Quiz 4 Solutions

(1) Write a regular expression for the language of the following NFA $M_1$. Make it as simple as possible. Use standard abbreviations, not writing the concatenation symbol or extra parentheses.

```
 p  a, b
   b   q
    b  r
```

\[(a \cup b)^*bb\]

(2) Suppose $L$ is accepted by a 10-state DFA $M_2$. Using constructions described in class, we could convert $M_2$ into a 20-state NFA $M'_2$ for $L'$ (the bar denoting the complement of the language). We could then convert $M'_2$ into a 20-state DFA for $L'$.

(3) Darken the box to indicate if the statement is True or False. Really make your mark dark.

**True** Every regular language can be accepted by a DFA with an odd number of states.

**True** Every regular language can be accepted by a DFA whose start state is never visited twice.

*Note: these were both taken from PS2.*

(4) Same instructions. Throughout, fix an NFA $M = (Q, \Sigma, \delta, q_0, F)$.

**True** Suppose there is a $q_0 \leadsto q$ path in the diagram for $M$ where $q \in F$ and the concatenation of edge-labels along the path is $s$. Then $M$ accepts $s$.

**False** Suppose there is a $q_0 \leadsto q$ path in the diagram for $M$ where $q \not\in F$ and the concatenation of edge-labels along the path is $s$. Then $M$ rejects $s$.

(5) Let’s begin a proof that $L = \{a^n b^n : n \geq 0\}$ is not regular:

Assume for contradiction that $L$ is regular. Then there is a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts $L$. Let $N = |Q|$. Consider the $N + 1$ strings $\varepsilon, a, aa, \ldots, a^N$. Each of these strings $w$ determines a state $\delta^*(q_0, w)$. By the pigeonhole principle (PHP), we know that some two of these states are the same. And so on ...