Problem Set 4 – Due Tuesday, February 7, 2012

Problem 1. Are the following statements true or false? Either prove the statement or give a simple counter-example.

(a) If $L \cup L'$ is regular then L and L' are regular.

(b) If L^* is regular then L is regular.

(c) If LL' is regular then L and L' are regular.

(d) If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

(e) If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

Problem 2. Define $A = \{x \in \{a, b, \sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or consecutive letters$\}$.

(a) Can you use the pumping lemma to prove that A is not regular? Explain.

(b) Prove that A is not regular.

Problem 3. Give a context free grammar for $L = \{a^n b^m : n \neq 2m\}$. Try to make your grammar unambiguous—and explain why it is unambiguous.

Problem 4. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) *Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.*

(b) *Is $L(G)$ regular? Prove your answer either way.*

(c) *Is G ambiguous? Prove your answer either way.*

(d) *Is $L(G)$ inherently ambiguous? Give a convincing argument either way.*

Problem 5. A *regular grammar* is a context-free grammar $G = (V, \Sigma, R, S)$ in which every rule is of the form $A \to \varepsilon$ or $A \to aB$, where a is a terminal and A and B are variables. Show that L is regular iff L is generated by a regular grammar.