Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **4** problems. Do your best to communicate your ideas *clearly* and *succinctly*. If you don't understand what a problem means, ask. Draw a picture if it is useful to explain something. Good luck.

— Earl Barr and Phil Rogaway

Name:

Signature:

On problem	you got	out of
1		35
2		30
3		15
4		20
Σ		100

1 Short Answer [35 points]

(A) Carefully define what it ***means*** if we say: *the regular languages are closed under intersection.* (Don't explain if this statement is true or false; just give a "mathematical translation" of the sentence).

(B) Carefully state the ***pumping lemma*** for **regular** languages. (Please don't use the word "pumps," at least not without defining it!) Be sure that your quantifiers are precise.

(C) Draw a DFA that accepts

$$L = \{w \in \{0,1\}^* : w \text{ is the binary representation of a number divisible by } 3\}\ .$$

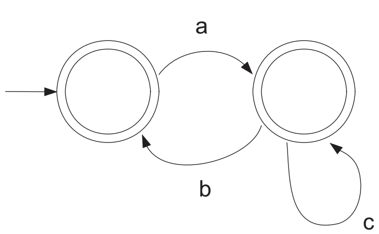
That is, $L = \{0\}^* \circ \{\varepsilon, 11, 110, 1001, \ldots\}$. Your DFA should have a minimum number of states.

(D) Complete the definition: A context free grammar is a 4-tuple $G = (\quad,\quad,\quad,\quad)$, where

(E) Give a CFG for $L = \{w \in \{a, b\}^* : \ w \text{ is a palindrome}\}$. (Recall that a string w is a palindrome if $w = w^R$ — the string reads the same forwards or backwards.)

(F) Prove that the language of Part (E), $L = \{w \in \{a, b\}^* : \ w \text{ is a palindrome}\}$, is **not** regular.

(G) Using the procedure given in class and in your book, convert the following NFA into a regular expression for the same language. Do not "simplify" your work.



2 Justified True or False [30 points]

Put an **X** through the **correct** box. When it says "Explain" provide a **brief** (but convincing) justification. Where appropriate, the justification should be a **counterexample**.

---

1. If F is a finite language and L is an arbitrary language then $L \cap F$ is a regular language. **True** **False**

Explain:

---

2. If L^* is regular, then L is regular, too. **True** **False**

Explain:

---

3. Every regular language can be accepted by an DFA $M = (Q, \Sigma, \delta, q_0, F)$ such that $\delta^*(q_0, x) \neq q_0$ for any string $x \in \Sigma^+$. *In case you've forgotten the notation, $\Sigma^+ = \Sigma\Sigma^*$ and $\delta^*(q, x)$ is the state one gets to after processing x, starting in state q.* **True** **False**

Explain:

---

4. If L_1 and L_2 are accepted by DFAs then $L_1 \oplus L_2 = (L_1 - L_2) \cup (L_2 - L_1)$ is accepted by a DFA. **True** **False**

Explain:

---

5. Suppose L is a regular language with arbitrarily long "gaps" — for every n there is an interval $[a_n, b_n]$ such that $b_n - a_n \geq n$ and L has no strings of any length $\ell \in [a_n \mathinner{..} b_n]$. Then L is finite. **True** **False**

Explain:

---

6. If a context-free grammar G is ambiguous then it can be converted to a context-free grammar G' where $L(G) = L(G')$ and G' is not ambiguous. **True** **False**

Explain:

---

3 A Decision Procedure [15 points]

Suppose you are given regular expressions α, β over the alphabet $\{a, b\}$. Describe a decision procedure (an algorithm) to answer the following question: is $L(\alpha) \cap L(\beta)$ infinite?

4 A Closure Property [20 points]

Suppose that L is a regular language over the alphabet $\Sigma = \{0, 1\}$. Define

$$\mathcal{B}(L) \quad = \quad \{x :\ xy \in L \text{ for some } y \in \Sigma^*\}$$

Part A. List the elements of $\mathcal{B}(\{101\})$:

$\mathcal{B}(\{101\}) = \{ \qquad\qquad\qquad\qquad\qquad\qquad \}$.

Part B. Prove that if L is regular then $\mathcal{B}(L)$ is regular.