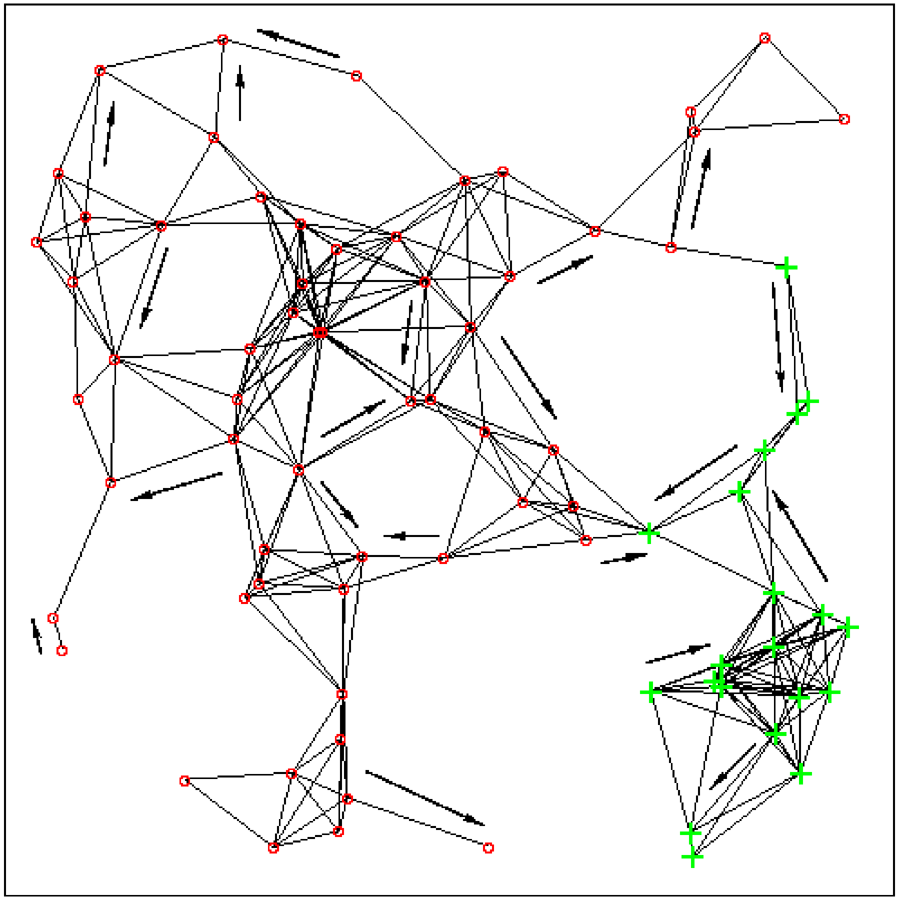
# MAE 298, Lecture 4

## April 11, 2006



# “Exploring network robustness”

# Recall: Preferential Attachment random graphs: The Barabási and Albert model

- A discrete time process.
- Start with single isolated node.
- At each time step, a new node arrives.
- This node makes $m$ connections to already existing nodes. (Why $m$ edges?)
- We are interested in the limit of large graph size.

# Probability

- Probability incoming node attaches to node j:

$$Pr(t+1 \rightarrow j) = d_j / \sum_j d_j.$$

- Probability incoming node attaches to any node of of degree $k$:

  (# nodes of degree k)/(# nodes) x (degree of that node)/(degree sum over all nodes) =

$$\boxed{\frac{kp_k}{\sum_k d_k} = \frac{kp_k}{2mn}}$$

# Network evolution
# Process on the degree sequence

- Note that $p_k$ will change in time!
  So we show denote this explicitly: $p_{k,t}$

- Also, when a node of degree $k$ gains an attachment, it becomes a node of degree $k+1$.

- When the new node arrives, it increases by one the number of nodes of degree $m$.

# Recursion for $p_m$

$$p_k = \frac{(k-1)(k-2)\cdots(m)}{(k+2)(k+1)\cdots(m+3)} \cdot p_m = \frac{m(m+1)(m+2)}{(k+2)(k+1)k} \cdot \frac{2}{(m+1)}$$

