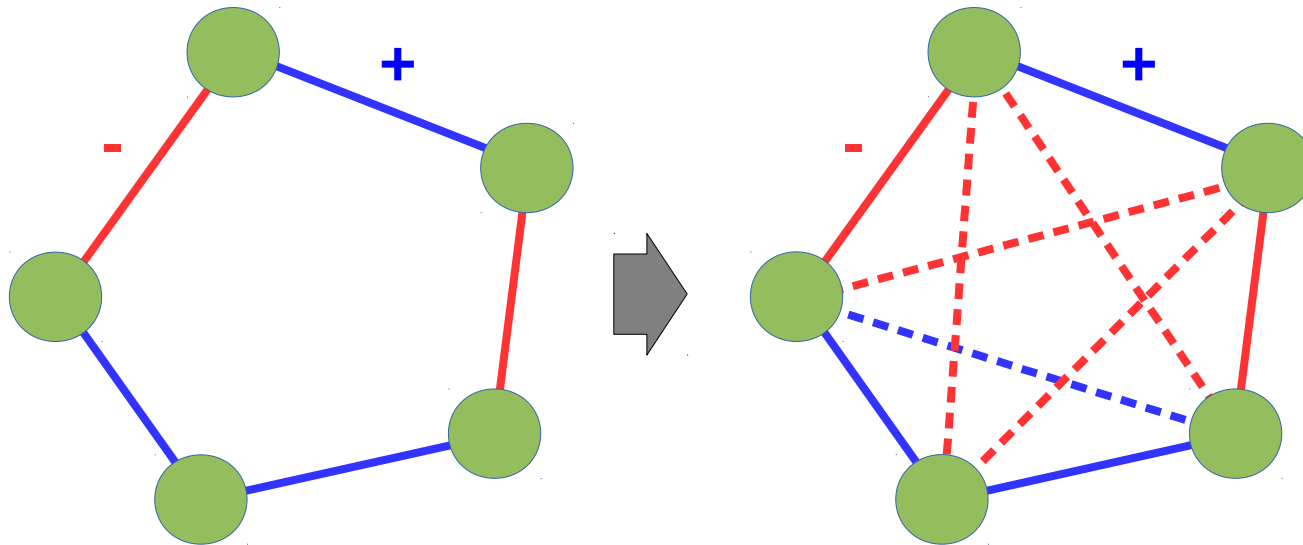
Two definitions equivalent

- 1. $\rightarrow$ 2.



