1. A **function** from A to B is an assignment of exactly one element of B to each element of A.
   Notation: $f : A \to B$, $f(a) = b$
   A is called the **domain** of F and B is the **codomain**.
   The set $\{f(a) \mid a \in A\}$ is called the **range** of f.

2. $f : A \to B$ is **one-to-one** or **injective** if $f(a) = f(b)$ then $a = b$.
   (If not injective it is *many-to-one.*)
   Examples[1]:

   - Is the function $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$ one-to-one?
   - Is the function $f : \{\text{US residents}\} \to \mathbb{Z}$, $f(x) = SSN$ one-to-one?

3. $f : A \to B$ is **onto**, or **surjective**, if for any $b \in B$, there is an $a \in A$ with $f(a) = b$.
   Examples:

   - Is the function $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x^2$ onto? (Is every $\mathbb{Z}$ the square of a number?)
   - Is the function $f(x) = x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$ onto?

4. f is **one-to-one correspondence**, or a **bijection**, if it is both one-to-one and onto.
   Examples:

   - Is $f(x) = x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$ bijective?
   - The identity function $\ell_A : A \to A$, $\ell_A(x) = x$, is a bijection?

5. If $f : A \to B$ is a bijection, then the function $f^{-1} : B \to A$ defined by $f^{-1}(b) = a$ if $f(a) = b$ is called the **inverse function** or f.
   Examples:

   - Let $f(x) = x + 1$, is f invertible? If yes, what is f^{-1}?
   - Let $f(x) = x^2$, is f invertible? If yes, what is f^{-1}?

[1]The notation $\mathbb{Z}$ for the set of integers comes from the German word "Zahlen"

6. Let f_1 and f_2 be two functions from A to $\Re$, then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to $\Re$ defined by

$$\begin{aligned}(f_1 + f_2)(x) &= f_1(x) + f_2(x),\\ (f_1 f_2)(x) &= f_1(x) f_2(x).\end{aligned}$$

Example: Let $f_1(x) = x^2$ and $f_2(x) = x - x^2$, then

$$\begin{aligned}(f_1 + f_2)(x) &= \\ (f_1 f_2)(x) &= \end{aligned}$$

7. If $g : A \to B$ and $f : B \to C$, then define $h : A \to C$, $h(a) = f(g(a))$. h is called the **composite function** of f and g, and written as $f \circ g$.
Examples:

- Let $f(x) = 2x + 3$ and $g(x) = 3x + 2$,
$(f \circ g)(x) = f(g(x)) = 2(3x + 2) + 3 = 6x + 7$
$(g \circ f)(x) = g(f(x)) = 3(2x + 3) + 2 = 6x + 11$
- Since $f^{-1}(b) = a$ when $f(a) = b$,
$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$.

8. Two frequently used functions in computer science (translating real values to binary):
$f : \Re \to Z$. Let $x \in \Re$,

- floor function: $\lfloor x \rfloor$ = the largest integer n such that $n \leq x$.
- ceiling function: $\lceil x \rceil$ = the smallest integer n such that $n \geq x$.

Example: How many 8-bit packets can be transmitted in a second if the rate is 80 bits/sec? How many 8-bit packets can be transmitted in a second if the rate is 60 bits/sec?