

Suppressing cascades of load in interdependent networks

PNAS Plus Author Summary

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Modern society depends on increasingly interdependent systems that are prone to widespread failure. Transportation, communication, power grids and other infrastructures support one another and the world's interconnected economies. Barrages of incidents large and small—downed power lines, grounded aircrafts, natural disasters and the like—cause avalanches of repercussions that cascade within and among these systems [1]. Although interdependence confers benefits, its effect on the risks of individual systems and on the collection of them remains poorly understood.

Here we analyze how the interconnectivity (interdependence) between networks affects the sizes of their cascades of load shedding. For networks derived from interdependent power grids, we show that interdependence can have a stable equilibrium. An isolated network suppresses its large cascades by connecting to other networks, but too many interconnections exacerbate its largest cascades—and those of the whole system. We develop techniques to estimate this optimal amount of interconnectivity, and we examine how differences among networks' capacity and load affect this equilibrium. Our framework advances the current mathematical tools for analyzing dynamics on interdependent (or modular) networks, and it improves our understanding of systemic risk in coupled networks.

In the basic process we consider, a system contains many elements that shed load to neighboring elements whenever they reach their capacity. This is captured by the classic sandpile model of Bak-Tang-Wiesenfeld, a paradigm for the power law statistics of cascades in many disciplines, from neuronal avalanches to financial instabilities to electrical blackouts [24]. In a basic formulation on a graph of nodes and edges, each node has a capacity for holding grains of sand (interpreted here as load or stress). Grains of sand are dropped randomly on nodes, and whenever a node has more sand than its capacity, it topples and sheds all its sand to its neighbors, which may in turn have too much sand and topple, and so on. Thus dropping a grain of sand can cause an avalanche (cascade) of topplings. These avalanches, like blackouts in power grids [6], occur in sizes characterized

by a power law: they are often small but occasionally enormous.

The Bak-Tang-Wiesenfeld model was originally formulated on a lattice. Given the relevance of networked systems, the dynamics have recently been studied on isolated networks, but not yet on interdependent (or modular) networks. Here we study it on two networks with sparse connections between them. Each network models an infrastructure (or a module of one), and the interconnections between them model their interdependence. We explicitly study networks extracted from two interdependent power grids in the southeastern USA and an idealization of them that is more amenable to mathematical study. In this idealization, each node is connected to a node in the other network with probability p (Fig. 1, inset).

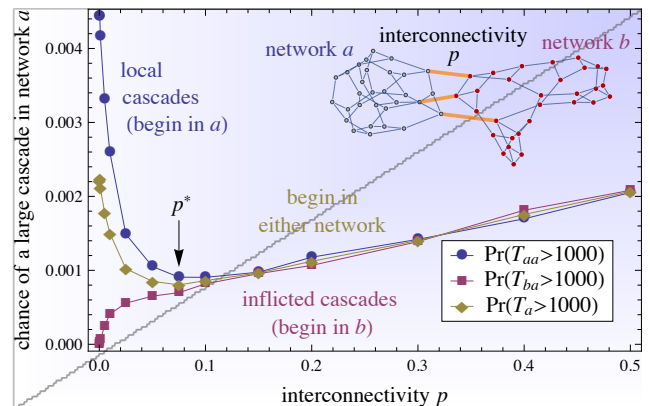


FIG. A1: The chance that a network a coupled to another network b suffers a cascade larger than half its network (gold curve) has a stable minimum at a critical amount of interconnectivity p^* . Networks seeking to mitigate their largest cascades would prefer to build or demolish interconnections to operate at this critical point p^* . The blue (red) curve is the chance that a cascade that begins in a (b) topples at least 1000 nodes in a . Increasing interconnectivity only exacerbates the cascades inflicted from b to a (red), but interestingly it initially suppresses the local cascades in a . (From simulations on coupled random 3-regular graphs; the inset depicts a small example with 30 nodes per network and $p = 0.1$.)

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of Fig. A1 shows that the chance of a large cascade in a network can be reduced by 70% by increasing the interconnectivity p from 0.0005 to 0.075. Too much interdependence, however, becomes detrimental for two reasons. First, new interconnections open pathways for the neighboring network to inflict additional load. Second, each interconnection augments the system's capacity, making available more load that fuels even larger cascades in each network. As a result, the chance of a large cascade in an individual network eventually increases with interconnectivity p , so p^* is a stable minimum.

This second factor above—that new interconnections increase the networks' capacity for load—has global consequences. With more load available, larger cascades in the system as a whole become possible. Therefore networks that interconnect to one another to mitigate their own cascades (Fig. A1) may unwittingly cause larger global cascades in the whole system. This is a warning for the interconnections under construction among, for example, different power grids to accommodate long-distance trade and renewable sources of energy [11].

The results in Fig. A1 show that networks suppressing their largest cascades would seek interconnectivity p^* . However, as shown in the the main article, building interconnections to operate at p^* increases the occurrence of small cascades. Conversely, networks can suppress their smallest cascades the most by seeking isolation, $p = 0$. But suppressing their smallest cascades exacerbates their largest ones (left side of Fig. A1), just as extinguishing

small forest fires can incite large ones and engineering power grids to suppress small blackouts can increase the risk of large ones [6].

Finally we determine how asymmetry among networks affects the optimal level of interconnectivity that each prefers. For instance, two interconnected power grids may differ in capacity, load, redundancies, demand, susceptibility to line outages, and ages of infrastructure. We capture these differences with a parameter that controls the rates at which cascades begin in either network. We show that in any asymmetric situation the equilibrium will be frustrated, with only one of the grids able to achieve its optimal level of interconnectivity.

Determining how interdependence affects the functioning of networks is critical to understanding the infrastructure so vital to modern society. Whereas others have recently shown that interdependence can lead to alarmingly catastrophic cascades of failed connectivity [20], here we show that interdependence also provides benefits, and these benefits can balance the detriments at stable equilibria. We expect that this work will stimulate calculations of critical points in interconnectivity among networks subjected to other dynamics. As critical infrastructures such as power grids, transportation, communication, banks and markets become increasingly interdependent, resolving the risks of large cascades and the incentives that shape them becomes ever more important.

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