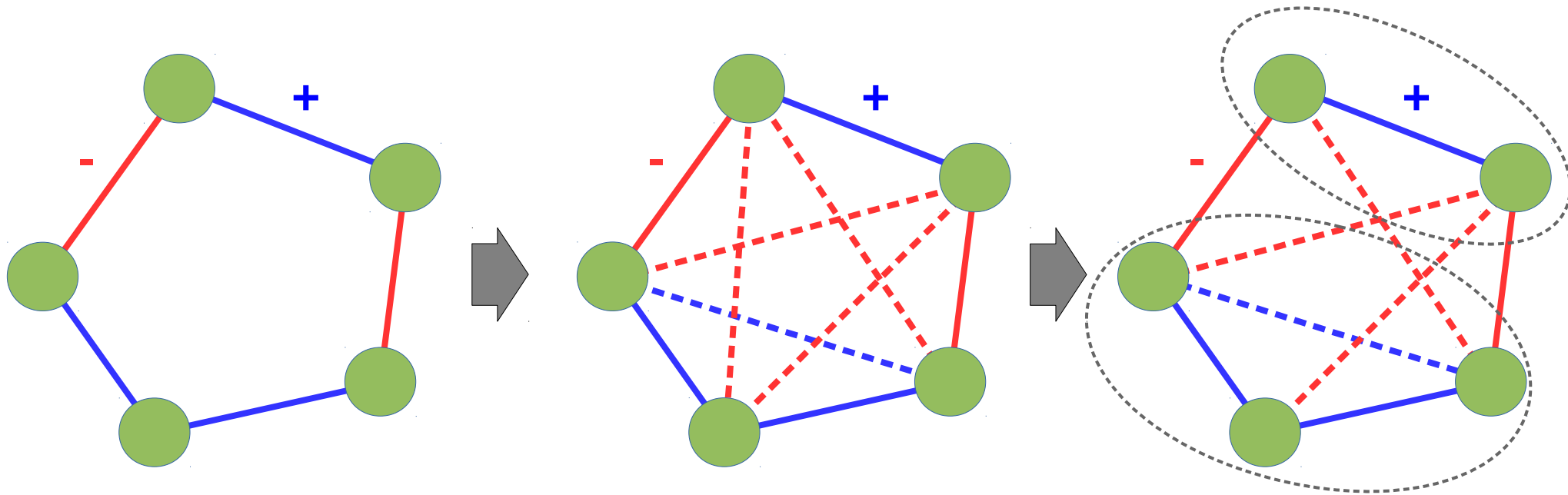
Two definitions equivalent

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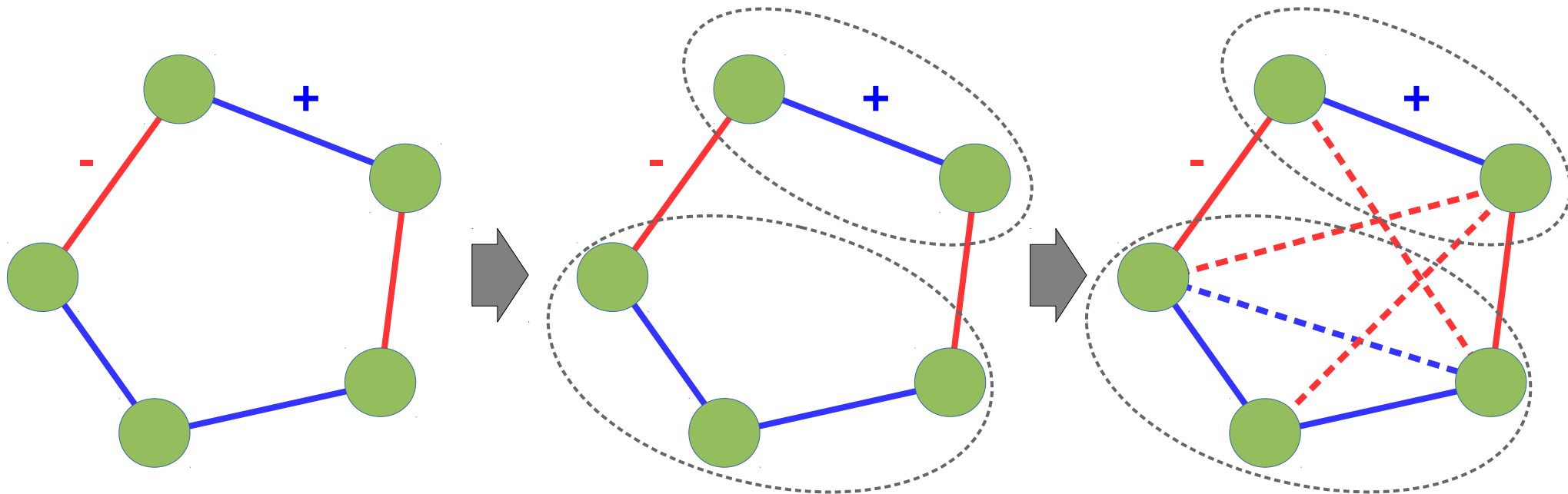
Two definitions equivalent

- 1. $\rightarrow$ 2.
 - We fill out missing links: now balanced complete network
 - According to Structural balance theorem (Harary, 1953): we can group nodes into two opposing factions



