# Problem Set 6 – Due Wednesday, 4:15 pm, November 13, 2013

1. In a cogent, pedantic paragraph, describe a solution to something you missed on the midterm that you now understand. Write the paragraph as though carefully explaining the idea to someone who was quite confused.

2. Prove or disprove each statement.
(a) If $f\colon A \to A$ is one-to-one, then $f$ is onto.
(b) If $A$ is finite and $f\colon A \to A$ is one-to-one, then $f$ is onto.
(c) If $f\colon A \to A$ is $f$ is onto, then $f$ is one-to-one.
(d) If $A$ is finite and $f\colon A \to A$ is $f$ is onto, then $f$ is one-to-one.

3. Answer if each of the following functions is a bijection onto its range. For any function that is a bijection, identify $f^{-1}(5)$. Justify all of your answers.
(a) $f(n) = 2n \bmod 10$. The domain is $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
(b) $f(n) = 2n \bmod 11$. The domain is $\mathbb{Z}_{11} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
(c) $f(n) = 2^n \bmod 10$. The domain is $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
(d) $f(n) = 2^n \bmod 11$. The domain is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

4. (a) Specify a bijection from $[0, 1]$ to $(0, 1]$. This shows that $\big|\,[0, 1]\,\big| = \big|\,(0, 1]\,\big|$.
(b)The Cantor-Bernstein-Schroeder (CBS) theorem says that if there's an injection from $A$ to $B$ and an injection from $B$ to $A$, then there's a bijection from $A$ to $B$ (ie, $|A| = |B|$). Use this to come to again show that $\big|\,[0, 1]\,\big| = \big|\,(0, 1]\,\big|$.

5. Given sets $A$ and $B$, say that $A \sim B$ (the sets are *equicardinal*) if $|A| = |B|$ (that is, there exists a bijection $f$ from $A$ to $B$.) Show that $\sim$ satisfies the three properties of an equivalence relation.

6. Let *BIG* be an uncountable set and let *Little* be a countable one. Prove that $\mathit{BIG}' = \mathit{BIG} - \mathit{Little}$ (set difference) is uncountable.

7. Show how to encode any pair of real numbers $x, y$ into a real number $z = \text{encode}(x, y)$: from the real number $z$, the real numbers $x$ and $y$ are uniquely determined. What does this say about $|\mathbb{R} \times \mathbb{R}|$?

8. (a) How many permutations (bijections) are there on the set $B = \{0, 1\}^8$ of bytes? (b) Prove that this set forms a *group* under the composition operation: $g \cdot f$ is defined by $(g \cdot f)(x) = g(f(x))$.