Problem Set 4 – Due Wednesday, February 2, at 5pm

Based on the polls, I have marked the hardest couple problems with a ∗. And there could be a hint or two lurking. Regardless, if you spend a reasonable amount of time but just can't get something, I suggest you explain where you're at and move on. As discussed in the syllabus, I expect most people won't solve every single problem. It's OK not to.

1. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is not. By $X \subsetneq Y$ we mean that X is a proper subset of Y: $X \subseteq Y$ and $X \neq Y$.

(a) $A \in B \wedge B \in C \implies A \in C$

(b) $A \subseteq B \wedge B \subseteq C \implies A \subseteq C$

(c) $A \subsetneq B \wedge B \subsetneq C \implies A \subsetneq C$

(d) $A \in B \wedge B \subseteq C \implies A \in C$

(e) $C \in \mathcal{P}(A) \iff C \subseteq A$

(f) $A = \emptyset \iff \mathcal{P}(A) = \emptyset$

2. Which of the following conditions imply that $B = C$? In each case, either prove or give a counterexample.

(a) $A \cup B = A \cup C$

(b) $A \cap B = A \cap C$

(c) $A \oplus B = A \oplus C$

(d) $A \times B = A \times C$

3. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is not.

(a) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$

(b) $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$

(c) $(A \oplus B) \times C = (A \times C) \oplus (B \times C)$

(d) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

4. **(a)** List, in lexicographic order, the first six strings, of $\{a, bc\}^* - \{abc\}^*$. (Assume $a < b < c$.) **(b)** Consider the language of all binary strings whose first two characters are the same as the string's last two characters. (Say that a string must have at least two character to satisfy this condition.) Find a way to write this language by combining finite sets (written {list-of-strings}) with union, concatenation, and star (written $\cup, \circ, *$). Of course you can use parentheses to make the order of operations clear. (The convention is star, then concatenation, then union.)

***5.** A stupid math teacher tells Zack to write down every positive integer from 1 to 99999. Numbers are to be written in the usual way, with no leading zeros. In carrying out this tedious exercise, how many times will Zach write the digit `1`? Find an easy way to get to the answer.

***6.** Find a picture-proof (in the spirit of those in Lecture 5) that $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots = \frac{1}{3}$.

Don't want/need any hints?!?! Then **Go away!**

Scram!

Get out of here!!

Hint for 5: Fixed-length strings.

Hint for 6: Divide a square. Again. And again.