Problem Set 7 – Due Wednesday, March 2, at 5pm

1. (a) Given an equal arm balance capable of determining only relative weights of two quantities, and given **8** coins, all of equal weight except possibly one that is lighter, explain how to determine if there is a light coin, and how to identify it, in just **2** weighings.

   (b) Given an equal arm balance as in (a), and given 80 coins, all of equal weight except possibly one that is lighter, show how to determine if there is a light coin and how to identify it with at most **4** weighings.

2. Sort the following functions into groups $G_1, G_2, \ldots$ such that all functions in a group have the same $\Theta(\cdot)$-complexity, and functions grow asymptotically faster as the group index increases.

$5n \lg n$	$6n^2 - 3n + 7$	1.5^n	$\lg n^4$	$10^{10^{10}}$	$\sqrt{n}$
$15n$	$\lg \lg n$	$9n^{0.7}$	$n!$	$n + \lg n$	$\lg^4 n$
$\sqrt{n} + 12n$	$\lg n!$	$\log n$	e^n	2^n	$n\lceil \lg n \rceil$

3. Compute the $\Theta(\cdot)$-running time for the following code fragment. Assume that `S` takes unit time to run.

```
for i = 1 to n do
  for j = 1 to i do
    for  k = 1 to j*j do
       for m = k to k+100 do
          S
```

4. Solve the following recurrence relations to get at $\Theta(g(n))$ result. Assume that all of the recurrence relations are a positive constant for all sufficiently small n. Show all of your work, not making use of any "Master" theorem you might have seen.

   (a) $T(n) = T(n-1) + n^2$.

   (b) $T(n) = 5T(n/5) + n$.

   (c) $T(n) = 2T(n/3) + n$

5. Five misanthropes (all computer science professors) live on a triangular island of the south Pacific. The island's dimensions are 2 miles $\times$ 2 miles $\times$ 2 miles. Show that some two of the misanthropes must live within a mile of one another. (They won't be happy about it.) (English usage: two people who live one mile apart *do* live "within a mile" of one another.)

6. In honor of twosday, calculate $\gcd(22022022, 222222)$. Show your work. Don't factor the numbers.

7. Prove that for any positive number n there is a nonzero multiple of n whose digits, base-10, are all 0s and 1s. *Hint: Pigeons* $1, 11, 111, \ldots$.