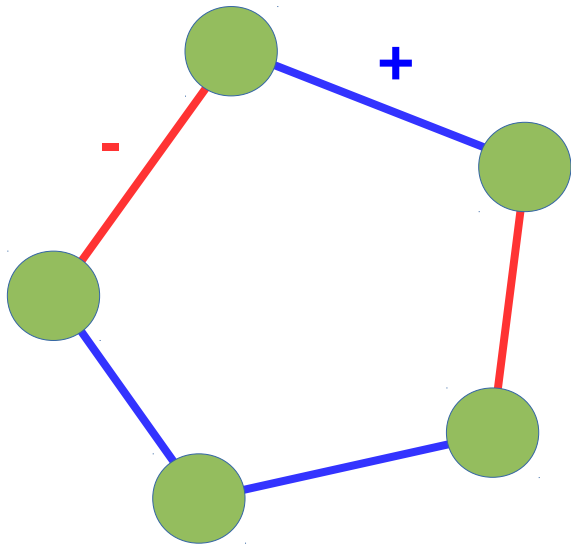
Two definitions equivalent

- 1. $\rightarrow$ 2.
- 2. $\rightarrow$ 1.
 - There exist a proper grouping of nodes
 - Rules to add edges: Inside group $\rightarrow +$; between groups $\rightarrow -$



