Problem Set 9 – Due Friday, 1:10 PM, December 5, 2008

Note the atypical due date. You may bring your solution to the 1:10 pm discussion section or you may leave your solution in the turn-in box in Kemper by 1:10 pm.

In each of the problems below, if you are calculating a real number, please give me an explicit approximation to it.

1. What's the chance that Alice, Bob, Charlie, and Dana were all born on different days of the week? Assume that each was born on a random day of the week.

2. Monika, a handsome capuchin monkey from Bolivia, throws darts at a dart-board having five concentric circles. The circles have radiuses of 10, 20, 30, 40, and 50 cm. Half the time Monika misses the dart-board completely (watch out) while the other half of the time she manages to get the dart to land on the the dart-board, but she has no particular control about where on the dart-board it lands.

   Describe a probability space (that is, a sample space S and a probability measure P) that models what is going on.

   Under your model, what is the probability that Monika gets 3 or more bull's-eyes in 5 throws? (A "bull's-eye" means that the dart lands in the centermost circle.)

3. Box A contains 70 red marbles and 30 black marbles, and box B contains 30 red marbles and 70 black marbles. A marble is drawn uniformly at random from each box. What is the probability that they are the same color?

4. (a) Ornery Prof. R asks five multiple choice questions, where there are five possibilities (ie., a, b, c, d, e) for each of the questions. He grades the questions: 4 points for a correct answer, 0 points for an absent answer, and -1 points for an incorrect answer. Suppose that Joe, who's rather clueless, randomly guesses on all five questions. What is his expected score?

   (b) Now suppose that Prof. R, in an uncharacteristic moment of generosity, decides to give everyone a score of *at least* zero on the multiple choice problem. That is, you get a zero even if you earn a total that is a negative number. Now what will be Joe's expected score?

5. Consider the following one-person game. Alice initially has \$300. A dealer has a deck of 52 cards, perfectly shuffled. The dealer turns over the cards one-by-one. If at some point Alice has $\$x$ then, just before the next card is turned over, Alice can bet any amount of money in $[0, x]$. The bet is "red" or "black". If the card turned over has the color Alice said, then Alice wins $\$x$; otherwise, she loses $\$x$.

   (a) Describe a strategy such that Alice is *guaranteed* to end the game with \$600.

   (b) Describe a strategy such that Alice is *guaranteed* to end the game with at least \$800.

   (c) (Thought problem—not graded—I don't know the answer.) What's the most money that Alice can be certain to win?