ECS 189A Final — Cryptography — Spring 2011

Hints for success: Good luck on the exam. I don’t think it’s all that hard (I do believe I could answer everything!). Please read the questions carefully, and think before you answer. Make your response legible, logical, and succinct. Nothing here needs more than 2–3 sentences.

Final grades should be ready around Tuesday. You should be able to retrieve them from my.ucdavis.edu in the usual way.

Hope to see some of you next year. (If you’re brave enough to take another Rogaway class, I’m scheduled to teach ecs188 (Ethics) in Fall, and ecs120 (Theory of Computation) and ecs227 (Cryptography) in Spring.)

Phillip Rogaway

Name: Deedie Goode

Signature:

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1. Ciphers

1. Draw a picture showing two rounds of a Feistel network. Denote the round functions for the two rounds as $F^1, F^2 : \mathcal{K} \times \{0, 1\}^n \rightarrow \{0, 1\}^n$ where $\mathcal{K}$ is the key space.

2. True or False, and briefly explain: DES would remain invertible—it would still be a blockcipher—even if its S-boxes were arbitrarily changed (the number of input and output bits remaining the same).

   True — A Feistel network is invertible regardless of the round function.

3. True or False, and briefly explain: AES would remain invertible—it would still be a blockcipher—even if its S-boxes were arbitrarily changed (the number of input and output bits remaining the same).

   False — An SP-cipher needs invertible S-boxes to be invertible

4. In a couple of sentences, give me a quick synopsis of Trivium.
5. Let $E: \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ be a blockcipher and let $A$ be an adversary. Carefully define $\text{Adv}^{\text{DTT}}_E(A)$, the advantage that an adversary $A$ gets in attacking the blockcipher $E$. Use notation following that used in class. Then, in a paragraph, carefully explain what the notation means.

$$\text{Adv}^{\text{DTT}}_E(A) = \Pr\left[K \leftarrow \mathcal{K} : A^{E_K(\cdot)} \Rightarrow 1\right] - \Pr\left[\pi \leftarrow \text{Perm}(n) : A^{\pi(\cdot)} \Rightarrow 1\right]$$

**Explanation:**

Choose a random permutation and let $A$ interact with an oracle $O$ that behaves according to the "real" blockcipher $E(K, \cdot)$. At the end, it outputs a prediction: 1 for "real" and 0 for "fake" ($\pi = \text{random permutation}$).

6. **True** or **false**, and explain: For a blockcipher like $E = \text{AES}$, we know that $\text{Adv}^{\text{DTT}}_E(A)$ is "small" for any reasonable adversary $A$—cryptographers have proven good upper bounds.

**False** — we don’t know this, but cryptographers generally believe it is true.
2 Attacks

1. Suppose you have a blockcipher with a 40-bit key: \( E : \{0,1\}^{40} \times \{0,1\}^{128} \rightarrow \{0,1\}^{128} \). Construct from \( E \) the blockcipher \( F : \{0,1\}^{40} \times \{0,1\}^{128} \rightarrow \{0,1\}^{128} \) by saying that

\[
F_{K_1,K_2}(X) = E_{K_2}(E_{K_1}(X))
\]

where \(|K_1| = |K_2| = 40\).

An adversary \( A \) has a single plaintext/ciphertext pair \((X,Y) = (X,F_{K_1,K_2}(X))\) for a random and secret key \( K = K_1K_2 \). **Describe a reasonably efficient attack** that will, most of the time, recover \((K_1, K_2)\). By “reasonably efficient” I mean far fewer than \(2^{80}\) times steps (with one time step being the amount of time to compute one \( E_K \) value).

**What is this attack called?**

Meet-in-the-middle:

- Compute \( E_{K_1}(X) \) for every key \( K_1 \in \{0,1\}^{40} \) to
- Compute \( E_{K_2}^{-1}(Y) \) for every key \( K_2 \in \{0,1\}^{40} \)

If some two of these coincide — say \( E_{K_1'}(X) = E_{K_2''}^{-1}(Y) \), then output \((K_1', K_2'')\).

Needs \(2^{40}\) queries.

