ECS 20 — Lecture 8 — Fall 2013 —22 Oct 2013

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Today:

- o Quiz 2
- o Some more operations on sets
- o How a computer might manipulate sets: dictionaries and disjoint-sets
  (INSERT/ IN/DELETE; UNION/FIND/MAKESET)

Various laws

Prove them by tracing through the definitions

De Morgan's laws:

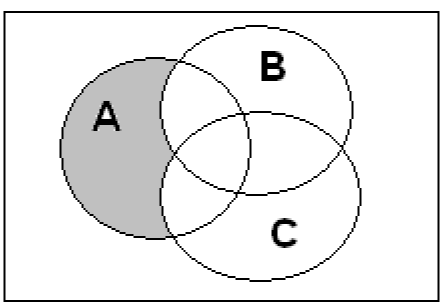
- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$

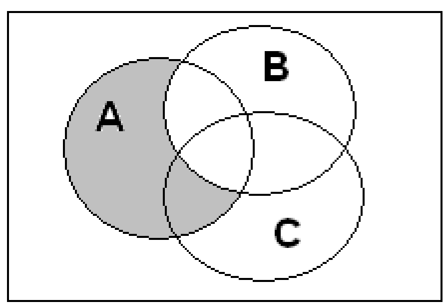
Proof (of first claim): $x \in (A \cup B)^c$

iff $\neg(x \in (A \cup B))$
iff $\neg(x \in A \;\vee\; x \in B)$
iff $\neg(x \in A) \wedge \neg(x \in B)$
iff $x \in A^c \wedge x \in B^c$

Be careful!!

$$(A \setminus B) \setminus C \stackrel{?}{=} A \setminus (B \setminus C)$$





Cartesian Product (= Cross product)

A × B = {(a,b): A∈A, B∈ B}

$\mathbf{R}^2$ points in the plane
An array of chessmen might be represented by BYTES^{64}

Unordered Product

A & B = {{a,b}: $A \in A$, $B \in B$}   // when I learned graph theory -- never saw it since!

Power Set

$\mathcal{P}$ – Power set operator, unary operator (takes 1 input). ***P***(x) is the "set of all subsets of x"

$\mathcal{P}(X) = \{A: A \subseteq X\}$

Example: X = {a, b, c}

Example:

Variant notation: $\mathcal{P}(X) = 2^X$

Notation is suggestive of size –

For X finite, $|\mathcal{P}(X)| = 2^{|X|}$

Dictionary ADT

and its realization with a list and with a hash table

Want to be able to **Insert** items into a dictionary and to **Lookup** if an item is already in the dictionary. (Sometimes want to be able to **Delete** an item, too.) For concreteness, think of the items we are inserting as strings.

Example: discover how many distinct words are in a book.

Implementation

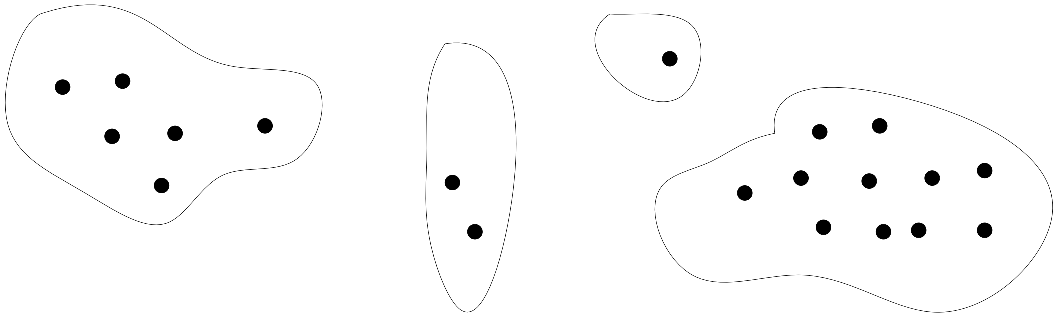
1) A **list** of words, each one appearing at most once.
2) A **hash table**.

Explain how each works.

Show how to modify the hash table to do a frequency count.

Representing a collection of sets in a computers

A different game – we are going to maintain a collection of **disjoint sets**. We want to be able to figure out if two things are in the same set, or in different sets. For example, each point in the set might represent a person and when we learn that person one and person two know one another – maybe one calls or emails the other – then we combine them. Each set then represents people that know one another through **some path** of knowing.



More interesting applications will come later, when we do graph theory.
You want to realize

- **find**(x) return a *canonical name* for the unique set containing *x*.
  x and y are in the same set iff find(x)=find(y)
- **union(x,y) merge the sets containing x and y.**
- **makeset** (x) create a set containing the element x. Return a canonical name for it

Naïve implementation: list of elements

Smarter – "union/find data structure"
Union by rank
Collapsing find.
Any sequence of n operations takes $n\,\alpha(n)$ time, for an incredibly slows growing function $\alpha(n)$. [Omit big-O because not yet introduced]

Tarjan (1975)

```
function MakeSet(x)
     x.parent := x
     x.rank   := 0
 function Union(x, y)
     xRoot := Find(x)
     yRoot := Find(y)
     if xRoot == yRoot
         return

     // x and y are not already in same set. Merge them.
     if xRoot.rank < yRoot.rank
         xRoot.parent := yRoot
     else if xRoot.rank > yRoot.rank
         yRoot.parent := xRoot
     else
         yRoot.parent := xRoot
```

```
        xRoot.rank := xRoot.rank + 1
```

The second improvement, called *path compression*, is a way of flattening the structure of the tree whenever *Find* is used on it. The idea is that each

```
function Find(x)
     if x.parent != x
        x.parent := Find(x.parent)
     return x.parent
```