Problem Set 4 – Due Tuesday, October 19, at 5pm

1. Complete the following table, answering whether the statement is true (T) or false (F) when the intended universe $\mathcal{U}$ is as indicated (the set of reals or the set of integers).

	$\mathbb{R}$	$\mathbb{Z}$
$\forall x \exists y (2x - y = 0)$		
$\exists y \forall x (2x - y = 0)$		
$\forall x \exists y (x - 2y = 0)$		
$\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$		
$\exists y \exists z (y + z = 100)$		
$\forall x \exists y (y > x \wedge \exists z (y + z = 100))$		

2. Translate the negation of the following statements into formulas of first-order logic, introducing predicates as needed.

   (a) There is someone in the freshman class who doesn't have a roommate.

   (b) Everyone likes someone, but no one likes everyone.

   (c) $(\forall a \in A)(\exists b \in B)(a \in C \leftrightarrow b \in C)$

   (d) $(\forall y > 0)(\exists x)(ax^2 + bx + c = y)$

3. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is not.

   (a) $A \in B \wedge B \in C \implies A \in C$

   (b) $A \subseteq B \wedge B \subseteq C \implies A \subseteq C$

   (c) $A \subsetneqq B \wedge B \subsetneqq C \implies A \subsetneqq C$

   (d) $A \in B \wedge B \subseteq C \implies A \in C$

   (e) $C \in \mathcal{P}(A) \iff C \subseteq A$

   (f) $A = \emptyset \iff \mathcal{P}(A) = \emptyset$

4. Which of the following conditions imply that $B = C$? In each case, either prove or give a counterexample.

   (a) $A \cup B = A \cup C$

   (b) $A \cap B = A \cap C$

   (c) $A \oplus B = A \oplus C$

   (d) $A \times B = A \times C$

5. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is not.

   (a) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

   (b) $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$

   (c) $(A \oplus B) \times C = (A \times C) \oplus (B \times C)$

   (d) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$