Problem Set 8 – Due Tuesday, May 25, 2010, at 4:15 pm

Problem 1. Classify each of the following languages as either (a) **recursive**—I see how to decide this language; (b) **r.e.**—I don't see how to decide this language, but I can see a procedure to accept this language; (c) **co-r.e.**—I don't see how to decide this language, but I can see a procedure to accept the complement of the language; or (d) **neither**: I don't see how to accept this language nor its complement. No justification is needed for your answers.

Part A. $\{\langle M\rangle : M$ is a TM that accepts some string of prime length$\}$.

Part B. $\{\langle M\rangle : M$ is a C-program that halts on $\langle M\rangle\}$.

Part C. $\{\langle G\rangle : G$ is a CFG and G accepts an odd-length string$\}$.

Part D. $\{\langle M\rangle : M$ is a TM and M has 150 states$\}$.

Part E. $\{\langle M\rangle : M$ is a TM and $L(M) = L(M)^*\}$.

Part F. $\{\langle M\rangle : M$ is a TM and $L(M) = \emptyset\}$.

Part G. $\{\langle M\rangle : M$ is a TM and $L(M)$ is r.e. $\}$.

Part H. $\{\langle G_1, G_2\rangle : G_1$ and G_2 are CFGs and $L(G_1) = L(G_2)\}$.

Part I. $\{\langle M\rangle : M$ is a TM and M will visit state q_{25} when run on some input $x\}$.

Part J. $\{\langle M\rangle : M$ is a TM and M that uses at most 17 tape cells when run on blank tape$\}$.

Problem 2. Prove whether each of the following languages is **recursive**, **r.e.** but not recursive, **co-r.e.** but not recursive, or **neither** r.e. nor co-r.e.

Part A. $L = \{\langle M, w\rangle : M$ is a TM that uses at most 17 tape squares when run on $w\}$.

Part B. $L = \{\langle M, k\rangle : M$ is a TM that accepts at least one string of length $k\}$.

Part C. $L = \{\langle M, k\rangle : M$ is a TM that diverges (loops) on at least one string of length $k\}$.

Part D. $L = \{\langle M, k\rangle : M$ is a TM that accepts a string of length k and diverges on a string of length $k\}$. Assume that the underlying alphabet has at least two characters.

Part E. $L = \{\langle M\rangle :\ M$ is a TM that accepts some palindrome$\}$.

Problem 3.

Part A. Give two languages L_1 and L_2, each r.e. but not recursive, with empty intersection.

Part B. Give two languages L_1 and L_2, each r.e. but not recursive, with union Σ^*.

Part C. Are there languages L_1 and L_2 meeting conditions (A) and (B) simultaneously? Why or why not?