Problem Set 5 – Due Monday, October 27, 2008

1. Draw a DFA for $L = \{a_1 \ldots a_n : \text{each } a_i \text{ is a bit and } a_i = 1 \text{ for all odd } i\}$. Assume that the empty string $\varepsilon$ is in $L$.

2. (a) Enumerate the first 6 strings of $L = 00^\ast 11^\ast$ in lexicographic order.\(^1\)
(b) Draw a DFA for this language. It should have no more than four states.
(c) Draw a DFA for $L^c$, the complement of $L$ relative to the alphabet $\{0, 1\}$. It should be obtained from your solution to (b) by generic means.
(d) Write a regular expression for $L^c$, trying to “read this off” of your DFA for (c).

3. Let $L = \{x \in \{0, 1\}^\ast : \text{the substring } 000 \text{ occurs an even number of times in } x\}$.
(a) Is $\varepsilon \in L$? Is $0000 \in L$?
(b) Draw a DFA for $L$. *Hint: states of the DFA might represent things like “I’ve seen an even number of 000’s so far and the longest strings of 0’s that I’ve just seen has length 0” (that would be the start state).*

4. Consider the relation $R = \{(1, 2), (1, 4), (2, 3), (2, 4), (4, 4)\}$ on the set $A = \{1, 2, 3, 4\}$. Write down, as sets: (a) the domain of $R$, (b) the range of $R$, (c) $R^{-1}$, (d) $R \circ R^{-1}$, (e) $R^{-1} \circ R$, and (f) the transitive closure of $R$. I suggest you draw directed graphs to help you see figure out (c)–(f).

5. Determine if the following relations $R$ are (i) reflexive, (ii) symmetric, and (iii) transitive.
   (a) $xRy$ if $x$ and $y$ are people who were born on the same day.
   (b) $xRy$ if $x$ and $y$ are strings that contain a common character.
   (c) $xRy$ if $x$ and $y$ are people and there exists a country $C$ that both have been in.
   (d) $xRy$ if $x$ and $y$ are points in the plane that are equidistant to the origin.

6. For $a, b \in \mathbb{R}$ define $a \sim b$ if $a - b \in \mathbb{Z}$.
   (a) Prove that $\sim$ defines an equivalence relation on $\mathbb{Z}$.
   (b) Describe the equivalence class containing 5. Describe the equivalence class containing 5.5.

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\(^1\) Recall that the lexicographic order of the strings in $L$ means: all strings in $L$ of length 0; then all strings in $L$ of length 1 (these in dictionary order, say 0 < 1); then all strings in $L$ of length 2 (these in dictionary order); and so on.