Brief Announcement: Brokerage and Closure in a Strategic Model of Social Capital

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ABSTRACT

This paper introduces a model of strategic network formation grounded in two disparate modes of acquiring social capital – brokerage and closure – through the unification of a dual-level view of interactions between individuals and between groups of individuals referred to as *structural autonomy*. After motivating and introducing the model, we establish the existence of equilibrium and propose interesting open questions and extensions to the basic model for future research.

Categories and Subject Descriptors

F.2.2 [**Theory of Computation**]: Analysis of Algorithms and Problem Complexity—*Nonnumerical Algorithms and Problems*; G.2.2 [**Mathematics of Computing**]: Discrete Mathematics—*graph theory, network problems*

Keywords

game theoretic models, network design

1. INTRODUCTION

This paper introduces a model of strategic network formation grounded in two disparate modes of acquiring social capital – brokerage and closure – referred to as *structural autonomy* [3]. Structural autonomy considers a dual-level view of individual and collective action, calling on individuals to cooperate within groups, and then for the groups themselves to compete against one another. Examples of such environments are prevalent in many social settings, from the internal cooperation among producers and external structure of the markets in which the producers are situated [5], to the cooperation among guild (team) members who collaborate to accomplish quests in massively multiplayer online role-playing games [14].

There are examples in the literature of network formation games where agents seek to maximize brokerage [6, 13] or closure [1], independently, but ours appears to be the first to involve agents pursuing both brokerage and closure simultaneously. This model, the Structural Autonomy Connections (SAC) game is formally defined in Section 2, and the existence of equilibrium is established in 2.1. Open problems and extensions to the basic SAC model are discussed in Section 3 as suggestions for future research. The remainder of Section 1 is devoted to a brief review of the necessary background on social capital, brokerage, closure, and structural autonomy.

1.1 Social Capital via Brokerage and Closure

Social capital accounts for the value that one gains from their relationship with society, often considered in terms of one's location in a social network. By introducing this notion of value, Coleman [7] brought social structure into the rational action paradigm, an area that had previously been dominated by a premise of extreme individualism. Coleman argued that *closure*, where an individual is connected to others who are themselves connected to one another, serves as one source of social capital. Through closure, mechanisms such as reputation, trust, and the establishment of norms and shared vision are enabled. Coleman [7] gives an example where the lack of closure in the social networks around high school students and their families is correlated to high school dropout rates, highlighting the ability of closure to act as an enforcement mechanism for social norms and reputation.

Another source of social capital is proposed by Burt's theory of *structural holes* (*cf.*, [2, 3]). This theory is based on the idea that individuals create value by filling "holes" in their social network, acting as a broker between two (or more) otherwise disconnected groups. The social capital afforded to such hole-spanning individuals, called *brokers*, can be described as a *vision advantage*, since exposure to diverse groups, each with their own sets of expertise, puts the broker "at risk of having good ideas" [3]. The brokerage advantage exists insofar that the lack of alternate means of connectivity (across the structural hole) provides the broker with sole access and control over the bridge they provide.

It should be clear that closure and brokerage operate on opposite ends of the same space – where closure thrives on the interconnectedness of third-parties, brokerage deteriorates. The *network constraint* [3] characterizes this space, and is defined for an individual $i \in N$ in a network $G = (N, E)$ as:

$$C_i(G) = \sum_{j \in N \setminus \{i\}} \left(p_{ij}(G) + \sum_{q \in N \setminus \{i,j\}} p_{iq}(G) p_{qj}(G) \right)^2 . \quad (1)$$

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Here $p_{ij}(G)$ is the proportion of time and energy that i invests in maintaining their link with j. We will make the assumption that i invests uniformly across their links, so $p_{ij}(G)$ equals the inverse of i's degree in G. Intuitively, the network constraint provides a summary index measuring the extent to which an individual's resources are constrained by their direct and indirect network relationships. Buskens and van de Rijt [6] show that (1) evaluates to a value in the range $[0, 9/8]$. Here, we will make use of a 0/1 *network constraint*, denoted by $\mathcal{C}_i(G) = \max\{C_i(G), 1\}$, to force the constraint to a value in the range $[0, 1]$. Also, we will use $\mathcal{C}(G) = \frac{1}{|N|}\mathcal{C}_i(G)$ to denote the average (normalized) network constraint among all agents N in G.

