ECS 20 Discrete Math: Discussion 3, Mon 10/14/2013

To express a Boolean expression in CNF from a truth table, for example

A	B	X
0	0	0
0	1	1
1	0	0
1	1	1

Write an equation in terms of what X cannot be using the 0-rows. In this example X cannot be $\bar{A}\bar{B}$ and it cannot be $A\bar{B}$:

$$X = \overline{(\bar{A}\bar{B})}\,\overline{(A\bar{B})}$$

Use De Morgan's Law $\overline{AB} = \bar{A} \vee \bar{B}$ on each clause to achieve OR relations:

$$X = (A \vee B)(\bar{A} \vee B)$$

Write a CNF formula for X and Y.

A	B	C	X	Y
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$X = \overline{(\bar{A}\bar{B}\bar{C})}\,\overline{(\bar{A}\bar{B}C)}\,\overline{(A\bar{B}C)}\,\overline{(AB\bar{C})}$$
$$X = (A \vee B \vee C)(A \vee B \vee \bar{C})(\bar{A} \vee B \vee \bar{C})(\bar{A} \vee \bar{B} \vee C)$$

$$Y = (A \vee B \vee C)(A \vee \bar{B} \vee C)(A \vee \bar{B} \vee \bar{C})(\bar{A} \vee B \vee C)$$

Complete the following table, answering T or F given the universe of discourse indicated. $\mathbb{N}$ denotes all positive integers. ($\mathbb{Z}$ on PS3 denotes all integers.)

	$\mathbb{R}$	$\mathbb{N}$
$\exists x \exists y(x + y = 10)$	T	T
$\forall x \forall y(x + y = 10)$	F	F
$\exists x \forall y(x + y = 10)$	F	F
$\forall x \exists y(x + y = 10)$	T	F
$\forall x \exists y(x + y < 0)$	T	F
$\exists x \forall y(x + y \geq 0)$	F	T
$\forall x(x < 5 \rightarrow \forall y(y < x \rightarrow y < 4))$	F	T

Write each of the following sentences into formulas of quantification logic, introducing predicates as needed.

a) There is someone in this discussion who did not have breakfast.

   Let the universe of discourse be people. D(x) be in this discussion. B(x) had breakfast.

$$\exists x(D(x) \land \neg B(x))$$

b) Every ECS 20 student likes programming, but some ECS 20 students don't like math.

   Let the universe of discourse be ECS 20 students. P(x) likes programming. M(x) likes math.

$$\forall x\, P(x) \land \exists y\, \neg M(y))$$

Negate each of the following statements. Recall that

$$\neg\forall x\, p(x) \equiv \exists x\, \neg p(x) \text{ and } \neg\exists x\, p(x) \equiv \forall x\, \neg p(x)$$

a) $\exists x \forall y\, p(x, y)$ negates to $\forall x \exists y \neg p(x, y)$

b) $\forall x \forall y\, p(x, y)$ negates to $\exists x \exists y \neg p(x, y)$

c) $\exists x \exists y \forall z\, p(x, y, z)$ negates to $\forall x \forall y \exists z \neg p(x, y, z)$

d) $\forall x \exists y\, \big(p(x, y)\ \land \neg q(x, y)\big)$ negates to $\exists x \forall y \neg\big(p(x, y)\ \land \neg q(x, y)\big)$
   Apply De Morgan's Law: $\exists x \forall y\big(\neg p(x, y) \lor q(x, y)\big)$

e) $(\exists x > 10)(\exists y)\ (x + y = p(x, y))$ negates to $(\forall x > 10)(\forall y)\neg(x + y = p(x, y))$
   Apply negation: $(\forall x > 10)(\forall y)(x + y \neq p(x, y))$

f) $\exists x \exists y\, \big(p(x) \leftrightarrow p(y)\big)$ negates and expands to $\forall x \forall y\, \neg\big[\big(p(x) \rightarrow p(y)\big) \land \big(p(y) \rightarrow p(x)\big)\big]$

   $p(x) \rightarrow p(y) \equiv \neg p(x) \lor p(y)$: $\forall x \forall y\, \neg[(\neg p(x) \lor p(y)) \land (\neg p(y) \lor p(x))]$

   Apply De Morgan's Law: $\forall x \forall y\, [\neg(\neg p(x) \lor p(y)) \lor \neg(\neg p(y) \lor p(x))]$

   Apply De Morgan's Law: $\forall x \forall y\, [(p(x) \land \neg p(y)) \lor (p(y) \land \neg p(x))]$

NAND (↑) is logically complete.

$A \uparrow B = \neg(A \wedge B)$. The truth table for ↑:

A	B	$A \wedge B$	$A \uparrow B$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Recall that {∧, ∨, ¬} is logically complete.

To represent ¬ with ↑, consider the fact that a literal is equivalent to it ANDing itself, i.e. $A = A \wedge A$:

$$\begin{aligned} \neg A &= \neg(A \wedge A) \\ &= A \uparrow A \end{aligned}$$

by definition of ↑; ¬ is achieved.

Now consider representing ∧ with ↑ :

$$\begin{aligned} A \wedge B &= \neg\neg(A \wedge B) \\ &= \neg(A \uparrow B) \\ &= (A \uparrow B) \uparrow (A \uparrow B) \end{aligned}$$

so ∧ is achieved.

Finally consider representing ∨ with ↑, starting with De Morgan's Law:

$$\begin{aligned} &A \vee B = \neg(\neg A \wedge \neg B) \\ &= \neg[(A \uparrow A) \wedge (B \uparrow B)] \\ &= (A \uparrow A) \uparrow (B \uparrow B) \end{aligned}$$

by definition of ↑. So ∨ is achieved. Hence {↑} is logically complete.

For what the NAND gate looks like, and for the equivalent NAND circuits of NOT, AND, and OR circuits, visit http://hyperphysics.phy-astr.gsu.edu/hbase/electronic/nand.html#c4