2. Don proposes a 128-bit blockcipher \( E \) that works like this. It has 16 S-boxes, \( S_1, \ldots, S_{16}\), each a permutation mapping 8-bits to 8-bits. It uses a 128-bit key that gets mapped into 32 subkeys, \( K_1, \ldots, K_{32}\), each 128 bits. To encrypt an input block \( X \), for each of 32 rounds \( i\):

1. Replace \( X \) by \( X \oplus K_i \);
2. Replace the \( j \)-th byte of \( X \), \( X[j] \), by \( S_j[X[j]] \) (for each \( 1 \leq j \leq 16 \));
3. Circularly rotate \( X \) by one byte position to the left.

When the above is complete, the ciphertext block is the final value of \( X \).

**What queries should you ask**—no more than a few hundred—to allow you to completely and efficiently break this cipher?

\[
\begin{align*}
\text{Ask} & \quad 0x0000 \ldots 00 \\
& \quad 0x0101 \ldots 01 \\
& \quad 0xFFFF \quad FF \\
\text{Can now encrypt/decrypt anything}
\end{align*}
\]
3 Math

1. How many permutations are there on the space of 128-bit strings?

\[ 2^{128} \]

2. An adversary \( A \) asks an \( n \)-bit to \( n \)-bit (uniform) random permutation \( \pi \) for the values of \( \pi(x_1), \ldots, \pi(x_q) \) for distinct values \( x_1, \ldots, x_q \). Then \( A \) outputs a pair \((x, y)\). The probability that this is a good forgery (that is, that \( x \) is none of \( x_1, \ldots, x_q \) and yet \( \pi(x) = y \)) is at most \( \frac{1}{2^{n-q}} \). (Give a tight value.)

3. The product of bytes

\[ \begin{align*}
1010111 & \quad (= \text{0xAF}) = x^7 + x^3 + x^2 + x + 1 \\
0000011 & \quad (= \text{0x3}) = x + 1
\end{align*} \]

and

\[ \begin{align*}
11101010 & \quad \text{in GF}(2^8)
\end{align*} \]

in GF(\(2^8\)) is \( \overline{1} \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \) \( \). Assume here that field elements are represented using the primitive polynomial

\[ g(x) = x^8 + x^4 + x^3 + x + 1 \]

and show your work below.

\[ \begin{align*}
& x^8 + x^6 + x^4 + x^3 + x^2 + x + 1 \\
& + x^7 + x^5 + x^3 + x^2 + x + 1 \\
& + x^9 + x^3 + x + 1 \\
& \Rightarrow x^7 + x^6 + x^5 + x^3 + x
\end{align*} \]

4. If \( n = pq \) is the product of distinct primes, \( |\mathbb{Z}_n^*| = \phi(n) = \frac{(p-1)(q-1)}{n/l_n(n)} \).

5. For large \( n \), there are roughly this many primes less than \( n \): \( n/\ln(n) \).
4 Encryption

1. Alice would like to privately send a single bit \( M \in \{0, 1\} \) to Bob. An adversary should get no information about \( M \). Alice and Bob share a uniformly random key \( K \in \{0, 1, 2\} \). How can Alice securely send her bit to Bob? Give a formula for the ciphertext \( C \):

\[
C = E_K(M) = M + K \pmod{3}
\]

2. Let \( E \) be the encryption algorithm of a symmetric encryption scheme. Recall that we define the ind-security of \( E \) to be

\[
Adv^{\text{ind}}_E(A) = \Pr[A^{E_K(\cdot)} = 1] - \Pr[A^{E_k(\cdot)} = 1]
\]

A secure blockcipher \( E \) (secure in the prp-sense) will always / sometimes / never (circle one) be a secure encryption method \( E \) (in the ind-sense).

3. The **decisional Diffie-Hellman assumption** is the assumption that:

\[
\left[ (g^a, g^b, g^{ab}) \right]_{a, b, c} \approx \left[ (g^a, g^b, g^c) \right]_{a, b, c}
\]

4. Let \( \Pi = (K, E, D) \) be a public-key encryption scheme. Can it be IND-secure\(^2\) if the encryption of a plaintext \( P \) leaks the identity of the key \( pk \) with which it is encrypted? Yes or No (choose one).

5. Fix a cyclic \( G \) of order \( p \) (that is, \(|G| = p\)) generated by \( g \in G \) (that is, \( (g) = G \)). Alice has a public key of \( A = g^p \) and a secret key \( a \). If Bob wants to encrypt a message \( m \in G \) to Alice using **ElGamal encryption**, he should choose a random \( b \in \mathbb{Z}_p \) and send Alice a ciphertext \( E_A(m) = \left[ g^{ab}, m \right] \).

