Problem Set 3 – Due Tuesday, January 31, 2012

Problem 1. Consider trying to show that the NFA-acceptable languages are closed under $*$ (Kleene closure) by way of the following construction: *add ε-arrows from every final state to the start state; then finalize the start state, too.* Show, by finding a small counterexample, that the proposed construction does not work.

Problem 2. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA. We say that M accepts a string x in the *all-paths sense* if *every* computation of M on x ends in a state in F. Let $L'(M)$ denote the set of all $x \in \Sigma^*$ such that M accepts x in the all-paths sense. Show that L is regular iff $L = L'(M)$ for some NFA M.

Problem 3. Prove that the following languages are not regular.

Part A. $L = \{www : \ w \in \{a, b\}^*\}$.

Part B. $L = \{a^{2^n} : \ n \geq 0\}$.

Problem 4. Decide if the following languages are regular or not, proving your answers either way.

Part A. $L = \{w \in \{0, 1\}^* : \ w$ is *not* a palindrome $\}$.

Part B. $L = \{w \in \{0, 1\}^* : w$ has an equal number of 01's and 10's$\}$.

Part C. $L = \{w \in \{0, 1, 2\}^* : w$ has an equal number of 01's and 10's$\}$.

Problem 5. Describe a decision procedure to solve the following problem: given a regular expression α, is there a shorter regular expression for the same language?

Problem X. *The following question is for, at most, the top 2–3 students in the class; other students should spend their time elsewhere. If you solve it, please email a solution directly to Prof. Rogaway.*

Show that if $L \subseteq 1^*$, then L^* is regular.