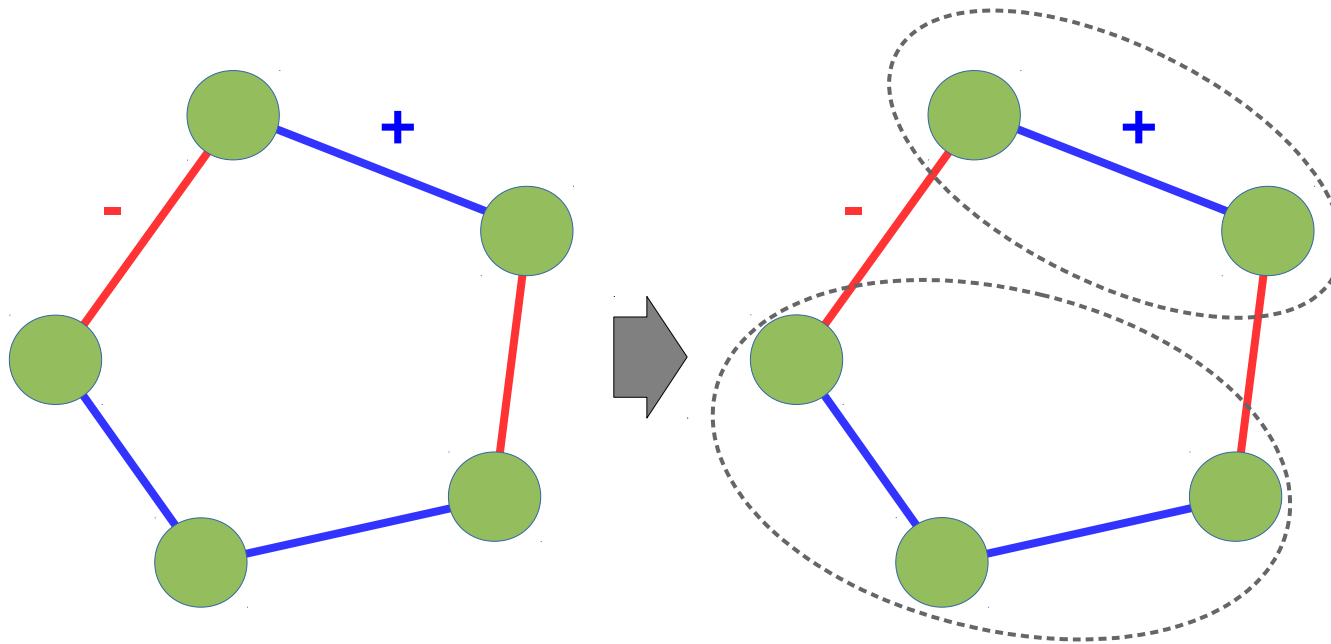
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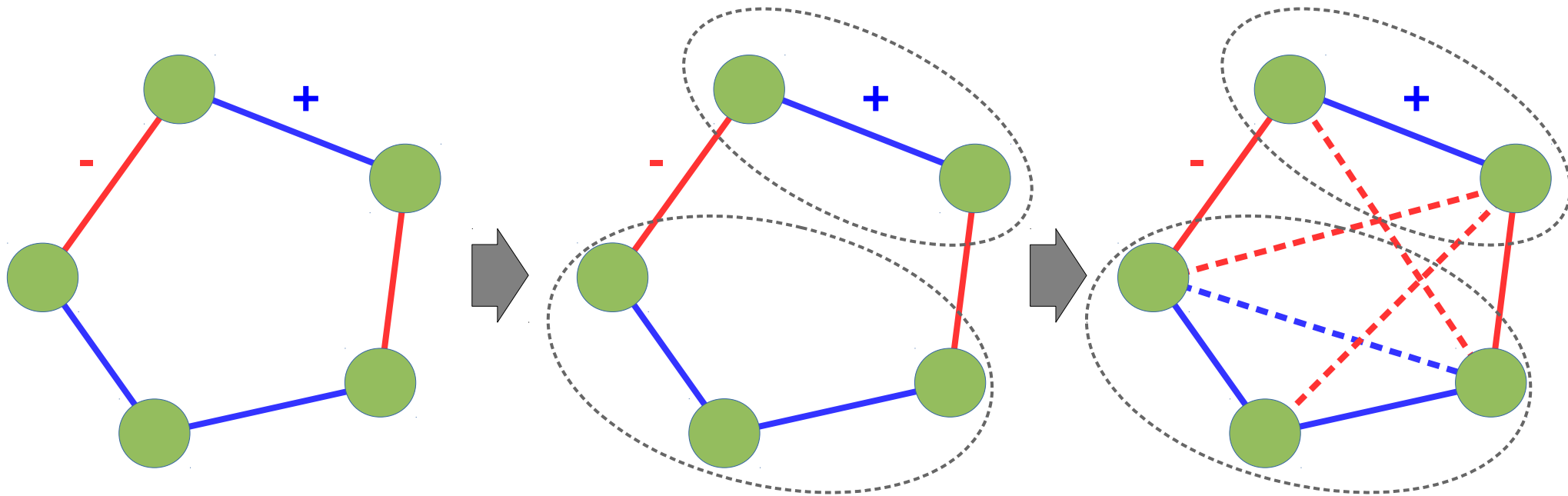
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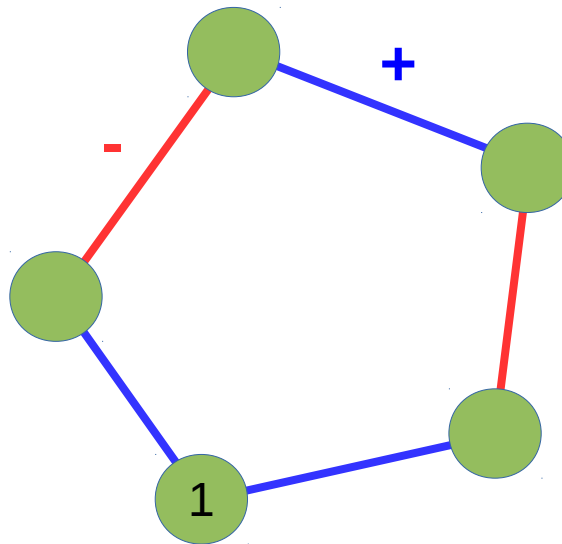
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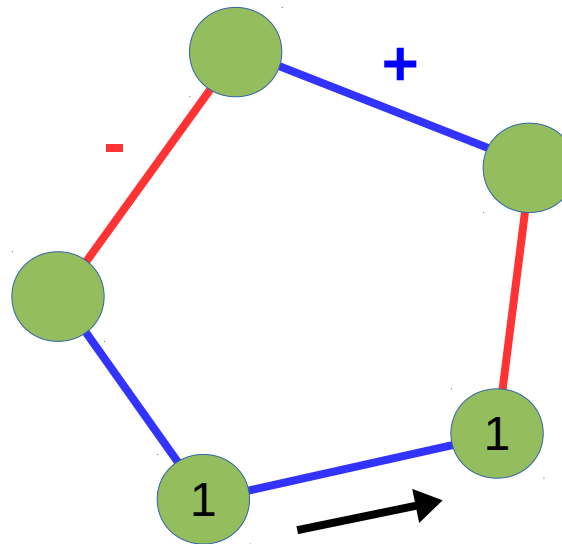
Characterizing balanced networks