1.2 Structural Autonomy

Structural autonomy is defined with a dual-level view of individuals and their connections in a network. We assume that group *membership* is given, so that each agent knows who is and is not on their *team*, but nothing is assumed about how these groups are organized – internally or externally.

Let $\mathcal{N} = (N_1, N_2, \ldots, N_m)$ be a partition of N agents into m groups. Given a network $G = (N, E)$ and a partition $\mathcal{N}$, let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be the *inter-group network* (or simply the *group network*) induced by collapsing the groups into single nodes. An edge $\{N_i, N_j\}$ is present in $\mathcal{E}$ if there exist a pair of individuals $i \in N_i$ and $j \in N_j$ such that $\{i, j\} \in E$. Burt [3] relates the performance of a group N_i to its structural autonomy, which is a non-linear combination of the closure among individuals within N_i and the brokerage beyond N_i; *i.e.*, Structural Autonomy $= \alpha\text{Closure}^{\beta}\text{Brokerage}^{\gamma}$. Here, we will use the normalized network constraint as a basis for both closure and brokerage, and define the structural autonomy of a group $N_i \in \mathcal{N}$ in a graph $G = (N, E)$ as

$$\Pi_{N_i}(G) = \alpha\mathcal{C}(G_{N_i})^{\beta}(1 - \mathcal{C}_{N_i}(\mathcal{G}))^{\gamma}, \tag{2}$$

where G_{N_i} is the *intra-group network* induced by the subgraph of G consisting only of vertices in N_i, $\mathcal{G}$ is the inter-group network induced by the partitioning $\mathcal{N}$, and α, β, γ are constants. With the β and γ terms assuming values greater than one, the structural autonomy decreases more rapidly with reductions from higher levels of intra-group closure, $\mathcal{C}(G_{N_i})$, or inter-group brokerage, $(1 - \mathcal{C}_{N_i}(\mathcal{G}))$ than from lower levels.[1] Throughout the remainder of this paper, we will simply assume that $\alpha = 1$.

2. MODEL

In this section we formally present the Structural Autonomy Connections (SAC) network formation game. A SAC instance is described by a partitioning $\mathcal{N} = (N_1, \ldots, N_m)$ over a set of agents N, where each group N_i is composed of n individual agents (and therefore, $|N| = mn$). The strategy space for each agent i is defined to be $S_i = \mathcal{P}(N \setminus \{i\})$, the power set of all other agents, which represents the set of all possible sets of links that agent i could form. Edge formation is *bilateral*, so a joint strategy profile $s = (s_1, \ldots, s_{mn})$ specifies an undirected graph $G_s = (N, E_s)$ with edges $E_s = \{\{i, j\} : j \in s_i \wedge i \in s_j\}$.

[1] In Burt's work [5, 3], structural autonomy is formulated slightly differently than Equation (2). See [3, Chapter 3.3.3] for details.

As a notational tool, we will use $\tau(i)$ to denote the group (or *team*) that agent i belongs to. This way, given an arbitrary agent $i \in N$, we can identify their group by $N_{\tau(i)} = \{j : \tau(j) = \tau(i)\}$.

Agents' strategy choices are motived by a utility function that is defined at the group level. Thus, each agent selects a strategy so as to maximize the utility of their respective group. Using a modified definition of structural autonomy as the starting point (see Equation (4) below), we define the utility for an agent i given a joint strategy profile $s = (s_1, \ldots, s_{mn})$ by

$$u_i(s) = \frac{1}{n}\bar{\Pi}_{N_{\tau(i)}}(G_s). \tag{3}$$

This utility function can be interpreted as splitting the gains that the group acquires through structural autonomy evenly among its n members – *i.e.*, the *equal split* rule (*cf.*, [11]).

