COMPLEX NETWORKS

A winning strategy

Introducing connections between two distinct networks can tip the balance of power — at times enhancing the weaker system. The properties of the nodes that are linked together often determine which network claims the competitive advantage.

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Cooperation and competition drive and shape the dynamical evolution of systems spanning ecology, business, human and animal societies, and genetic selection. The interplay between these mechanisms can give rise to conflict[1], discontinuous percolation transitions[2] and even enable new technologies for wireless communications[3]. Problems devised in game theory, such as the 'prisoner's dilemma', provide a framework to assess the best strategy available to a self-interested decision-maker when interacting (by cooperating or competing) with another individual or entity.

Taking a step out in scale, we see that these players are only two individuals in a larger network of interactions between many players, and that network may have intermediate levels of structure such as clusters and modules (Fig. 1a). We might thus envisage that an individual's position in a network can profoundly impact the outcome of its interactions. In reality, we see that no individual network lives in isolation[4] and, instead, whole networks may choose to cooperate or compete with other networks. Writing in *Nature Physics*, Jacobo Aguirre and colleagues have made an important advance in shaping this broader perspective by using measures of network structure to analyse how one network can maximize the cumulative benefit it gains when adding connective edges to other networks[5].

The structure of a network is encoded through an adjacency matrix indicating how nodes are connected to other nodes by edges, which can either be weighted or unweighted. The properties of the eigenvalues and eigenvectors of the adjacency matrix — referred to as the spectral properties — directly relate to many important network functions, such as whether a particular network structure supports the decentralized onset of synchronization between network elements[6]. Likewise, we can use the notion of 'eigenvector centrality' to rank the importance of individual nodes in a network, similar to the celebrated PageRank algorithm for ranking pages on the World Wide Web[7] — in which the more central nodes have higher rank.

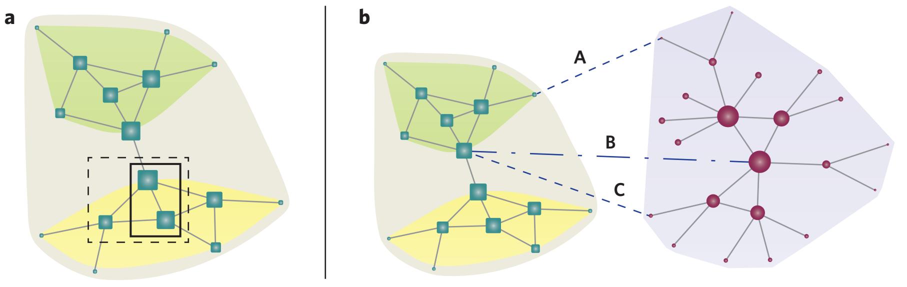

Figure 1 | Connecting two networks. **a**, An individual network may comprise multiple levels of scale, from an individual edge (solid box), to clusters of three nodes (dashed box), to modular structure (highlighted in yellow and green). Here, the size of the node is proportional to its eigenvector centrality value. **b**, The network in **a** may connect to a second network by different classes of edges. Edge A connects two low-ranked nodes, edge B, two high-ranked nodes, and edge C links a high-ranked node with a low-ranked node. If edge C is chosen, the right-hand network loses its initial dominance, even though the individual node connected in that network gains significantly in importance.

Aguirre *et al.* considered the cumulative sum of eigenvector centrality over all nodes in a network as a measure of the centrality of the network, and calculated how network centrality depends on the way in which edges are added to other networks[5]. They restricted the study to the regime in which edges connecting nodes across two distinct networks are much weaker than edges internal to the networks, allowing them to build a quantitative approach based on perturbation theory.

There are different strategies for how edges can be added between two initially distinct networks (see Fig. 1b). A high-ranked node in the first network may be connected with a high-ranked node in the second network. An alternative method might involve connecting a high-ranked node with a low-ranked one, or even linking two low-ranked nodes. Aguirre *et al.* showed that each distinct connection strategy carries with it different advantages[5]. They found that connecting the low-ranked nodes typically enhanced the centrality (or importance) of the network that was initially 'stronger'. By contrast, linking high-ranked nodes generally had the effect of increasing the centrality of the 'weaker' network. But network size and the density of edges also factored into the equation, with the larger, denser network usually claiming the competitive advantage (or higher centrality).

In general, the centrality of a network was shown to be extremely sensitive to exactly which edges were added to a second network. Even if only one connecting edge was added between two networks, depending on the connection strategy used, the resulting variation in network centrality could span orders of magnitude.

A network may not necessarily be able to achieve a higher centrality than another network, but by choosing connective edges wisely, it can attain the maximum centrality accessible and hence maximize the benefit gained through interconnectivity. In some cases, higher centrality is not always desirable. For instance, a more central autonomous system in the World Wide Web is likely to have to transmit more data, which can cause congestion. Similarly, a more central network is more susceptible to contagion from other networks.

Even in cases where centrality is beneficial, there are trade-offs that manifest

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at different scales. Although a network may gain some competitive advantage by connecting to a second network, individual nodes within that network may actually lose importance due to the new connections (see for instance Fig. 3 in ref. 5). At the scale of networks, Aguirre *et al.* considered the impact of new edges on an individual network, and not on the system as a whole[5]. A related study showed that although an individual network may benefit from adding edges to a second network, the system as a whole (the collection of networks) may be worse off[8]. From a network-centric perspective, interconnectivity may be beneficial, but, from a system-wide perspective, increased interconnectivity is often accompanied by increased systemic risk.

The past year has seen a flurry of activity on interdependent and layered networks. Aguirre *et al.* studied structural competition, but there have also been several advances in game-theoretic formulations of cooperation and competition, concerning how interdependence can enhance the extent of cooperation[9–11] and also exhibit trade-offs at different scales[11].

These papers usher in an era focused on quantifying the effects of network interdependence. It is my expectation that we will witness exciting developments as the different types of competition possible between two networks are further explored. For instance, in economics, two goods (or in our case networks) can be viewed either as substitutes or complements for one another. If they act as substitutes, the networks are in direct competition (say, for nodes) and if one network is enhanced the other is depleted. If, however, the networks are complementary, enhancement in one network also enhances the second.

I also expect that incorporating a diversity of edge types will lead to new developments. Edges can be (among other things) cooperative, competitive, activating, inhibitory or purely connective. Edges that express a dependency themselves have multiple classes — such as direct, logistic or latent — each one with a different timescale of impact. Different benefits, detriments and trade-offs will result from coupling together previously distinct networks in this myriad of ways. And we must bridge the scales considering competition and cooperation at the scale of individual agents, at the scale of coalitions or communities or modules of networked agents, and at the scale of coupling together distinct networks. ❐

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