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Discrete Maths

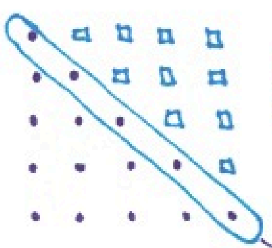
Example: $1+2+3+\dots+100$

$1+2+3+\dots+n$

$$S(n) = \sum_{i=1}^{n}$$

$$
\begin{array}{l}
S(n) = 1 + 2 + 3 + \dots + (n-1) + n \\
\quad\;\; n + (n-1) + (n-2) + \dots + 2 + 1 \\
\hline
\underbrace{(n+1) + (n+1) + (n+1) + \dots + (n-1) + (n+1)}_{n}
\end{array}
$$

$$S(n) = \frac{n(n+1)}{2} = \frac{100 \cdot 101}{2} = 50 \cdot 101 = 5050$$



$$\frac{n^2-n}{2} = \frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$$

$\to$ n on diagonal

Example 2: $\sqrt{2}$ is an irrational

Def: A real number x is rational if $x = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$, $b \neq 0$

Def: A real number x is irrational if it is not rational

Integer

Assume for Contradiction that x is rational

$\exists\; a, b \in \mathbb{Z}$ a, b have no common division

$\sqrt{2} = \frac{a}{b}$ $b \neq 0$

if it is not true $2 = \frac{a^2}{b^2}$, $\underline{2b^2 = a^2}$

a^2 is even then,
a is even and b is odd

a even $a = 2\alpha$ for $\alpha \in \mathbb{Z}$

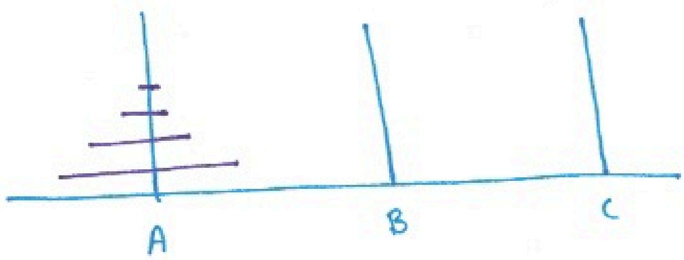
$b = 2\beta + 1$ for some $\beta \in \mathbb{Z}$

$$(2\beta+1)^2 = 2\alpha^2$$

$$4\beta^2 + 4\beta + 1 = 2\alpha^2$$

$$\underbrace{4(\beta^2+\beta)+1}_{\substack{\uparrow \\ \text{odd} \\ \text{cuz it says}+1}} = \underbrace{2\alpha^2}_{\substack{\uparrow \\ \text{Even}}}$$

Example 3:



We need to move — from a to b or c on ring at a time and big ring can't go on small

Let T_n = ~~moves~~ min # of moves to move the n rings from one peg to another

Algorithm:

$$T_n \leq T_{n-1} + 1 + T_{n-1}$$

$$\leq 2T_{n-1} + 1 \quad \text{if } n \geq 2$$

$$T_n = 1 \quad \text{if } n = 1$$

						formula
$n =$	1	2	3	4	5	n
$=$	1	3	7	15	31	$2^n - 1$
						guess

Fix any scheme that moves the n rings from one peg to another

lets name Scheme "M"

After alot of shuffle we got the bottom ring on c on top

M moved the biggest ring from a peg to c for the last time

- M Spent $\geq T_{n-1}$ moves to clear the $n-1$ rings off of the biggest ring
- M spent $\geq T_{n-1}$ moves to transport the final $n-1$ ring to c

$$T_{n-1} \geq 2T_{n-1} + 1$$

$$T_n = 2T_{n-1} + 1$$
$$= 2(2T_{n-2}+1)+1$$
$$= 2^2 T_{n-2} + 1 + 2$$
$$= 2^2 (2T_{n-3}+1) + (1+2)$$
$$= 2^3 T_{n-3} + 1 + 2 + 2^3$$
$$= 2^4 T_{n-4} + (1+2+2^2+2^3)$$
$$= 1+2+2^2+2^3+ \dots + 2^{n-1}$$

Example Cards Shuffle:

1 2 3 | 4 5 6 7 8

We need idea to solve big puzzles and that idea called rising Sequence

So we have one Rising Sequence in main number order

4 1 5 6 | 2 7 8 3

2 4 1 7 8 5 3 6

rising Sequence: A maximal $\underbrace{\text{Consective Subsequence}}_{\text{Sequence "2, 4, 6, 8, 10"}}$ of numbers

1 2 3 ..... 52 1 rising sequence

52 51 50 ..... 2 1 52 rising sequence

Shuffle	# of rising sequences
0	1
1	2
2	4
3	8
4	16
5	32
6	52