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\(^1\) The correct answer is that *something* is computationally indistinguishable from *something*.

\(^2\) I refer here to the definition given in class—that \( Adv^{\text{IND}}_E(A) = \Pr[A^{E_K(\cdot)}(pk) = 1] - \Pr[A^{E_K(\cdot)}(pk) = 1] \) is “small” for all “reasonable” \( A \).
5 Message authentication and digital signatures

1. Draw a picture that illustrates the CBC MAC of a message \( P = P_1P_2P_3 \) where \( |P_i| = n \). The underlying blockcipher is \( E : K \times \{0, 1\}^n \rightarrow \{0, 1\}^n \). Make sure it is clear from your picture what string is the MAC, and how it depends on \( E, K, \) and \( P \).

![CBC MAC Diagram](image)

2. We have seen that the CBC MAC is not secure across strings of varying lengths. Describe a simple way to "fix" it (changing the CBC MAC as little as possible) so that it will (under reasonable assumptions) be secure across strings of varying lengths.

\[ \text{XOR in a key } K' \text{ just before the last blockcipher call.} \]

3. Consider signing with "raw" RSA: the signature of a message \( m \in \mathbb{Z}_n^* \) is \( m^d \mod n \) (where \( e \in \mathbb{Z}_{\phi(n)}^* \) and \( ed \equiv 1 \mod \phi(n) \)). True or False, and briefly explain: we showed that this signature scheme is correct (it is existentially unforgeable under an adaptive chosen-message attack) if RSA is a secure trapdoor permutation.

False.

Eg, you can forge 1 without asking any queries: its signature is 0=1.
6 Hash functions, authenticated encryption, and esoterica

1. Briefly describe a theorem we covered that helps justify the use of the Merkle-Damgård construction in schemes like SHA1.

   Merkle-Damgård Theorem: if a compression function of an ideal hash is collision resistant, then so is the hash function built from it.

2. Describe a correct algorithm or approach we discussed for making an authenticated encryption scheme—a symmetric encryption scheme that achieves both privacy and authenticity.

   - OAEP - Kind of complex ECS-like mode
   - Encrypt-then-MAC - Generic comb of a private-only enc. scheme and a MAC

3. Describe what is a 1-out-of-2 oblivious transfer.

4. Recall that in the problem 2-party Secure Function Evaluation, Alice has a private input of \( a_1 a_2 \cdots a_n \) (each \( a_i \) a bit) and Bob has a private input of string \( b_1 b_2 \cdots b_m \) (each \( b_j \) a bit). Bob should learn \( C(a_1, a_2, \cdots, a_n, b_1, \cdots, b_m) \), and Alice should learn nothing, where \( C \) is some fixed a boolean circuit. In solving this problem, we used 1-out-of-2 oblivious transfer. Please explain how.
7 A reduction

We argued in class that every pseudorandom function is also a good message authentication code (MAC). Formalize and prove this result.

Let $F$ secure as a PRF $\Rightarrow$ $F$ secure as a MAC

$F: \{0,1\}^n \rightarrow \{0,1\}^n$ $F$ insecure as a PRF $\Leftarrow$ $F$ insecure as a MAC

$\exists A_{PRF}$ break $F$ as a PRF $\Leftarrow$ $\exists A_{MAC}$ break $F$ as a MAC

Definition of $A_{PRF}$:

Run $A_{MAC}$. When $A_{MAC}$ acts a query $M$, return $O_{PRF}(M)$.

When $A_{MAC}$ halts, outputting $(M^*, T^*)$.

Then return 1 if $M^*$ was never asked in an oracle query and $T^* = O_{PRF}(M^*)$; return 0 otherwise.

$Adv^+_{A_{PRF}} = \Pr[A_{PRF}^{F_X(\cdot)} \Rightarrow 1] - \Pr[A_{MAC}^{O_{PRF}(\cdot)} \Rightarrow 1]$

$\uparrow$

$Adv^+_{A_{MAC}} \; \uparrow \; 2^{-n}$

So $1 - 2^{-n}$ is large if $n$ is.