- How to determine if a network is balanced?
- Balanced = All cycles contain even number of negative links
 - There exist a proper grouping of nodes
 - Rules to add edges: Inside group $\rightarrow +$; between groups $\rightarrow -$



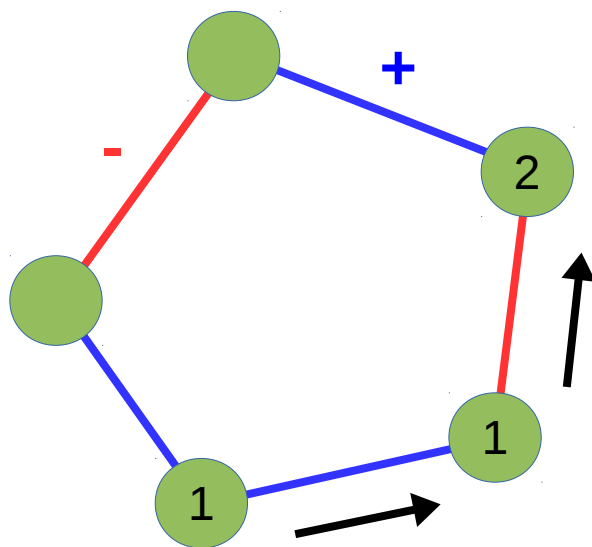
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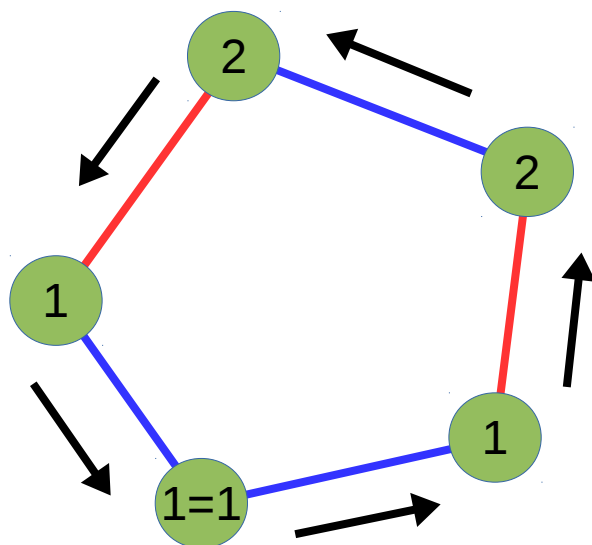
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Characterizing balanced networks

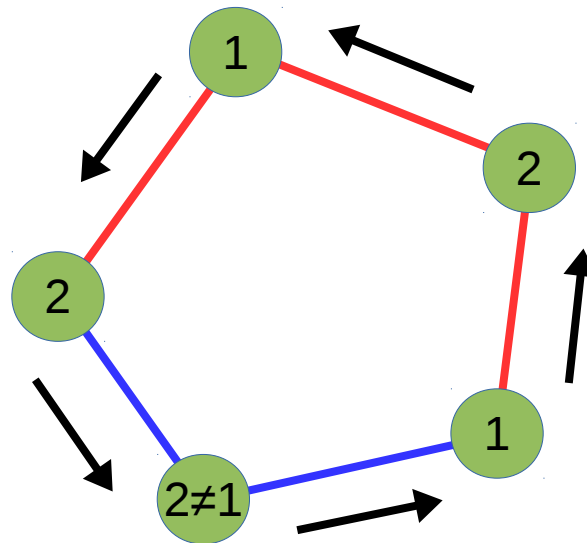
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Consistent labeling $\rightarrow$ Balanced

