Problem Set 7 — Due Wednesday, November 10, at 5pm

1. Express the *twin-prime conjecture* in the minimalistic language, NT, that we used for Peano arithmetic. NT (for *number theory*) is first-order logic, with equality, that supports a constant 0, the unary function S, binary functions $+$, $\cdot$, and E, and a binary operator $<$. (Everything else should be defined from these basic elements.) The *twin-prime conjecture* is the following: *that there are infinitely many pairs of primes* $p,\ p+2$.

2. Recall that a number n is divisible by d, written $d \mid n$, if there exists an integer k such that $kd = n$. Prove that $n^3 + 2n$ is divisible by 3 for every $n \geq 1$.

3. Prove that for any integer $n \geq 1$, if $x_1, \ldots, x_n$ are distinct real numbers, then, no matter how the parentheses are inserted into their product, the number of multiplications used to compute the product is $n - 1$.

4. Considered a lucky number, the Thai government decides to issue coins of 9 baht. Show that, for all sufficiently large numbers N (just how large?), you can dispense N baht using only 9 baht and 10 baht coins.

5. Prove or disprove: for every $N \geq 2$ (not just for powers of 2, like we did in class), there is a punctured $N \times N$ grid (that is, an N by N grid with one cell removed) that can be tiled by triominoes. (A triominoe, you will recall, are three adjacent squares in the shape of an L.)