From the definition of network constraint given in Equation (1), it would seem that isolated nodes experience the lowest network constraint possible (*i.e.*, zero). In empirical studies, isolates are usually discarded [4, Appendix B]. However, in our strategic model, we cannot simply throw them out; so instead we penalize isolates in order to incentivize them to build links. Since our definition of structural autonomy, Equation (2), simultaneously "rewards" a high network constraint (in the closure term, $\mathcal{C}(G_{N_i})^{\beta}$) and a low network constraint (in the brokerage term, $(1 - \mathcal{C}_{N_i}(\mathcal{G}))^{\gamma}$), we will have to choose our penalty carefully so that it incentivizes isolates to connect (thus increasing brokerage) but does not incentivize connected agents to disconnect (thereby decreasing closure).[2] Our solution to this dilemma is to modify our definition of structural autonomy to incorporate penalties for brokerage isolates while leaving the network constraint definition alone;

$$\bar{\Pi}_{N_i}(G) = \begin{cases} 0 & \text{if } N_i \text{ is isolated in } \mathcal{G} \\ \Pi_{N_i}(G) & \text{otherwise.} \end{cases} \tag{4}$$

2.1 Existence of Equilibrium

In settings with bilateral edge formation, *pairwise stability* [11] is a commonly used solution concept, and is the one that we employ for the SAC model. A network G_s is pairwise stable if:[3]

1. $\forall\{i, j\} \in G_s$, $u_i(G_s) \geq u_i(G_s - \{i, j\})$ and $u_j(G_s) > u_j(G - \{i, j\})$, and
2. $\forall\{i, j\} \notin G_s$, if $u_i(G_s + \{i, j\}) > u_i(G_s)$ then $u_j(G_s + \{i, j\}) < u_j(G_s)$.

Pairwise stability stipulates that a network is stable so long as no two agents can benefit by building a link between themselves and no individual agent would be better off by severing one of their existing connections.

Theorem 2 establishes the existence of pairwise stable equilibrium for the SAC model – an important first step

[2] For example, if we penalize isolates by giving them a network constraint of 1 in the hope of incentivizing the formation of brokerage links in the inter-group network, we would find that this "penalty" would simultaneously incentivize all the individuals in each group N_i to isolate themselves from their fellow group members in G_{N_i}.

[3] Given a graph $G = (N, E)$, the shorthand $G + \{i, j\}$ refers to the graph $G' = (N, E')$ obtained from G where $E' = E \cup \{i, j\}$. $G - \{i, j\}$ is similarly defined.

attesting to its validity and applicability. Our construction relies upon the following result of Buskens and van de Rijt [6] as well as a couple of properties regarding trees and the network constraint (the proofs of which are omitted).

THEOREM 1 ([6]). *All multipartite inter-group networks are pairwise stable if the parts are of equal size and each has more than a single agent.*

LEMMA 1. *The average network constraint for a tree with l leaves and maximum degree d is at most*

$$\frac{n - l - dl}{nd}.$$

COROLLARY 1. *Among tree networks, the average network constraint is maximized in a star topology.*

COROLLARY 2. *As $n \to \infty$, the average network constraint of an n-node star approaches 1.*

THEOREM 2. *For every SAC instance with $n \geq 5$ agents per group there exists at least one pairwise stable outcome.*

PROOF SKETCH. Consider the following construction: For each group $N_i \in \mathcal{N}$ build a star among the n agents in N_i. Next, build edges $\{i, j\}$ between nodes $i \in N_i$ and $j \in N_j$ so that the resulting inter-group graph $\mathcal{G}$ is a balanced multipartite network.