$$\boxed{p_k = \frac{2m(m+1)}{(k+2)(k+1)k}}$$

For $k \gg 1$

$$\boxed{p_k \sim k^{-3}}$$

# Simulating PA

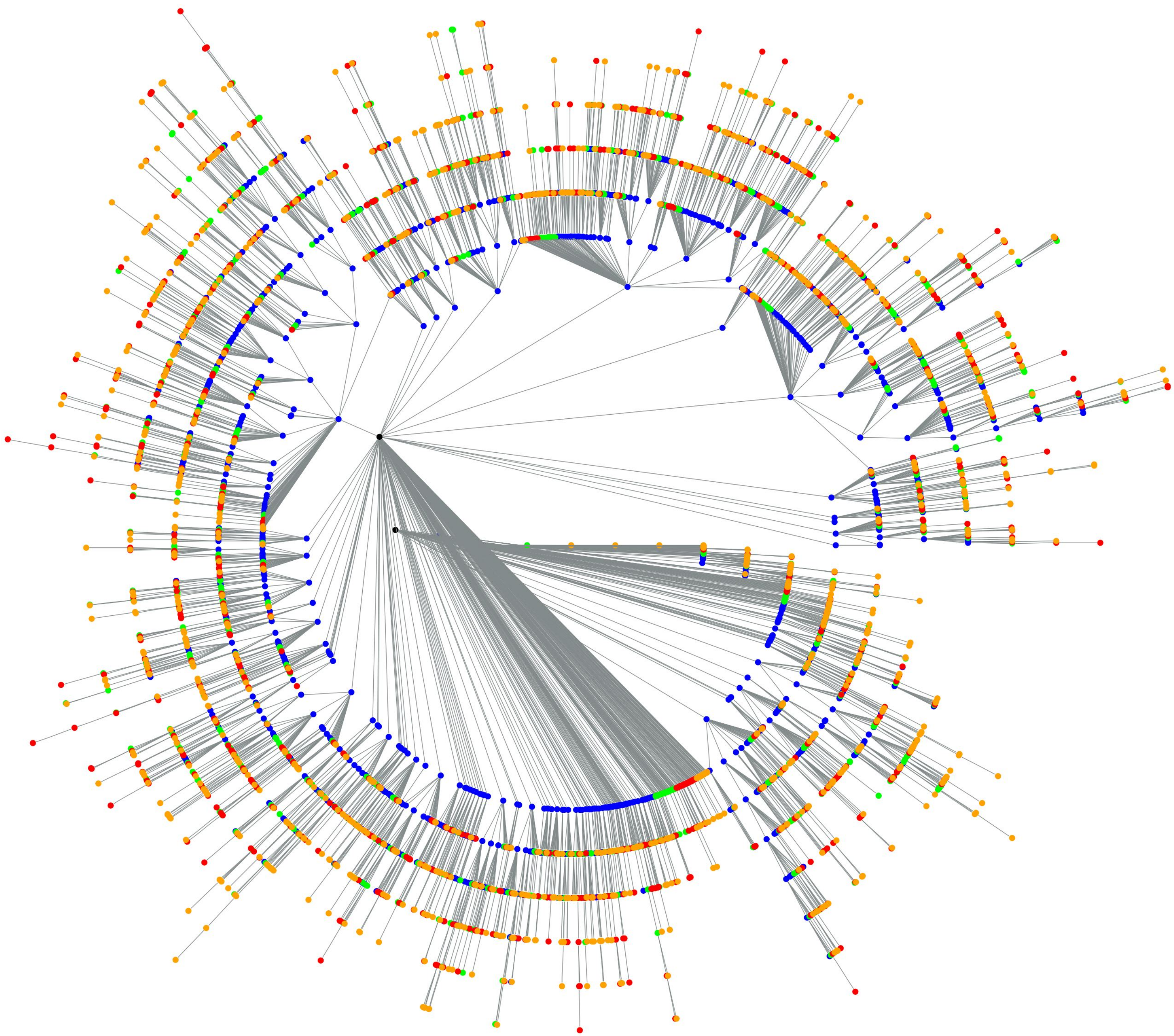
Basic code for simulating PA with $m = 1$ using R:

- runPA ← function(N=100)

```
{
    # outLink[i] is the parent of i
    outLink ← numeric(N)
    # numlinks[i] is number total-links (in and out) for node i
    numLinks ← numeric(N)+1

for(i in 2:N)
{
p ← sample(c(1:(i-1)),size=1,prob=numLinks[1:(i-1)])
outLink[i] ← p
numLinks[p] ← numLinks[p]+1
}
return(list(outLink, numLinks))
}
```

# Visualizing a PA graph ($m = 1$) at $n = 5000$



# Exploring PA networks

Demo:

- R
- Graphviz

We will use these as tools to explore Network robustness (defined here as maintaining connectivity despite node and edge deletion).

- Rest of lecture will be interactive demo....

# Robustness of power law random graphs to node and edge deletion

- Extremely resilient to random failure.
- Extremely sensitive to targeted attack.

The "achilles' heel" of the Internet

- "Scientists spot Achilles heel of the Internet", CNN, July 26, 2000.
- R. Albert, H. Jeong and A.-L. Barabási, "Error and attack tolerance of complex networks", *Nature* **406** 2000. (Correction: **409** 2001).