Characterizing balanced networks

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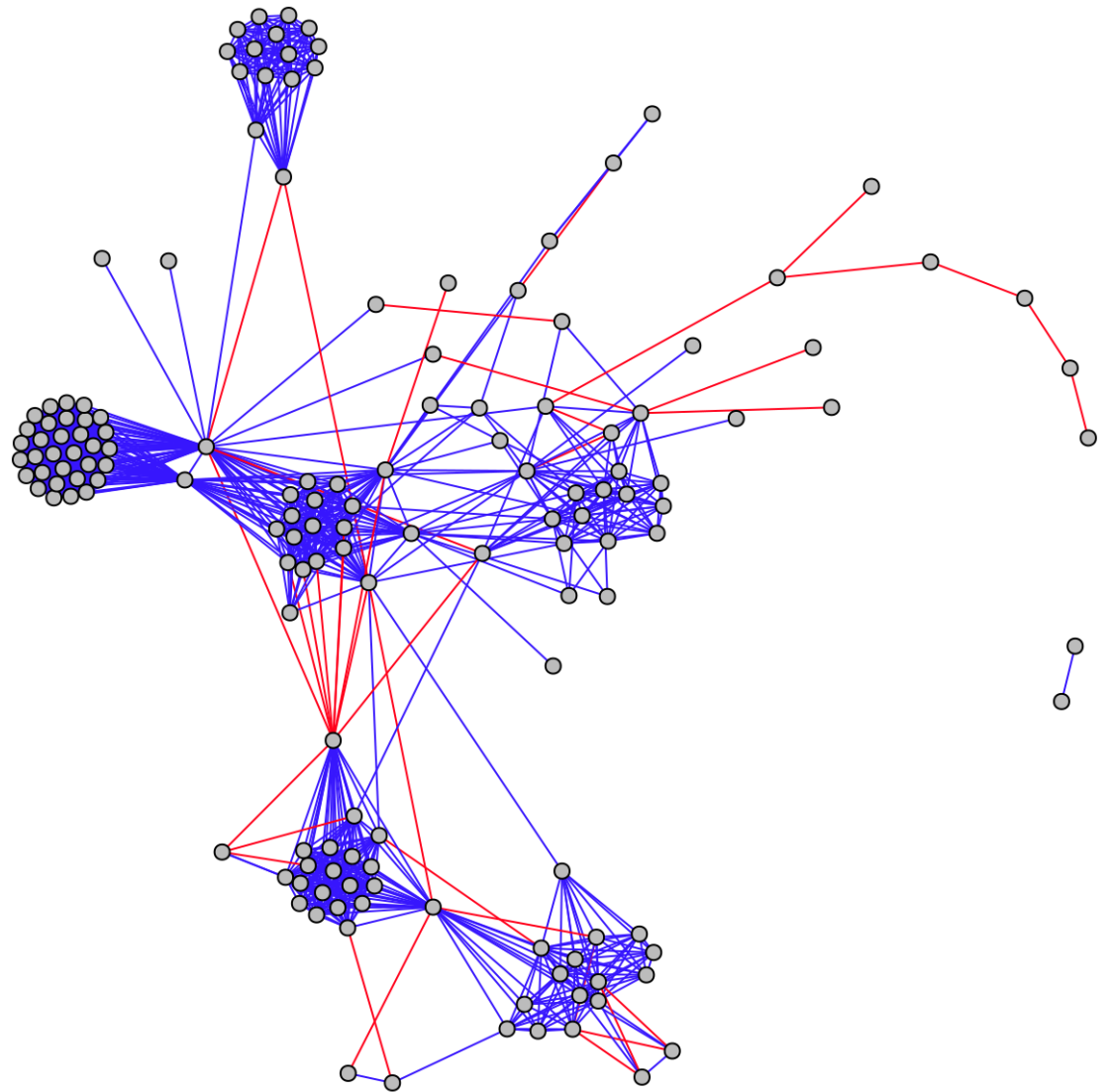


Contradiction $\rightarrow$ Unbalanced

Online tutorial

- International relations network from 2000-2015

- Triads: 9285
- Unbalanced: 42
- Example: USA-Iran-Pakistan



- Vincent Traag (<http://www.traag.net/>)
<https://github.com/vtraag/4TU-CSS/blob/master/presentations/traag/notebook/Network.ipynb>

Summary

- Previous classes: Links: binary relations
- Weighted networks
 - Links: positive weight, strength of interaction
 - Important: broad distribution, not typical value
 - Strength of weak ties
- Signed networks
 - Links: positive and negative relations
 - Social Balance Theory
 - SBT in networks
- Further reading:

Easley, David, and Jon Kleinberg.

Networks, crowds, and markets: Reasoning about a highly connected world.

Cambridge University Press, 2010.

Chapters: 3 and 5