By Corollary 2, the intra-group star networks achieve the maximum amount of closure, and by Theorem 1, the inter-group brokerage network is pairwise stable. □

3. FUTURE RESEARCH

We believe that the SAC model of strategic network formation gives rise to a wealth of interesting questions and quantities that can be the subject of future research. In this section we highlight a handful of those that we find most interesting.

Open problems concerning the basic SAC model (as presented in Section 2) involve investigating the structures of equilibrium strategies to determine: (i) the variety of intra- and inter-group network topologies that are supported in equilibrium; (ii) how different values of β and γ effect equilibrium structures, and whether values can be determined that mirror observed social structures (like the above mentioned massively multiplayer online role-playing games); (iii) the range of utility values realized in equilibrium; and (iv) whether there exists an appropriate definition of *social value* function in the SAC model, and if so, what is the *Price of Anarchy* and *Price of Stability* given this function? Another interesting problem is to determine the effect that the addition of supplemental edges will have on an equilibrium networks.

An interesting extension to the basic SAC model is to incorporate elements of a *coalition game* and require that, instead of agents evenly splitting the value afforded by their groups structural autonomy (as is the case with the utility function given in Equation (3)), the agents within a group must choose how to allocate the group's value among themselves (*cf.*, [10, 11]). This extension opens a rich array of research possibilities. For example, will brokerage agents demand higher payoffs than those who do not contribute any inter-group edges? Under what conditions can an agent with only closure ties demand a larger allocation than an agent with brokerage ties? To what extent can inequality exist (*cf.*, [9]) and can it lead to groups splitting apart (*cf.*, [12])?

More generally, since many complex social and technical networks are in fact interacting networks-of-networks, the authors believe that an examination of such networks in a strategic, game theoretic setting is of fundamental interest, especially as multiple scales of analysis are needed (*cf.*, [8]).

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5. REFERENCES

[1] M. J. Burger and V. Buskens. Social context and network formation: An experimental study. *Social Networks*, 31(1):63–75, January 2009.

[2] R. S. Burt. Autonomy in a social topology. *American Journal of Sociology*, 85(4):892–925, January 1980.

[3] R. S. Burt. *Brokerage and Closure: An Introduction to Social Capital*. Oxford University Press, Great Clarendon Street, Oxford OX2 6DP, 2005.

[4] R. S. Burt. *Neighbor Networks: Competitive Advantage Local and Personal*. Oxford University Press, Oxford, 2010.

[5] R. S. Burt, M. Guilarte, H. J. Raider, and Y. Yasuda. Competition, contingency, and the external structure of markets. In P. Ingram and B. S. Silverman, editors, *Advances in Strategic Management*, volume 19, pages 167–217. Emerald Group Publishing Limited, 2002.

[6] V. Buskens and A. van de Rijt. Dynamics of networks if everyone strives for structural holes. *American Journal of Sociology*, 114(2):371–407, September 2008.

[7] J. S. Coleman. Social capital in the creation of human capital. *American Journal of Sociology*, 94:S95–S120, 1988.

[8] R. M. D'Souza. Complex networks: A winning strategy. *Nature Physics*, 9:212–213, April 2013.

[9] R. M. D'Souza and S. D. Johnson. Inequality and network formation games. arXiv:1303.1434, March 2013.

[10] M. O. Jackson. Allocation rules for network games. *Games and Economic Behavior*, 51(1):128–154, April 2005.

[11] M. O. Jackson and A. Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, 1996.

[12] W. Kets, G. Iyengar, R. Sethi, and S. Bowles. Inequality and network structure. *Games and Economic Behavior*, 73(1):215–226, September 2011.

[13] J. Kleinberg, S. Suri, Éva Tardos, and T. Wexler. Strategic network formation with structural holes. In *EC '08: Proceedings of the 9th ACM conference on Electronic commerce*, pages 284–293, New York, NY, USA, 2008. ACM.

[14] C. Shen, P. Monge, and D. Williams. Virtual brokerage and closure: Network structure and social capital in a massively multiplayer online game. *Communication Research*, August 2012.